Quantum tunneling of magnetization in dipolar spin-1 condensates under external fields

Limin Yang[1] and Yunbo Zhang[1,2,*]

[1]*Institute of Theoretical Physics, Shanxi University, Taiyuan 030006, People's Republic of China*
[2]*Laboratory of Optics and Spectroscopy, Department of Physics, University of Turku, 20014 Turku, Finland*
(Received 9 February 2006; published 5 October 2006)

We study the macroscopic quantum tunneling of magnetization of the $F=1$ spinor condensate interacting through dipole-dipole interaction with an external magnetic field applied along the longitudinal or transverse direction. We show that the ground state energy and the effective magnetic moment of the system exhibit an interesting macroscopic quantum oscillation phenomenon originating from the oscillating dependence of thermodynamic properties of the system on the vacuum angle. Tunneling between two degenerate minima are analyzed by means of an effective potential method and the periodic instanton method.

I. INTRODUCTION

Rapid experimental progresses in realization of spinor condensates [1,2] have generated fascinating opportunities to study the spin dynamics and magnetic properties of condensate atoms. The properties of spinor condensates under external magnetic field are investigated both experimentally [3] and theoretically [4]. The spin-exchange interaction plays an important role in these works, which is reminiscent of the exchange interaction responsible for interesting magnetic properties in solid. Other interactions, due to much weaker than the exchange interaction, are in most cases ignored.

Since the spin degree of freedom becomes accessible in an optical trap, the magnetic dipole-dipole interaction which arises from the intrinsic or field-included magnetic dipole moment [5,6] should be taken into account. The dipolar coupling was first considered between atoms at different sites, based on the assumption that the spin-exchange interaction dominates the on-site interaction [7]. Due to its long range and vectorial characters, more and more attentions are paid to this spinor dipolar condensate, for example, the ground state structure and spin dynamics were examined for this novel quantum system in a single trap [8] and in deep optical lattices [9].

Recently a major experimental breakthrough [10] has been achieved in the condensation of chromium atoms ^{52}Cr which possesses a large magnetic dipole moment of $6\mu_B$ (μ_B is the Bohr magneton) in its ground state. The dipolar interaction in this condensate is a factor of 36 higher than that for alkali metal atoms, which makes possible the study of many dipolar phenomena and new kinds of quantum phase transitions predicted by theory. Indeed, the long-range and anisotropic magnetic dipole-dipole interaction in degenerate quantum gases has been shown to lead to an anisotropic deformation of the expanding Cr-BEC which depends on the orientation of the atomic dipole moments [11].

In the present paper we mainly consider a rich set of macroscopic quantum phenomena occurred in different quantum phases of spinor dipolar condensate. These phases may be tuned via modifying the trapping geometry so that various effective strengths of the dipolar interaction can be achieved. Based on the ground state structure of a spin-1 condensate with dipole-dipole interaction at zero temperature, macroscopic quantum tunneling, and oscillations occur in different phases, which are quite similar to what happens in molecular magnets [12,13].

The paper is organized as follows. In Sec. II, we introduce the dipolar spinor BEC model for $T=0$ under the single mode approximation, taking as a starting point the spin system analogous to the magnetization tunneling in magnetic particles. The ground state structure of the system is summarized in Sec. III for zero, longitudinal and transverse fields, respectively. Sections IV and V are devoted to analyze the macroscopic quantum phenomena in the ground state for different phases under longitudinal or transversal external fields, in which cases tunneling between magnetizations arise naturally. Finally a brief summary is given in Sec. VI.

II. SPINOR CONDENSATE WITH DIPOLAR INTERACTION

Our starting point is the many-body Hamiltonian H proposed in Ref. [7], which describes a $F=1$ spinor condensate at zero temperature trapped in an axially symmetric potential V_{ext}. Without loss of generality, the symmetry axis is conveniently chosen to be the quantization axis $\hat{z}$. We consider here two atomic interaction terms, the short-range collisional interaction and the long-range magnetic dipolar interaction, the competition of which gives rise to different quantum phases. Under an external magnetic field B, the second quantized Hamiltonian of the system reads

$$H = \int d\mathbf{r}\, \hat{\psi}_\alpha^\dagger(\mathbf{r}) \left[\left(-\frac{\hbar^2\nabla^2}{2M} + V_{\text{ext}} \right) \delta_{\alpha\beta} - g\mu_B \mathbf{B} \cdot \mathbf{F}_{\alpha\beta} \right] \hat{\psi}_\beta(\mathbf{r})$$

