Three-particle nuclear reactions in the five-body bimuonic systems

Alexei M. Frolov
Department of Applied Mathematics, University of Western Ontario, London, Ontario, Canada N6A 5B7

(Received 25 August 2006; published 27 October 2006)

The energies and some other bound state properties of the ten five-body bimuonic ions $pdt\mu_2$, $ppt\mu_2$, $pdd\mu_2$, etc. are determined numerically with the use of our variational procedure. The bound state structure and stability of these Coulomb five-body systems are also discussed. By using the computed expectation values of the binary and three-particle δ functions we make a few predictions about possible experimental observations of the three-particle nuclear reactions in some of these five-body bimuonic systems.

DOI: 10.1103/PhysRevA.74.042515 PACS number(s): 36.10.Dr, 36.10.−k

In this paper we present the results of variational calculations of a number of five-body bimuonic systems or ions such as $pdt\mu_2(=p^+d^+t^+\mu_2^-)$, $ppt\mu_2(=p^+p^+t^+\mu_2^-)$, etc. These systems are of great interest, since in some of them one can observe the direct three-particle nuclear reactions between three hydrogenic nuclei. On the other hand, it is very interesting to discuss the bound state properties of actual five-body systems which, in general, differ significantly from the analogous properties of well known four-electron beryllium-like atoms and ions, i.e., one-center five-body systems. The main difference between, e.g., the $pdt\mu_2$ ion and beryllium atom can be found in their geometrical structure and single-particle kinematical properties.

Our main computational goal in this study is to determine the bound state solution of the five-body Schrödinger equation $H\Psi=E\Psi$, where H is the Hamiltonian of the few-body system, $E<0$ is its eigenvalue, and the unknown wave function Ψ has the finite and/or unit norm. The nonrelativistic Hamiltonian of the Coulomb five-body system takes the form

$$H=-\frac{\hbar^2}{2m_1}\nabla_1^2-\frac{\hbar^2}{2m_2}\nabla_2^2-\frac{\hbar^2}{2m_3}\nabla_3^2-\frac{\hbar^2}{2m_4}\nabla_4^2-\frac{\hbar^2}{2m_5}\nabla_5^2+\frac{q_1q_2e^2}{r_{12}}+\frac{q_1q_3e^2}{r_{13}}+\frac{q_1q_4e^2}{r_{14}}+\frac{q_1q_5e^2}{r_{15}}+\frac{q_2q_3e^2}{r_{23}}+\frac{q_2q_4e^2}{r_{24}}+\frac{q_2q_5e^2}{r_{25}}+\frac{q_3q_4e^2}{r_{34}}+\frac{q_3q_5e^2}{r_{35}}+\frac{q_4q_5e^2}{r_{45}}, \quad (1)$$

where m_i $(i=1,2,3,4,5)$ are the particle masses, while q_i are their (electric) charges expressed in the electron charge e. By choosing the system of units in which $\hbar=1$, $|e|=1$, and $m=\min(m_1,\ldots,m_5)=1$, we can simplify Eq. (1) and all formulas below. In fact, for all systems considered in this study $m=m_\mu$ and, therefore, our present system of units coincides with the muon-atomic units $\hbar=1$, $|e|=1$, $m_\mu=1$. Also, in Eq. (1) $r_{ij}=|\mathbf{r}_i-\mathbf{r}_j|=r_{ji}$ are the ten relative interparticle coordinates. Note that in this study we shall always assume that particles 4 and 5 designate the two negatively charged muons μ^-, while the notations 1, 2, and 3 stand for the three hydrogenic nuclei, i.e., p, d and/or t, where p is the protium nucleus, d designates the deuterium nucleus, and t stands for the tritium nucleus. Also, by using the $abc\mu_2$ notation we shall always assume that $m_a\leq m_b\leq m_c$, i.e., the $abc\mu_2$ notation starts from the lightest nucleus.

