

Entanglement versus energy in the entanglement transfer problem

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(Received 7 August 2006; published 25 October 2006)

We study the relation between energy and entanglement in an entanglement transfer problem. We first analyze the general setup of two entangled qubits (“ a ” and “ b ”) exchanging this entanglement with two other independent qubits (“ A ” and “ B ”). Qubit “ a ” (“ b ”) interacts with qubit “ A ” (“ B ”) via a spin-exchange-like unitary evolution. A physical realization of this scenario could be the problem of two-level atoms transferring entanglement to resonant cavities via independent Jaynes-Cummings interactions. We study the dynamics of entanglement and energy for the second pair of qubits (tracing out the originally entangled ones) and show that these quantities are closely related. For example, the allowed quantum states occupy a restricted area in a phase diagram entanglement vs energy. Moreover, the curve which bounds this area is exactly the one followed if both interactions are equal and the entire four qubit system is isolated. We also consider the case when the target pair of qubits is subjected to losses and can spontaneously decay.

DOI: [10.1103/PhysRevA.74.042328](https://doi.org/10.1103/PhysRevA.74.042328)

PACS number(s): 03.67.Mn, 03.65.Yz, 42.50.–p

I. INTRODUCTION

Entanglement is one of the most studied topics at present. The large interest for this issue relies mainly on the fact that entangled systems can be used to perform some tasks more efficiently than classical objects [1]. It is then natural to look for a good understanding of this resource not only from a purely mathematical point of view, i.e., formalizing the theory of entanglement, but also from a more practical approach, i.e., studying its role and manifestations in realistic systems. For example, recent works have been able to connect entanglement to thermodynamical properties of macroscopic physical systems [2]. In a distinct venue, other works study physical manifestations of quantum correlations by suitably choosing particular purity and entanglement quantifiers and restricting allowed quantum states according to these quantities [3]. In these studies, concepts like maximally entangled mixed states (MEMS) are discussed. A very recent study also adds energy to entanglement and purity as a third parameter to characterize certain quantum states [4]. In particular, the authors discuss the physically allowed states according to the possible values of entanglement, purity, and energy for a system composed of two qubits or two Gaussian states, and also study the entanglement transfer between them.

In the present paper, we study the connection between entanglement and energy that appears naturally in a swapping process involving two systems of two qubits. In the model investigated, we consider a simple form of interaction between two pairs of qubits labeled as aA and bB . The system ab is prepared in an entangled state while the pair AB is prepared in a factorable state. We analyze the dynamical relations between energy and entanglement of qubits AB when

exchanging energy and coherence with qubits ab . In particular, for any given time t , we calculate the full quantum state of qubits $abAB$ and then we trace out qubits ab to calculate energy and entanglement of the remaining pair AB . We show that this dynamics yields paths in an entanglement-energy diagram, and that these paths are contained in a very restricted region. Moreover, we identify the frontiers of this region from the general form of the density operator that represents the state of the subsystem AB . We also propose a physical system to realize such entanglement transfer and investigate how the dynamics of the entanglement swapping is modified if the AB system is open and allowed to dissipate energy to an external reservoir. In some sense, this work is complementary to the sequence [5] in which the authors study the problem of entanglement transfer from continuous-variable entangled states to qubits, although in those works the authors do not pay particular attention to the relation between energy and entanglement.

The paper is organized as follows. In Sec. II we introduce the general physical system that we will study and the basic setup from which we will approach it. We also define the quantities that will be analyzed throughout the paper and finally we discuss the dynamics of this system. Section III is devoted to study the entanglement and the energy of system AB under a particular unitary evolution. We then propose a physical implementation for the studied Hamiltonian, and generalize the time evolution in Sec. IV by considering the problem of a dissipative, nonunitary evolution. In Sec. V we conclude by reviewing the main points we have discussed and suggesting possible extensions of this study.