$$+ \frac{c_0}{2} \int d\mathbf{r}\, \hat{\psi}_\alpha^\dagger(\mathbf{r}) \hat{\psi}_\beta^\dagger(\mathbf{r}) \hat{\psi}_\alpha(\mathbf{r}) \hat{\psi}_\beta(\mathbf{r})$$

$$+ \frac{c_2}{2} \int d\mathbf{r}\, \hat{\psi}_\alpha^\dagger(\mathbf{r}) \hat{\psi}_\delta^\dagger(\mathbf{r}) \mathbf{F}_{\alpha\beta} \cdot \mathbf{F}_{\delta\gamma} \hat{\psi}_\beta(\mathbf{r}) \hat{\psi}_\gamma(\mathbf{r})$$

$$+ \frac{c_d}{2} \int\int \frac{d\mathbf{r}\, d\mathbf{r}'}{|\mathbf{r} - \mathbf{r}'|^3} [\hat{\psi}_\alpha^\dagger(\mathbf{r}) \hat{\psi}_\delta^\dagger(\mathbf{r}') \mathbf{F}_{\alpha\beta} \cdot \mathbf{F}_{\delta\gamma} \hat{\psi}_\beta(\mathbf{r}) \hat{\psi}_\gamma(\mathbf{r}')$$

*Electronic address: ybzhang@sxu.edu.cn

$$-3\hat{\psi}_\alpha^\dagger(\mathbf{r})\hat{\psi}_\delta^\dagger(\mathbf{r}')(\mathbf{F}_{\alpha\beta}\cdot\mathbf{e})(\mathbf{F}_{\delta\gamma}\cdot\mathbf{e})\hat{\psi}_\beta(\mathbf{r})\hat{\psi}_\gamma(\mathbf{r}')], \qquad (1)$$

where $\hat{\psi}_\alpha(\mathbf{r})(\alpha=0,\pm1)$ are the field annihilation operators for an atom in the hyperfine state $|F=1,m_F=\alpha\rangle$. The two coefficients $c_0=4\pi\hbar^2(a_0+2a_2)/3M$ and $c_2=4\pi\hbar^2(a_2-a_0)/3M$ characterize the density-density and spin-spin collisional interactions, respectively. Here a_f ($f=0$ or 2) being the s-wave scattering length for spin-1 atoms in the combined symmetric channel of total spin f. The dipolar interaction parameter is $c_d=\mu_0 g^2\mu_B^2/4\pi$ with g being Landé g factor. The V_{ext} represent the external trapping potential which is spin independent for a far off-resonant optical trap. And $\mathbf{e}=(\mathbf{r}-\mathbf{r}')/|\mathbf{r}-\mathbf{r}'|$ is a unit vector. In this study, the external field $\mathbf{B}$ is assumed to be spatially uniform.

In order to simplify the Hamiltonian (1), we usually adopt the single mode approximation (SMA): $\hat{\psi}_\alpha(\mathbf{r})\simeq\phi(\mathbf{r})\hat{a}_\alpha$ [14], where $\phi(\mathbf{r})$ is the spin independent spatial wave function of the condensate, $\hat{a}_\alpha$ is the annihilation operator for $m_F=\alpha$ component. It is always safe to use this approximation for ferromagnetic interactions, however, it may break down for antiferromagnetic interactions if both the atomic number N and the magnetization $\mathcal{M}$ are large. The interaction parameters in our case of study are such that $|c_2|\ll c_0$ and $c_d\lesssim0.1|c_2|$. Under these conditions, the single mode approximation is expected to be valid if the trapping potential is axially symmetric. All integral terms in the long range interactions involving $e^{\pm i\varphi_e}$ vanish after integrating over the azimuthal angles of $\mathbf{r}-\mathbf{r}'$. We then obtain a much simpler Hamiltonian by assuming that the mode function $\phi(\mathbf{r})$ hence possesses the axial symmetry and dropping the spin-independent constant terms [7]

$$H=(c_2'-c_d')\hat{\mathbf{L}}^2+3c_d'(\hat{L}_z^2+\hat{n}_0)-g\mu_B\mathbf{B}\cdot\hat{\mathbf{L}}, \qquad (2)$$