In this study, the five-body wave function Ψ of the ground $S(L=0)$ state is approximated by the variational expansion written in the five-body gaussoids of ten relative coordinates [1]

$$\Psi_{L=0}=B_s^{(\mu)}B_s^{(N)}\sum_{i=1}^{N}C_i\,exp(-\alpha_{12}^ir_{12}^2-\alpha_{13}^ir_{13}^2-\alpha_{14}^ir_{14}^2-\alpha_{15}^ir_{15}^2-\alpha_{23}^ir_{23}^2-\alpha_{24}^ir_{24}^2-\alpha_{25}^ir_{25}^2-\alpha_{34}^ir_{34}^2-\alpha_{35}^ir_{35}^2-\alpha_{45}^ir_{45}^2), \quad (2)$$

where C_i are the linear (or variational) parameters, while α_{kl}^i are the corresponding nonlinear parameters. Also, in this equation $B_s=B_s^{(\mu)}B_s^{(N)}$ is the projector operator which produces the trial wave function of the correct permutation symmetry. In fact, the bimuonic part of this projector always takes the "singlet" form $B_s^{(\mu)}=\frac{1}{\sqrt{2}}(1+\hat{P}_{45})$. The nuclear part of the total projector B_s [i.e., $B_s^{(N)}(1,2,3)$] is the corresponding projector for the three hydrogenic nuclei. The explicit form of this projector depends upon the system considered. The computation of matrix elements in the basis of five-body gaussoids is a relatively simple procedure which has been proposed 25 years ago [1] and extensively described in the literature. Here we do not want to repeat the explicit formulas for all matrix elements needed in computations.

The particle masses used in this study have been chosen from [2]: $m_p=1836.152\,672\,61m_e$, $m_d=3670.482\,965\,2m_e$, $m_t=5496.921\,58m_e$, and $m_\mu=206.768\,264m_e$. With this muonic mass we have 1 ma.u.$=206.768\,264$ a.u. $\approx 5626.453\,132\,6$ eV. Note that each of the systems considered in this study contains three hydrogenic nuclei p,d,t and two negatively charged muons μ^-. Therefore, there are ten such ions total $(abc\mu_2)$ with different combinations of the hydrogenic nuclei p,d, and/or t. The total variational energies of these ten ions are presented in Table I. The bound state properties for two of these ions (the $pdt\mu_2$ and $ppt\mu_2$ ions) can be found in Tables II and III. All energies and other bound state properties are given in these three tables in muon-atomic units.

Now, consider the stability of the five-body bimuonic systems $abc\mu_2$. In contrast with the analogous three- and four-body muonic systems this question is not trivial. From the general point of view it is clear that the lowest-by-energy dissociation channel must be one of the two-particle

TABLE I. The energies E of the ground $S(L=0)$ states of the five- and four-body bimuonic systems (in muon-atomic units $m_\mu=1$, $\hbar=1$, $e=1$, where 1 ma.u.=206.768 264 a.u.≈5626.453 1326 eV). The total energies of four-body systems coincide with the stability thresholds for the corresponding five-body bimuonic systems. Also, the total energy of the ground state in the $tt\mu_2$ system is −1.066 1545 ma.u..

	$ppd\mu\mu$	$ppt\mu\mu$	$ppp\mu\mu$	$pdd\mu\mu$	$ddt\mu\mu$
E	−1.0841437	−1.0956015	−1.0563781	−1.1128005	−1.1531423
	$ptt\mu\mu$	$dtt\mu\mu$	$ttt\mu\mu$	$pdt\mu\mu$	$ddd\mu\mu$
E	−1.1362505	−1.1651523	−1.1777431	−1.1240695	−1.1412544
	$pd\mu\mu$	$pt\mu\mu$	$dt\mu\mu$	$pp\mu\mu$	$dd\mu\mu$
E	−0.9995703	−1.0124221	−1.0509535	−0.9654742	−1.0366033

(=binary) channels in which one of the newly formed particles is a neutral particle and/or cluster. For instance, for the $pdt\mu_2$ ion there are six similar dissociation channels. The three following binary dissociation channels for the $pdt\mu_2$ ion include the formation of one three-body ion and one two-body neutral system (=muonic atom)

$$pdt\mu_2 = pd\mu + t\mu, \quad pdt\mu_2 = pt\mu + d\mu, \quad pdt\mu_2 = dt\mu + p\mu. \tag{3}$$

By using the known variational energies for the three-body ions $pd\mu$, $pt\mu$, and $dt\mu$ systems [5] and the total energies of the three muonic atoms one finds that the lowest-by-energy channel for the $pdt\mu_2$ system is the first one, i.e., the system $pdt\mu_2$ must be stable against dissociation into the two fragments $pd\mu+t\mu$, i.e., one (lightest) muonic ion ($pd\mu$)+one (heaviest) neutral muonic atom ($t\mu$). The total energy (=threshold energy) of this channel is $E_{tot}\approx$ −0.994 585 9 ma.u.