II. PHYSICAL SCENARIO

Let us start by describing the system we are interested in. Suppose a system of four qubits a , b , A , and B interacting via a spin-exchange-like Hamiltonian:

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$$H = H_{aA} + H_{bB}, \quad (1)$$

where

$$H_{aA} = \frac{\hbar\omega_a}{2}\sigma_z^a + \frac{\hbar\omega_A}{2}\sigma_z^A + \hbar g_{aA}(\sigma_-^a\sigma_+^A + \sigma_+^a\sigma_-^A) \quad (2a)$$

and

$$H_{bB} = \frac{\hbar\omega_b}{2}\sigma_z^b + \frac{\hbar\omega_B}{2}\sigma_z^B + \hbar g_{bB}(\sigma_-^b\sigma_+^B + \sigma_+^b\sigma_-^B). \quad (2b)$$

For each qubit, the relevant Pauli operators are defined by

$$\sigma_z = |1\rangle\langle 1| - |0\rangle\langle 0|, \quad (3a)$$

$$\sigma_+ = |1\rangle\langle 0|, \quad (3b)$$

$$\sigma_- = |0\rangle\langle 1|, \quad (3c)$$

and the interaction operators like $\sigma_-^a\sigma_+^A$, for example, can be viewed as annihilating an excitation of subsystem a and creating an excitation in subsystem A . The constants g_{aA} and g_{bB} give the strength of the interaction between these subsystems. One important feature in understanding such Hamiltonians is that the total number of excitations is a conserved quantity. The eigenvectors of Eq. (2a) [similarly to Eq. (2b)] are given by $|00\rangle_{aA}$, with eigenvalue $E_{00}^{aA} = -\hbar\omega$, $|11\rangle_{aA}$, with eigenvalue $E_{11}^{aA} = \hbar\omega$, and $|\Lambda_{\pm}\rangle_{aA} = (|01\rangle \pm |10\rangle)/\sqrt{2}$, with eigenvalue $E_{\pm}^{aA} = \pm\hbar g_{aA}/2$, where $\omega = (\omega_a + \omega_A)/2$.

As the initial state, let us suppose that the entire $abAB$ system is prepared in the form

$$|\phi(t=0)\rangle = |\psi(\theta)\rangle_{ab} \otimes |00\rangle_{AB}, \quad (4)$$

where $|\psi(\theta)\rangle_{ab} = \sin\theta|01\rangle + \cos\theta|10\rangle$, which means that subsystem AB is prepared in its ground state and subsystem ab is usually prepared in some entangled state with one excitation (except if $\theta = n\frac{\pi}{2}$, $n \in \mathbb{Z}$, when the state is factorable). Note that this initial state is pure and it is chosen so that the bipartition $ab \otimes AB$ does not present any initial entanglement. From now on, we will study the time evolution of this initial state when subjected to Hamiltonian (1) for different coupling constants g_{aA} and g_{bB} . We will concentrate our analysis on the subsystem AB by tracing out the degrees of freedom of systems a and b . Another simplifying assumption we made is to consider the complete resonance condition $\omega_a = \omega_A = \omega_b = \omega_B = \omega$.

A special case of this dynamics happens when $g_{aA} = g_{bB} = g$, in which case state (4) evolves into state

$$|\phi(t)\rangle = \cos(gt)|\psi\rangle_{ab} \otimes |00\rangle_{AB} - i \sin(gt)|00\rangle_{ab} \otimes |\psi\rangle_{AB}. \quad (5)$$

Note that in this simple case, for $t = n\frac{\pi}{2g}$, with n odd, the subsystems exchange their states, the entanglement initially present in subsystem ab is completely transferred to subsystem AB and with respect to the bipartition $ab \otimes AB$ the state becomes again separable. However, for $t \neq n\frac{\pi}{2g}$, the whole system is entangled (as long as $\theta \neq n\frac{\pi}{2}$) and subsystem AB will be in some mixed state.