where $\hat{\mathbf{L}}=\hat{a}_\alpha^\dagger\mathbf{F}_{\alpha\beta}\hat{a}_\beta$ characterizes the total many-body angular momentum operator, $\hat{n}_0=\hat{a}_0^\dagger\hat{a}_0$ is the number operator for $m_F=0$ atoms. The new parameters are $c_2'=(c_2/2)\int d\mathbf{r}|\phi(\mathbf{r})|^4$ and $c_d'=(c_d/4)\int d\mathbf{r}d\mathbf{r}'|\phi(\mathbf{r})|^2|\phi(\mathbf{r}')|^2(1-3\cos^2\theta_e)/|\mathbf{r}-\mathbf{r}'|^3$, with θ_e being the polar angle of $(\mathbf{r}-\mathbf{r}')$. Equation (2) may be put into the following dimensionless form by rescaling it in energy unit $|c_2'|$:

$$H=(\pm1-c)\hat{\mathbf{L}}^2+3c(\hat{L}_z^2+\hat{n}_0)-\mathbf{B}'\cdot\hat{\mathbf{L}}, \qquad (3)$$

where $+$ and $-$ correspond to $c_2>0$ and $c_2<0$, respectively. The new parameter $c=c_d'/|c_2'|$ thus measures the relative strength of dipolar interaction with respect to the spin-exchange interaction. The dimensionless magnetic field $\mathbf{B}'=g\mu_B\mathbf{B}/|c_2'|=B'(\sin\theta,0,\cos\theta)$ is assumed to lie in the xz plane with an angle θ relative to the z axis.

III. GROUND STATE STRUCTURE UNDER EXTERNAL FIELD

We summarize the ground state structure which has been described in Ref. [7]. In the absence of an external field, the ground state of our system is divided into three distinct regions A, B, and C in the $c_2'-c_d'$ parameter plane (see Fig. 1). We denote here the simultaneous eigenstate of $\hat{\mathbf{L}}^2$ and $\hat{L}_z$ as
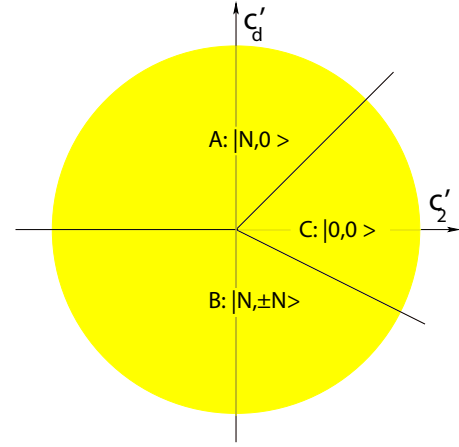

FIG. 1. (Color online) Magnetic phase diagram of dipolar spinor condensate parametrized in the $c_2'-c_d'$ plane. Corresponding ground states are shown for zero external field. The two tunneling models studied in this paper are in phase A with a longitudinal field and phase B with a transverse field.

$|l,m\rangle$ with eigenvalues $l(l+1)\hbar^2$ and $m\hbar$, respectively. In region A ($c_d'>0$ and $c_d'>c_2'$) the ground state is given by $G=|N,0\rangle$, a quantum superposition of a chain of Fock states $|N/2-k,2k,N/2-k\rangle$ in which the numbers of atoms in the spins 1 and -1 are equal. In region B ($c_d'<0$ and $c_d'<-c_2'/2$), $G=|N,\pm N\rangle$ is a Fock state with all the population in either $m_F=1$ or -1. In these two regions, the $\hat{n}_0$ term is at least a factor of $1/N$ smaller than the rest and therefore can be neglected safely. In region C, however, the ground state is a little more complicated because the $\hat{n}_0$ term is expected to be important. In general it is expressed as $G=\Sigma_l g_l|l,0\rangle$, a superposition of different angular momentum states with $\langle\hat{L}_z\rangle=0$.

When we apply a longitudinal field to the system along the z axis ($\theta=0$), the condensate in different parameter regions behave quite differently. In region A, the system can be mapped onto an easy-plane anisotropic particle with the transverse xy plane being the easy plane. The ground state is $G=|N,m\rangle$ with m depends on the field strength linearly, with steps. Region B corresponds to a uniaxial anisotropic magnetic spin model with the easy axis z. The presence of the external magnetic field simply removes the twofold degeneracy and forces the atoms into the fully polarized state $G=|N,N\rangle$. The ground state in region C is changed into $G=\Sigma_{l\geq m_0}g_l|l,m_0\rangle$ while m_0 increases with the field strength.