Three other two-particle channels include formation of the neutral four-body system and emission of one positively charged hydrogenic nucleus, i.e.,

$$pdt\mu_2 = pd\mu_2 + t, \quad pdt\mu_2 = pt\mu_2 + d, \quad pdt\mu_2 = dt\mu_2 + p. \tag{4}$$

It is clear that the last channel has the lowest energy possible. Indeed, the total energy of the $dt\mu_2$ quasimolecule is ≈−1.050 953 5 ma.u. (see the last line in Table I), i.e., it is much lower than the total energy in the $pd\mu+t\mu$ channel mentioned above. This means that an arbitrary five-body bimuonic system $abc\mu_2$ must be stable against dissociation into the heaviest four-body bimuonic system $bc\mu_2$ and lightest hydrogen nucleus a^+. The energies of all ten possible five-body bimuonic ions $abc\mu_2$ and six four-body bimuonic "molecules" $ab\mu_2$ can be found in Table I. By using the total energies of the four-body systems $ab\mu_2$ one can evaluate the binding energies for any of the five-body ions $abc\mu_2$ from Table I. This discussion also allows us to predict the approximate structure of an arbitrary five-body bimuonic system. For instance, the structure of the $dtp\mu_2$ system can be repre-

TABLE II. The expectation values in muon-atomic units of basic properties for the ground $S(L=0)$ state of the $ppt\mu\mu(=ppt\mu_2)$ ion. The notations 1, 2, and 3 stand for the hydogenic p, p and t nuclei, respectively. The notations 4 and 5 designate the two negatively charged muons.

$\langle r_{12}\rangle$	2.94712	$\langle r_{12}^{-1}\rangle$	0.395711	$\langle r_{12}^{-2}\rangle$	0.19111
$\langle r_{13}\rangle$	2.54691	$\langle r_{13}^{-1}\rangle$	0.450822	$\langle r_{13}^{-2}\rangle$	0.24070
$\langle r_{14}\rangle$	2.34358	$\langle r_{14}^{-1}\rangle$	0.629975	$\langle r_{14}^{-2}\rangle$	0.83554
$\langle r_{34}\rangle$	2.05183	$\langle r_{34}^{-1}\rangle$	0.723554	$\langle r_{34}^{-2}\rangle$	1.09452
$\langle r_{45}\rangle$	2.67774	$\langle r_{45}^{-1}\rangle$	0.476797	$\langle r_{45}^{-2}\rangle$	0.34309
$\langle r_{12}^{2}\rangle$	9.85228	$\langle r_{12}^{3}\rangle$	36.640	$\langle r_{12}^{4}\rangle$	149.38
$\langle r_{13}^{2}\rangle$	7.30084	$\langle r_{13}^{3}\rangle$	23.211	$\langle r_{13}^{4}\rangle$	80.902
$\langle r_{14}^{2}\rangle$	6.99719	$\langle r_{14}^{3}\rangle$	24.761	$\langle r_{23}^{4}\rangle$	99.827
$\langle r_{33}^{2}\rangle$	5.40489	$\langle r_{23}^{3}\rangle$	16.982	$\langle r_{14}^{4}\rangle$	61.069
$\langle r_{45}^{2}\rangle$	8.56802	$\langle r_{24}^{3}\rangle$	31.578	$\langle r_{24}^{4}\rangle$	130.91
$\langle -\frac{1}{2}\nabla_1^2\rangle$	0.440811	$\langle \delta_{12}\rangle$	1.4070×10^{-4}	$\langle \delta_{123}\rangle$	1.6758×10^{-7}
$\langle -\frac{1}{2}\nabla_3^2\rangle$	0.636571	$\langle \delta_{13}\rangle$	1.4158×10^{-4}		
$\langle -\frac{1}{2}\nabla_4^2\rangle$	0.487026	$\langle \delta_{14}\rangle$	0.09678		

TABLE III. The expectation values in muon-atomic units of basic properties for the ground $S(L=0)$ state of the $pdt\mu\mu$ ion. The notations 1, 2, and 3 stand for the hydogenic p, d, and t nuclei, respectively. The notations 4 and 5 designate the two negatively charged muons.

$\langle r_{12}\rangle$	2.73294	$\langle r_{12}^{-1}\rangle$	0.42155	$\langle r_{12}^{-2}\rangle$	0.21222
$\langle r_{13}\rangle$	2.62233	$\langle r_{13}^{-1}\rangle$	0.43748	$\langle r_{13}^{-2}\rangle$	0.22667
$\langle r_{14}\rangle$	2.35765	$\langle r_{14}^{-1}\rangle$	0.61929	$\langle r_{14}^{-2}\rangle$	0.80202
$\langle r_{23}\rangle$	2.28545	$\langle r_{23}^{-1}\rangle$	0.49110	$\langle r_{23}^{-2}\rangle$	0.27629
$\langle r_{24}\rangle$	2.09081	$\langle r_{24}^{-1}\rangle$	0.70127	$\langle r_{24}^{-2}\rangle$	1.01849
$\langle r_{34}\rangle$	2.01920	$\langle r_{34}^{-1}\rangle$	0.72695	$\langle r_{34}^{-2}\rangle$	1.09289
$\langle r_{45}\rangle$	2.58165	$\langle r_{45}^{-1}\rangle$	0.49301	$\langle r_{45}^{-2}\rangle$	0.36605
$\langle r_{12}^{2}\rangle$	8.40594	$\langle r_{12}^{3}\rangle$	28.6035	$\langle r_{12}^{4}\rangle$	106.257
$\langle r_{13}^{2}\rangle$	7.72491	$\langle r_{13}^{3}\rangle$	25.1650	$\langle r_{13}^{4}\rangle$	89.537
$\langle r_{14}^{2}\rangle$	7.00868	$\langle r_{14}^{3}\rangle$	24.4812	$\langle r_{23}^{4}\rangle$	96.705
$\langle r_{23}^{2}\rangle$	5.77902	$\langle r_{23}^{3}\rangle$	15.9889	$\langle r_{14}^{4}\rangle$	47.964
$\langle r_{24}^{2}\rangle$	5.55427	$\langle r_{24}^{3}\rangle$	17.4665	$\langle r_{24}^{4}\rangle$	62.524
$\langle r_{34}^{2}\rangle$	5.18933	$\langle r_{34}^{3}\rangle$	15.8116	$\langle r_{34}^{4}\rangle$	54.906
$\langle r_{45}^{2}\rangle$	7.93976	$\langle r_{45}^{3}\rangle$	28.0450	$\langle r_{45}^{4}\rangle$	111.129
$\langle -\frac{1}{2}\nabla_1^2\rangle$	0.42919	$\langle\delta_{12}\rangle$	7.093×10^{-7}		
$\langle -\frac{1}{2}\nabla_2^2\rangle$	0.61052	$\langle\delta_{13}\rangle$	6.320×10^{-5}		
$\langle -\frac{1}{2}\nabla_3^2\rangle$	0.67305	$\langle\delta_{23}\rangle$	3.675×10^{-5}		
$\langle -\frac{1}{2}\nabla_4^2\rangle$	0.50990	$\langle\delta_{123}\rangle$	2.118×10^{-7}		

sented as the motion of the lightest hydrogenic nucleus (p^+) in the field of stable four-body bimuonic quasimolecule $dt\mu_2$ which is neutral.

It is interesting to note that the five-body bimuonic systems can be used to observe and study some three-particle nuclear reactions between three light (hydrogenic) nuclei. In general, direct three-particle nuclear reactions are hard to observe, since in many cases one of the binary channels substantially dominates. For light nuclei the experimental situation is even worse and any observation of the three-particle nuclear reactions between such nuclei seems to be almost impossible. However, it is shown below that such reactions may proceed and can be observed in the five-body bimuonic systems. Furthermore, as follows from the results of this study some five-body bimuonic systems, e.g., the $ppd\mu_2$ and $ptt\mu_2$ ions, can be considered as ideal bound systems for observing various three-particle nuclear reactions. It is clear, however, that competition between three- and two-particle (=binary) nuclear reactions will take place in these ions also. Nevertheless, by selecting the three hydrogenic nuclei which form such five-body bimuonic systems one may shift the balance between the binary and three-particle nuclear reactions. Finally, the probability of the three-particle nuclear reaction can be quite comparable and even larger than the corresponding probability of the binary reaction.