Now we check the situation when a transverse field is applied along the x axis, i.e., $\theta=\pi/2$. Due to the easy plane anisotropy in region A, the ground state of the condensate is fully polarized, however, in this case along the x axis, $G=|N,0\rangle_{m_x=N}$. The situation in region B is more interesting because the ground state here $G=\Sigma_m g_m|N,m\rangle$ is twofold degenerate and it provides another example that can exhibit macroscopic quantum tunneling of magnetization. Stepwise magnetization curve will appear in region C—each step means the breaking of one spin singlet pair.

For clarity, we choose two of above models which admit extensive study of tunneling of magnetization, i.e., region A

with a longitudinal field and region B with a transverse field. The dipolar spinor condensate thus provides another platform for the investigation of macroscopic quantum phenomena.

IV. MACROSCOPIC QUANTUM OSCILLATION OF MAGNETIZATION

We first consider the spinor dipolar condensate under longitudinal field along the z axis in region A, in which case the transverse xy plane corresponds to an easy-plane anisotropy. After dropping the unimportant $\hat{n}_0$ and constant $\hat{L}^2$ terms, the effective Hamiltonian can be obtained

$$H_{LA} = 3c\hat{L}_z^2 - B'\hat{L}_z, \qquad (4)$$

where $c > 0$. The model is precisely the same as that for a ferromagnetic particle [12,13] with easy-plane anisotropy and a magnetic field along hard axis. The Hamiltonian is exactly diagonal in terms of the eigenstate $\hat{L}_z$ and with the eigenvalue $E_m = 3 \text{ cm}^2 - B'm$. For zero temperature the magnetization increases stepwise as a consequence of the fact that m take only integer values. This model, on the other hand, provides a perfect manifestation of the Θ vacuum effect originating from the oscillating dependence of thermodynamic properties of the system on the "vacuum angle" [15]. The concept of the Θ vacuum was developed mainly for the models of modern quantum field theory [16], but also in condensed media the nonperturbative vacuum is not a mathematical abstraction. The vacuum angle is nothing but the factor with which the total time derivative enters the Lagrangian. For Aharonov-Bohm problem in conductors with charge density waves, Θ is the normalized magnetic flux [17]. In the Josephson junction of mesoscopic sizes, the vacuum angle depends on the voltage applied to the junction [18]. We see shortly the magnetic field enters the Lagrangian and plays the role of vacuum angle in our spinor dipolar condensate system.

We express the partition function as a spin coherent state path integral for large number of atoms $N \gg 1$

$$Z = \text{Tr} \exp(-\beta H_{LA}) = \int D\{\mu(\mathbf{n})\}\exp(-S_E), \qquad (5)$$

where the measure of the integration is decomposed into $D\{\mu(\mathbf{n})\} = \Pi_{k=1}^{N-1}(N \sin \theta_k d\phi_k d\theta_k / 2\pi)$. The semiclassical approximation of the partition function turns out to be the transition amplitude between two spin configurations connected by periodic orbits with fixed imaginary time period β. Following the usual procedure [19], we represent the state vector of the system as coherent states and slice the integral into $\mathcal{N}$ identical pieces of length $\epsilon = \beta / \mathcal{N}$. Inserting complete sets of states gives

$$Z = \langle \mathbf{n}_F | \exp(-\beta H_{LA}) | \mathbf{n}_I \rangle$$
$$= \prod_{k=1}^{\mathcal{N}-1} \int D\{\mu_k(\mathbf{n})\} \prod_{k=0}^{\mathcal{N}-1} \langle \mathbf{n}_{k+1} | (1 - \epsilon H_{LA}) | \mathbf{n}_k \rangle. \qquad (6)$$

After some algebraic evaluation and finally passing to the

time continuum limit $\mathcal{N} \to \infty$ we obtain the Euclidean action in imaginary time $\tau = it$ as (overdots now denote τ derivatives)

$$S_E = \int_0^{\beta\hbar} d\tau [iN\dot{\phi}(1 - \cos \theta) + 3cN(N+1)\cos^2 \theta - B'N \cos \theta] \qquad (7)$$

with $\beta = 1/k_B T$ and T the temperature. Integrating over $\cos \theta$ we map the magnetic system onto a particle problem with Lagrangian

$$\mathcal{L} = \frac{m_{\text{eff}}\dot{\phi}^2}{2} + i\Theta\dot{\phi}, \qquad (8)$$

where the effective mass $m_{\text{eff}} = 1/6c$ and $\Theta = N(1 - B'/6cN)$. This Lagrangian is exactly the one for Θ vacuum of the non-Abelian gauge field in Refs. [16,18]. We noticed that the second term of $\mathcal{L}$ is the total imaginary time derivative and has no effect on the classical equation of motion, while it indeed alerts the canonical momentum into $\Pi_\phi = m_{\text{eff}}\dot{\phi} + i\Theta$. In order to minimize the Euclidean action S_E, we try to find the classical configuration, that is, the periodic instanton solution under the boundary condition $\phi_n(\tau + \beta) = \phi_n + 2\pi n$. The result is $\phi_n = 2\pi n\tau / \beta$ and the Euclidean action for this solution is $S_E = s_0 n^2 + i2n\pi\Theta$, where n is the winding number characterizing homotopically nonequivalent classes and $s_0 = \pi^2 k_B T / 3c$. The Euclidean functional integral of the partition function contains thus an additional summation over the homotopic number and detailed calculation leads to

$$Z = \sum_{n=-\infty}^{\infty} Z_n = \vartheta_3(\Theta, \exp(-s_0)), \qquad (9)$$

where $\vartheta_3(v, q)$ is the Jacobi theta function oscillating with v. By means of the well known asymptotics of the Jacobi theta function, the ground state energy can be shown clearly oscillating with Θ, i.e., the external magnetic field B

$$E_0 = -k_B T \ln Z = -\frac{(B')^2}{12c} + \frac{1}{2m_{\text{eff}}}\{\{\Theta\}\}^2, \qquad (10)$$

where $\{\{x\}\}$ is the difference between x and its nearest integer. According to this, the external magnetic field induces quantum oscillations in the dipolar spinor condensate. The period of oscillation is shown to be $\delta B = 6c'_d / g\mu_B$ (see Fig. 2).

The topological term in the Lagrangian (8) leads to the oscillation behavior of our system. We emphasize here the ground state (or in the language of quantum field theory, the instanton vacuum) of the condensate acquires a vacuum angle owing to the presence of the external magnetic field, which breaks the symmetry and changes the topology of the system. In a charge density wave ring-shaped conductor placed in an external vector potential field the magnetic susceptibility and the electrical conductivity oscillate as a function of the flux [17], while the voltage applied on the Josephson junction induces the oscillation of the effective capacitance of the junction [18]. In our system of investigation the macroscopic quantum effect manifests itself by
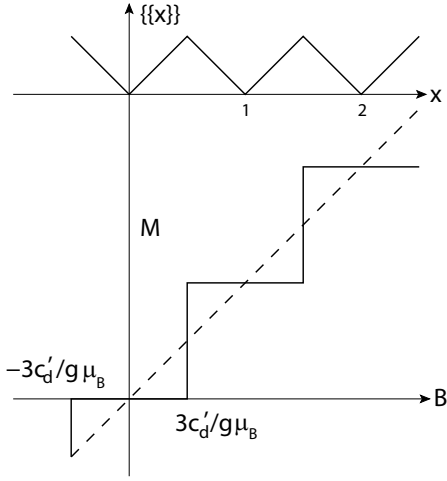
FIG. 2. The mathematical function $\{\{x\}\}$ and the oscillation of the magnetization with period δB.

the oscillation of the effective magnetic momentum $M = -k_B T \partial \ln Z / \partial B$ as a function of the applied magnetic field. The magnetization curve at zero temperature is stepwise due to the macroscopic quantum oscillation of the effective magnetic moment and the period of oscillation depends on the strength of the dipolar interaction c_d' uniquely. We show this oscillation period in Fig. 2, together with the mathematical function $\{\{x\}\}$. The functional integral approach presented here becomes necessary when environmental friction or dissipation is included, which amounts to the introduction into the Euclidean Lagrangian of an additional term describing the nonlocal interaction of the instantons [20]. This is, however, beyond the scope of this paper. It is interesting to estimate the modulation period in the typical laboratory experiment on spinor condensates. For a condensate with $N \sim 5 \times 10^5$ sodium atoms ^{23}Na and density $\rho \sim 10^{14}$ cm^{-3} [23] we obtain the regular interval of magnetic field $\delta B \simeq 1.4$ G. This oscillation period can be as small as 1.5×10^{-3} G for 2×10^4 rubidium atoms ^{87}Rb with density 2.6×10^{12} cm^{-3} as in the first experimental achievement of BEC in JILA [24] and can be as large as 10.5 G for 5×10^4 chromium atoms ^{52}Cr with density 10^{14} cm^{-3} as in the latest dipolar condensate experiments in Stuttgart [10,11]. Moreover, it can be adjusted flexibly by changing the trapping potential geometry.