Consider, e.g., all possible nuclear reactions in the $ppt\mu_2$ system. In fact, only the two following nuclear reactions can be observed in this system

$$p + t = {}^4\text{He} + \gamma + 19.814\ \text{MeV}, \quad (5)$$

$$p + p + t = {}^4\text{He} + p. \quad (6)$$

The proton which forms in the second (=three-particle) reaction is the fast particle $E_p \approx 15.85$ MeV. The reaction, Eq. (6), is an example of the three-particle reaction in the bimuonic system. The first reaction is the binary (or two-particle) reaction between the proton and tritium nuclei. For the rates of the binary (=nuclear-nuclear) and three-nuclear reactions in any one- and/or bimuonic system one can write the two following expressions [3,4]:

$$R_2^{(ij)} = \langle\sigma_{ij}v_{ij}\rangle = A_2^{(ij)}\langle\delta_{ij}\rangle, \quad R_3^{(123)} = A_3^{(123)}\langle\delta_{123}\rangle, \quad (7)$$

where $A_2^{(ij)}$ is the reaction constant (in $\text{cm}^3\,\text{sec}^{-1}$) of the nuclear reaction between nucleus i and nucleus j [in our present notation $i\neq j=(1,2,3)$]. The $A_3^{(123)}$ value is the analogous reaction constant of the nuclear reaction between the three hydrogenic nuclei which are designated by the indexes 1, 2, and 3. Also, in this equation the $\langle\delta_{ij}\rangle$ and $\langle\delta_{123}\rangle$ values are the expectation values of the nuclear-nuclear and three-nuclear δ functions.

The binary reaction constants $A_2^{(ij)}$ can be taken either from the tables of astrophysical data, or approximately computed and/or evaluated directly. The reaction constants of three-nuclear reactions A_3 are currently unknown values. The general theory of three-particle nuclear reactions is extremely complicated and currently does not exist. In the first approximation, however, it is possible to make some predictions about the $A_3^{(123)}$ reaction constants for the five-body bimuonic systems discussed in this study. In particular, for the $abc\mu_2$ systems in which one binary nuclear reaction substantially dominates we may assume that the $A_3^{(123)}$ reaction constant must be approximately equal to the reaction constant of the most fast binary nuclear reaction, i.e., $A_3^{(123)} \approx \max_{(ij)} A_2^{(ij)}$. This means that the ratio of actual and/or observed reaction rates, e.g., in the $pdt\mu_2$ systems, can be evaluated as the ratio of the corresponding expectation value of the binary (e.g., deuteron-triton) and three-nuclear δ functions, i.e., as the ratio of the $\langle \delta_{dt} \rangle$ and $\langle \delta_{pdt} \rangle$ values.

Since all hydrogenic nuclei are positively charged, then it is easy to predict that the expectation value of any two-nuclear δ function is always much larger, than the expectation value of the three-nuclear δ function computed for the same system. In other words, even in the five-body bimuonic molecules $abc\mu_2$ the three-particle nuclear reactions are hard to observe. For instance, in 99.9% of all cases one can see the emission of the fast 14.1 MeV neutron from the $pdt\mu_2$ system, and only in $\approx 0.1\%$ of all cases it is possible to detect the instantaneous emission of the fast proton and fast neutron.

In some $abc\mu_2$ systems, however, the reaction constant for the binary nuclear reaction $A_2^{(ij)}$ can be very small in comparison to the analogous reaction constant $A_3^{(123)}$ for the three-particle reaction. For instance, $A_3^{(123)} \gg A_2^{(ij)}$ in those cases when the three-particle nuclear (a,b,c) reaction in the $abc\mu_2$ system shows a sharp resonance at relatively low energies, while any of three possible binary nuclear reactions either does not have low-energy resonances, or these resonances are very small. Another possibility is the binary nuclear reaction in which only one nucleus is formed and one high-energy γ quantum is emitted. This process is slow in comparison to analogous nuclear reactions in which the two nuclear fragments are formed at the final stage. Formally, the reaction in which one high-energy γ quantum is emitted must be in $\alpha^{-2} \approx (137)^2 \approx 18\,769$ times slower, than the analogous reaction with the two final nuclear fragments in the final stage. In reality, this factor often differs from α^{-2}, e.g, it can be 1000–20 000, rather than $(137)^2$ exactly. The idea to detect the three-nuclear reactions in the five-body bimuonic $abc\mu_2$ systems is based on the following fact. The actual binary and three-particle nuclear reaction rates in some $abc\mu_2$ systems can be quite comparable with each other. In other words, in some $abc\mu_2$ systems one finds $A_2^{(ij)}\langle \delta_{ij} \rangle \approx A_3^{(123)} \langle \delta_{123} \rangle$ and even $A_2^{(ij)}\langle \delta_{ij} \rangle \ll A_3^{(123)} \langle \delta_{123} \rangle$, despite the fact that in all such systems $\langle \delta_{ij} \rangle \gg \langle \delta_{123} \rangle$.