V. QUANTUM TUNNELING OF MAGNETIZATION BETWEEN TWO LOCAL MINIMA

Now we prepare the system properly in Region B and apply a transverse field along the x axis to the condensate. Again we drop the constant terms and are left with the following effective Hamiltonian:

$$H_{TB} = -3d\hat{L}_z^2 - B'\hat{L}_x, \qquad (11)$$

where $d = |c|$. The model describes a quantum spin system with the easy-axis anisotropy while the external field is along x axis. This model has been extensively studied in the context of spin tunneling. Classically under the influence of a weak transverse field along the x direction, the two energy minima move away from the zero filed positions ($+z$ or $-z$) and towards the x axis while remaining in the xz plane. For $0 \le B' \le 6dN$, they are located at $\gamma_- = \arcsin(B'/6dN)$ and $\gamma_+ = \pi - \theta_-$, respectively, with the angle γ between $\hat{\mathbf{L}}$ and z axis. The degeneracy is removed when $B' \ge B'_{sat} = 6dN$, where the system is completely polarized by the external field and the two minima merge along the x axis. Quantum mechanically, the degeneracy is lifted before the magnetic field reaches B'_{sat} due to the magnetization tunneling. A well-known consequence of the tunneling between two degenerate states is the lifting of their degeneracy: The two new eigenstates are a symmetric and an antisymmetric superposition of the original states characterized by an energy difference (or tunneling splitting) ΔE_0 inversely proportional to the tunneling rate. The quantity of interest to determine the occurrence of tunneling is therefore this energy difference between the two lowest eigenstates of the Hamiltonian.

To calculate analytically this energy splitting, we use the effective potential method [21] which maps the spin system onto a particle system and the result can be easily obtained with the periodic instanton method. The Schrödinger equation $H_{TB}|\Psi\rangle = E|\Psi\rangle$ in the $\hat{L}_z$ representation takes the form

$$2(E + 3dm^2)C_m + B'\sqrt{(N-m)(N+m+1)}C_{m+1}$$
$$+ B'\sqrt{(N+m)(N-m+1)}C_{m-1} = 0, \qquad (12)$$

where $m = -N, -N+1, \ldots, N$ and $C_m = 0$ for $|m| > N$.

Let us introduce the generating function

$$\Phi = \sum_{m=-N}^{N} \frac{C_m}{\sqrt{(N-m)!(N+m)!}} \exp(mx). \qquad (13)$$

Multiplying Eq. (12) by a factor $\exp(mx)/\sqrt{(N-m)!(N+m)!}$ and summing all terms with $-N \le m \le N$, we can transform Eq. (12) into

$$3d\Phi'' - B' \sinh x \Phi' + (E + B'N \cosh x)\Phi = 0. \qquad (14)$$

In order to remove the first derivative term, let us define a new function $\Psi = \Phi \exp(-\frac{1}{2}B' \cosh x)$. When we replace Φ with Ψ, the new Schrödinger equation can be written after dividing by N^2 in the form

$$N^{-2}\Psi'' + \Psi(\kappa - U) = 0, \qquad (15)$$

where the corresponding parameter κ describe the dimensionless energy $\kappa = E/3dN^2$, and $U = (a \cosh x - 1)^2 - a^2$ is the effective potential well with $a = B'/6dN$.