Briefly, we have to answer the following principal question: what are the expectation values of the two-nuclear and three-nuclear δ functions in the five-body bimuonic systems? It is clear a priori that for an arbitrary $abc\mu_2$ system in our present notations we must always have $\tau = \frac{\langle \delta_{ij} \rangle}{\langle \delta_{123} \rangle} \gg 1$ (see above), where $i \neq j = (1,2,3) = (a,b,c)$. But, if such a ratio τ is $\approx 100 - 1{,}000 (\ll \alpha^{-2})$, then we can hope to find some five-body bimuonic systems in which the overall rates of the two- and three-particle reactions are very comparable to each other. If, however, the ratio τ is very large, e.g., $\tau \gg 15\,000$ $(\approx \alpha^{-2})$, then the three-particle nuclear reactions cannot be observed, in principle, in the five-body bimuonic systems considered in this study. To answer this question one needs to compute the expectation values of all binary nuclear-nuclear and three-nuclear δ functions for each of the ten $abc\mu_2$ ions.

In this study all these δ functions have been determined numerically. Our main interest below is related to the $ppd\mu_2$ and $ppt\mu_2$ five-body systems in which all binary reactions have relatively small probabilities. As follows from Tables II and III in the $ppd\mu_2$ and $ppt\mu_2$ systems considered in this study, the ratio $\tau = \frac{\langle \delta_{ij} \rangle}{\langle \delta_{123} \rangle} \approx 200 - 1000$ rather than 18 000. This means that there are the $abc\mu_2$ systems in which the actual (=observed) rate of three-particle nuclear reaction equals and even exceed the corresponding two-particle reaction rate. A very good example of such a system is the $ppt\mu_2$ ion (see Table II). Indeed, in this case the two-particle nuclear reaction, Eq. (5), is slow. The $\langle \delta_{pt} \rangle$ and $\langle \delta_{ppt} \rangle$ expectation values computed for this ion differ from each other in ≈ 800 times. The probabilities of the binary and three-particle nuclear reactions in the $ppt\mu_2$ system are evaluated below.

In general, in an arbitrary five-body $abc\mu_2$ system the three binary and one three-nuclear reactions are possible. In addition to this, the μ^- muon is an unstable particle. This means that all ground states of the $abc\mu_2$ systems considered in this study have nonzero widths, i.e., their lifetimes $\tau = \frac{1}{\Gamma}$ are finite. The total width Γ equals to the sum of all partial widths and, e.g., for the $pdt\mu_2$ system one finds

$$\Gamma = \Gamma_\mu + \Gamma_{pd} + \Gamma_{pt} + \Gamma_{dt} + \Gamma_{pdt} = \frac{1}{\tau_\mu} + \frac{1}{\tau_{pd}} + \frac{1}{\tau_{pt}} + \frac{1}{\tau_{dt}} + \frac{1}{\tau_{pdt}}, \tag{8}$$

where $\tau_\mu \approx 2.20 \times 10^{-6}$ sec is the muon lifetime, while the τ_{ij} and τ_{pdt} are the corresponding reaction times. Note that in the $pdt\mu_2$ ion we have for the total width $\Gamma \approx \Gamma_{dt}$, since in this system the binary (d,t) reaction dominates substantially.

In fact, by considering all $abc\mu_2$ systems mentioned in this study one finds that the $ppt\mu_2$ ion is, probably, the best choice. Indeed, in any $abc\mu_2$ system which contains at least two deuterium and/or tritium nuclei (or one dt combination) the rate of the most fast binary nuclear reaction is much larger than the actual rate of the three-particle reaction. In some such systems three-particle reactions may occur, but it will be very hard to detect. In the $ppp\mu_2$ system no nuclear reaction can be observed at all. Indeed, the muon lifetime $\approx 2.0 \times 10^{-6}$ sec is much shorter than the corresponding mean times $(\approx 1.4 \times 10^{10}$ yr) for the nuclear $p+p = d+e^+ + \nu_e$ and $p+p+p = {}^3\mathrm{He} + e^+ + \nu_e + \gamma$ reactions. The only systems in which the three-particle nuclear reactions can really be observed are the five-body $ppd\mu_2$ and $ppt\mu_2$ ions. The three-particle nuclear reaction in the $ppd\mu_2$ ion is $p+p+d = {}^3\mathrm{He} + p + 5.494$ MeV. In the competing binary (p,d) reaction, one high-energy γ quantum is emitted, i.e., this reaction is slow. In the case of three-particle reaction in the $ppt\mu_2$ system the emitted fast proton has significantly larger energy ($E_p \approx 14$ MeV), and this simplifies its registration. In general, the direct observation of three-particle nuclear reactions can be performed in the same way as described in Ref. [6]. The overall probabilities of such reactions can be measured quite accurately, if the liquid hydrogen (protium) with $\approx 1\%$ of added amount of tritium (or deuterium) is used (see discussion in Ref. [6]). Another possibility is to apply the method from Ref. [7]. Note, however, that both these methods have been developed to detect the repetitive formation of the three-body muonic ions, e.g., the $pd\mu$ and/or $dd\mu$ ions. The actual situation with the five-body bimuonic systems can be much more complicated. It should be mentioned here that our present idea has nothing in common with the notorious muon-catalyzed fusion of nuclear reactions (see, e.g., Ref. [8,9]). In particular, it is absolutely unrealistic to consider any repetitive formation of the five-body, bimuonic systems $abc\mu_2$ in liquid hydrogen, since the overall probabilities of such processes are quite small.

To illustrate the idea of detection of the three-particle nuclear reactions in the $abc\mu_2$ systems let us consider the three following five-body bimuonic ions: $pdt\mu_2$, $ppd\mu_2$, and $ppt\mu_2$. The consideration of other five-body bimuonic systems can be performed in an analogous manner. The rate of the nuclear (d,t) reaction in the $dt\mu$ ions is 1.25×10^{12} sec^{-1} [3]. Therefore, the rate of the binary (d,t) reaction in the four-body system $dt\mu_2$ is $\approx 4.49 \times 10^{13}$ sec^{-1}, while in the five-body system $pdt\mu_2$ such a rate is $\approx 5.18 \times 10^{13}$ sec^{-1}. By assuming that $A_3^{(pdt)} \approx A_2^{(dt)}$ in the $pdt\mu_2$ system, one finds that the three-nuclear reaction rate in this ion is $\approx 2.98 \times 10^{11}$ sec^{-1}. This value is much smaller than the rate of the (d,t) reaction 5.18×10^{13} sec^{-1}. In other words, it is very difficult to detect the three-particle (p,d,t) reaction in the $pdt\mu_2$ ion.

Consider now the $ppd\mu_2$ five-body system and/or ion. For this ion we have found in computations that $\langle \delta_{13} \rangle \approx 1.4158 \times 10^{-4}$ and $\langle \delta_{123} \rangle \approx 4.5119 \times 10^{-7}$. The rate of the (p,d) reaction in the three-body $pd\mu$ ion is $\approx 2.35 \times 10^5$ sec^{-1} (see, e.g, Ref. [3]). Therefore, the rate of the same reaction in the four-body $pd\mu_2$ system is $\approx 3.67 \times 10^6$ sec^{-1} (the δ function expectation values is taken from Ref. [10]), while in the five-body $ppd\mu_2$ ion the rate of this reaction equals $\approx 1.69 \times 10^6$ sec^{-1}. Now, let us evaluate the rate of the three-particle reaction $p+p+d={}^3\text{He}+p$. We can evaluate the corresponding reaction constant by using the numerical value for the reaction constant for the analogous binary reaction $d+d={}^3\text{He}+n$. However, to make our approximation more realistic we introduce a small factor λ $(\lambda \ll 1)$ in the following formula: $A_3^{(ppd)} \approx \lambda A_2^{(dd)} \approx \lambda \times 0.922 \times 10^{16}$ cm^3 sec^{-1}. The three different values of this factor λ are considered below $\lambda=0.1$, $0.02(=\frac{1}{50})$, and $0.005(=\frac{1}{200})$. For $\lambda=0.1$, one finds $\approx 4.16 \times 10^8$ sec^{-1}, while for $\lambda=0.02$ we have $\approx 8.31 \times 10^7$ sec^{-1} and $\approx 2.08 \times 10^7$. Thus, in all such cases we have $R_3^{(ppd)} \gg R_2^{(pd)}$ in the $ppd\mu_2$ five-body ion.