The value N^{-1} plays the role of the Planck constant $\hbar$, the potential takes the form of a double well for $a < 1$, i.e., for external magnetic field not exceeding the saturation value $B'_{sat} = 6dN$. The two local minima thus play the role of the degenerate classical states and the energy splitting of the lower states takes the following form according to Ref. [22]:

$$\Delta E_n = \Delta E_0 q^n / n! \qquad (16)$$

with the splitting for the ground state

$$\Delta E_0 = \sqrt{\hbar\omega/\pi}\, C \exp(-S/\hbar). \tag{17}$$

Here S_E is the Euclidean action evaluated along the trajectory from the left minimum $x_- = -\cosh^{-1}(1/a)$ to the right $x_+ = \cosh^{-1}(1/a)$, $\omega = \sqrt{1-a^2}$ is the small oscillator frequency near the bottom of potential well $x_\pm$. The asymptotic form of the instanton trajectory determined the constant C and $q = C^2/2\hbar\omega$. For the potential U we have

$$S_E = \ln\left(\frac{1+\sqrt{1-a^2}}{a}\right) - \sqrt{1-a^2}. \tag{18}$$

Including the prefactor we have finally

$$\Delta E_0 = \frac{24d}{\sqrt{\pi}} N^{3/2} \frac{(1-a^2)^{5/4}a^{2N}}{(1+\sqrt{1-a^2})^{2N}} \exp[(2S_E N + 1)\sqrt{1-a^2}]. \tag{19}$$

Experimentally this level splitting can be measured by means of the resonance measurement developed by Awschalom [25]. Tunneling between two degenerate orientations of magnetization leads to the splitting of the "nontunneling" ground state energy level into two levels separated by ΔE_0. Correspondingly a very weak ac field of frequency $\Delta E_0/\hbar$ will induce transitions between the two levels, which should result in the resonant absorption of the energy of the field. The atoms in the condensate are utterly identical and we do not have the problem of distribution of particle sizes and shapes. The level splitting (19) is expressed in units of c_2'. It is easily shown that for the sodium condensate in Ref. [23] the dipole-dipole interaction, i.e., the anisotropic energy in our model, is estimated as $c_d' = 1.69 \times 10^5$ Hz or 11.5 μK, a quantity much smaller than the anisotropy energy of molecular magnets $Mn_{12}Ac$ or Fe_8 [26]. The level splitting can be greatly enhanced by a smaller number of atoms in the condensate and also by a stronger dipolar-dipolar interaction.

Taking, as an example, sodium atoms ($N=38$) under an external field of $a=0.6$, we have $\Delta E_0' = 1.14 \times 10^2$ Hz. For the condensate of ^{52}Cr, the anisotropic energy is enhanced to $c_d' = 2.44 \times 10^6$ Hz or 166 μK. For $N=39$, we have $\Delta E_0' = 3.15 \times 10^2$ Hz. The level splitting remains in the same magnitude order because it depends very sensitively on the total number of atoms. These data are easily accessible in the present ultracold atom experiments.

VI. SUMMARY

Inspired by the macroscopic quantum tunneling in the magnetic system, we have investigated the macroscopic quantum tunneling in the dipolar spinor condensates at zero temperature and obtained some interesting results by analyzing different phase areas with applied external fields. We found that the ground state energy and the effective magnetic moment oscillate with the external magnetic field in region A under a longitudinal field and the oscillating period depends on the strength of the dipolar interaction as $\delta B = 6c_d'/g\mu_B$. This model provides a condensed media realization of the Θ vacuum in quantum field theory. The model in region B with a transverse field provides an example where quantum tunneling of magnetization occurs between two local minima. We estimated the level splitting to be at the reach of current ultracold atom experiments.

[1] T.-L. Ho, Phys. Rev. Lett. **81**, 742 (1998); T. Ohmi and K. Machida, J. Phys. Soc. Jpn. **67**, 1822 (1998).

[2] D. M. Stamper-Kurn, M. R. Andrews, A. P. Chikkatur, S. Inouye, H.-J. Miesner, J. Stenger, and W. Ketterle, Phys. Rev. Lett. **80**, 2027 (1998); J. Stenger, S. Inouye, D. M. Stamper-Kurn, H.-J. Miesner, A. P. Chikkatur, and W. Ketterle, Nature (London) **396**, 345 (1998).

[3] H. Schmaljohann, M. Erhard, J. Kronjäger, M. Kottke, S. van Staa, L. Cacciapuoti, J. J. Arlt, K. Bongs, and K. Sengstock, Phys. Rev. Lett. **92**, 040402 (2003); M.-S. Chang, C. D. Hamley, M. D. Barrett, J. A. Sauer, K. M. Fortier, W. Zhang, L. You, and M. S. Chapman, *ibid.* **92**, 140403 (2004); T. Kuwamoto, K. Araki, T. Eno, and T. Hirano, Phys. Rev. A **69**, 063604 (2004); J. M. Higbie, L. E. Sadler, S. Inouye, A. P. Chikkatur, S. R. Leslie, K. L. Moore, V. Savalli, and D. M. Stamper-Kurn, Phys. Rev. Lett. **95**, 050401 (2005); M.-S. Chang, Q. Qin, W. Zhang, L. You, and M. S. Chapman, Nat. Phys. **1**, 111 (2005).