The five-body ion $ppt\mu_2$ can be discussed analogously. The rate of the (p,t) reaction in the three-body ion $pt\mu$ is $\approx 1.33 \times 10^7$ sec^{-1}. From here, one finds the corresponding reaction constant $A_2^{(pt)} \approx 1.48 \times 10^{12}$ cm^3 sec^{-1}, i.e., for the reaction rate of (p,t) reaction in the four-body bimuonic system $pt\mu_2$ we obtain $R_2^{(pt)} \approx 2.23 \times 10^8$ sec^{-1}. For the five-body system $ppt\mu_2$ the rate of this nuclear reaction is $R_2^{(pt)} \approx 1.10 \times 10^8$ sec^{-1} (the $\langle \delta_{13} \rangle$ expectation value is taken from Table II). The rate of the three-particle nuclear reaction $p+p+t={}^4\text{He}+p$ can be evaluated by using the following approximate expression $R_3^{(ppt)} \approx \xi A_2^{(dt)} \langle \delta_{123} \rangle$, where ξ is a small factor (in this case $\xi \approx 1$) and $A_2^{(dt)} \approx 1.409 \times 10^{18}$ is the reaction constant of the (d,t) reaction. By using this expression and $\xi=0.5$ one finds $R_3^{(ppt)} \approx 6.75 \times 10^{10}$ sec^{-1}. Analogously for $\xi=0.2$ we have $R_3^{(ppt)} \approx 2.70 \times 10^{10}$ sec^{-1} and for $\xi=0.1$ we have $R_3^{(ppt)} \approx 1.35 \times 10^{10}$ sec^{-1}. In any of these cases we have $R_3^{(ppt)} \gg R_2^{(pt)}$, i.e., the three-particle nuclear (p,p,t) reaction can be detected in the five-body bimuonic $ppt\mu_2$ system.

Thus, in this work we have considered the bound state properties of the five-body bimuonic systems. This is a detailed study of the bound states in real five-body systems in which all particle masses are finite. Note that these systems are quite different from one-center atoms and/or two-center molecular-type ions. The ground state energies for all these systems have been determined to relatively high numerical accuracy. However, such an accuracy can be improved in the future studies. Note also that some of the ten bimuonic $abc\mu_2$ ions may have a few excited states. Currently, nothing is known about such states in the $abc\mu_2$ systems. It is shown that some of the five-body bimuonic ions, e.g., the $pdt\mu_2$, $ppt\mu_2$, and others ions, can be used to observe the three-body nuclear reactions with three light (hydrogenic) nuclei. In fact, analogous bimuonic systems can be formed by many other light nuclei, e.g., by the helium and lithium nuclei. Therefore, various three-particle nuclear reactions, which include such nuclei, can be studied also. In general, a possible experimental detection of the three-particle nuclear reactions from the bound states of some bimuonic systems will be a great step forward in the physics of nuclear fusion, few-body physics, and nuclear physics.

In our earlier work [10] we have shown that the nuclear reaction rates in the four-body bimuonic systems, e.g., in the $dd\mu_2$ and $dt\mu_2$ systems, are in $\approx 40-50$ times larger than in corresponding three-body ions $dd\mu$ and $dt\mu$. This idea still is waiting for its experimental confirmation. Note that in another of our work [11] it was shown that bimuon atomic systems, e.g, the ${}^3\text{He}\mu_2$ and ${}^4\text{He}\mu_2$ atoms, are almost perfect objects to study the effects of vacuum polarization. The lowest-order correction (=Uehling correction) in such systems can be seen and measured directly from their optical spectra. This study emphasizes another interesting aspect of bimuonic systems. The main problem now is to start theoretical and experimental research on bimuonic systems. The first goal is to understand how such systems can be created and studied in actual experiments.

[1] N. N. Kolesnikov and V. I. Tarasov, Yad. Fiz. **35**, 609 (1982) [Sov. J. Nucl. Phys. **35**, 354 (1982)].

[2] The NIST Reference on Constants, Units and Uncertainty, see http://physics.nist.gov/cuu/Constants/index.html

[3] J. D. Jackson, Phys. Rev. **106**, 330 (1957).

[4] S. Cohen, D. L. Judd, and R. J. Riddell, Phys. Rev. **119**, 384 (1960).

[5] D. H. Bailey and A. M. Frolov, J. Phys. B **34**, 3813 (2001).

[6] L. W. Alvarez *et al.*, Phys. Rev. **105**, 1127 (1957).

[7] H. Bossy *et al.*, Phys. Rev. Lett. **59**, 2864 (1987).

[8] A. K. Bhatia and R. J. Drachman, Comments At. Mol. Phys. **22**, 281 (1989).

[9] L. I. Men'shikov and L. N. Somov, Usp. Fiz. Nauk **160**, 47 (1990) [Sov. Phys. Usp. **33**, 616 (1990)].

[10] A. M. Frolov and V. H. Smith, Jr., Phys. Rev. A **55**, 2435 (1997).

[11] A. M. Frolov, Phys. Lett. A **350**, 241 (2006).