[4] H. Pu, S. Raghavan, and N. P. Bigelow, Phys. Rev. A **61**, 023602 (2000); T.-L. Ho and S. K. Yip, Phys. Rev. Lett. **84**, 4031 (2000); C. K. Law, H. Pu, and N. P. Bigelow, *ibid.* **81**, 5257 (1998); M. Koashi and M. Ueda, *ibid.* **84**, 1066 (2000).

[5] S. Yi and L. You, Phys. Rev. A **61**, 041604 (2000); S. Yi and L. You, *ibid.* **63**, 053607 (2001).

[6] K. Góral, K. Rzazewski, and T. Pfau, Phys. Rev. A **61**, 051601 (2000).

[7] S. Yi, L. You, and H. Pu, Phys. Rev. Lett. **93**, 040403 (2004); S. Yi and H. Pu, Phys. Rev. A **73**, 023602 (2006); S. Yi and H. Pu, Phys. Rev. Lett. **97**, 020401 (2006).

[8] R. Cheng, J.-Q. Liang, and Y. Zhang, J. Phys. B **38**, 2569 (2005).

[9] H. Pu, W. Zhang, and P. Meystre, Phys. Rev. Lett. **87**, 140405 (2001); W. Zhang, H. Pu, C. Search, and P. Meystre, *ibid.* **88**, 060401 (2002); K. Gross, C. P. Search, H. Pu, W. Zhang, and P. Meystre, Phys. Rev. A **66**, 033603 (2002).

[10] A. Griesmaier, J. Werner, S. Hensler, J. Stuhler, and T. Pfau, Phys. Rev. Lett. **94**, 160401 (2005).

[11] J. Stuhler, A. Griesmaier, T. Koch, M. Fattori, T. Pfau, S.

Giovanazzi, P. Pedri, and L. Santos, Phys. Rev. Lett. **95**, 150406 (2005).

[12] A. Garg, Phys. Rev. B **51**, 15161 (1993).

[13] S. P. Kou, J. Q. Liang, Y. B. Zhang, and F. C. Pu, Phys. Rev. B **59**, 11792 (1999).

[14] S. Yi, Ö. E. Müstecaphoğlu, C. P. Sun, and L. You, Phys. Rev. A **66**, 011601 (2002).

[15] I. V. Krive and A. S. Rozhavsky, Theor. Math. Phys. **89**, 1069 (1991).

[16] R. Rajaraman, *Solitons and Instantons* (North-Holland, Amsterdam, 1982).

[17] E. N. Bogachek, I. V. Krive, I. O. Kulik, and A. S. Rozhavsky, Phys. Rev. B **42**, 7614 (1990).

[18] I. V. Krive and A. S. Rozhavsky, Int. J. Mod. Phys. B **6**, 1255 (1992).

[19] A. Pereromov, *Generalized Cohernet States and Their*

[20] A. O. Caldeira and A. Leggett, Phys. Rev. Lett. **46**, 211 (1981); Ann. Phys. (N.Y.) **149**, 374 (1983).

[21] O. B. Zaslavskii, Phys. Lett. A **145**, 471 (1990).

[22] U. Weiss and W. Haeffner, Phys. Rev. D **27**, 2916 (1983).

[23] K. B. Davis, M.-O. Mewes, M. R. Andrews, N. J. van Druten, D. S. Durfee, D. M. Kurn, and W. Ketterle, Phys. Rev. Lett. **75**, 3969 (1995).

[24] M. H. Anderson, J. R. Ensher, M. R. Matthews, C. E. Wieman, and E. A. Cornell, Science **269**, 198 (1995).

[25] D. D. Awschalom, M. A. McCord, and G. Grinstein, Phys. Rev. Lett. **65**, 783 (1990).

[26] H. Hennion, L. Pardi, I. Mirebeau, and E. Suard, Phys. Rev. B **56**, 8819 (1997); C. Sangregorio, T. Ohm, C. Paulsen, R. Sessoli, and D. Gatteschi, Phys. Rev. Lett. **78**, 4645 (1997).

Applications (Springer, Berlin, 1986).