In a more general situation (different coupling constants), state (4) will evolve into

$$|\phi(t)\rangle = \cos\theta[\cos(g_{aA}t)|1000\rangle - i \sin(g_{aA}t)|0010\rangle] + \sin\theta[\cos(g_{bB}t)|0100\rangle - i \sin(g_{bB}t)|0001\rangle]. \quad (6)$$

Note that for generic times t , state (6) presents, again, multipartite entanglement among all its individual components (a , b , A , and B). Studying this multipartite entanglement may also prove intriguing and enlightening. However, this is not the purpose of this paper where, as mentioned above, we will concentrate our analysis in the subsystem AB .

Our goal is to investigate the relation between energy and entanglement in subsystem AB as a function of coupling constants and time. In order to study entanglement we will use the *negativity* (N) which can be defined for two qubits as two times the modulus of the negative eigenvalue of the partial transposition of the state ρ , ρ^{TA} [6], if it exists. For short:

$$N(\rho) = 2 \max\{0, -\lambda_{min}\}, \quad (7)$$

where λ_{min} is the lowest eigenvalue of ρ^{TA} . Our choice is motivated by the facts that the negativity is easy to calculate and provides full entanglement information for a two-qubit system. For the energy of subsystem AB , U , we will consider the mean value of the relevant restriction of the free Hamiltonian:

$$U = \text{Tr}\{\rho H_{AB}\}, \quad (8)$$

where

$$H_{AB} = \frac{\hbar\omega}{2}(\sigma_z^A + \sigma_z^B). \quad (9)$$

III. ENTANGLEMENT AND ENERGY

After tracing out the degrees of freedom of systems a and b in the global quantum state (6), the reduced state for the pair AB is described by

$$\rho_{AB} = \begin{pmatrix} a & 0 & 0 & 0 \\ 0 & b & d & 0 \\ 0 & d^* & c & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \quad (10)$$

where $a+b+c=1$ (from the normalization of ρ_{AB}), with a , b , c , and d given by the following functions of the coupling constants and time:

$$a = \cos^2\theta \cos^2(g_{aA}t) + \sin^2\theta \cos^2(g_{bB}t), \quad (11a)$$

$$b = \sin^2\theta \sin^2(g_{bB}t), \quad (11b)$$

$$c = \cos^2\theta \sin^2(g_{aA}t), \quad (11c)$$

$$d = \cos\theta \sin\theta \sin(g_{aA}t)\sin(g_{bB}t). \quad (11d)$$

Following Eq. (8), the energy of state (10) is

$$U = -a, \quad (12)$$

which, by means of Eqs. (11), becomes

$$U = -\cos^2 \theta \cos^2(g_{aA}t) - \sin^2 \theta \cos^2(g_{bB}t). \quad (13)$$

Note that $-1 \leq U \leq 0$, which means that there is at most one excitation on the AB system. This is expected since the chosen initial state contains only one excitation for the entire $abAB$ system and this set of qubits is isolated, i.e., it cannot be excited by external sources. A simple calculation gives for the entanglement (negativity) of state (10)

$$N = \sqrt{a^2 + 4d^2} - a. \quad (14)$$

A. Entanglement and energy versus time

In Fig. 1 we have plotted the temporal behavior of N and U for several values of the coupling constants g_{aA} and g_{bB} . The entanglement transferring process can be followed in those pictures. The simplest one is for equal coupling constants $g_{aA} = g_{bB}$, in which the state $|\psi\rangle$ is cyclically “bouncing” between the two pairs of qubits. The chosen ratios between coupling constants indicate a very important behavior of the system: the complete entanglement transferring process can only happen if $mg_{aA} = ng_{bB}$ for m and n odd integers. Equation (11a) supports this conclusion, since both $g_{aA}t$ and $g_{bB}t$ must be odd multiples of $\frac{\pi}{2}$ simultaneously (remember that $\cos^2 \theta$ and $\sin^2 \theta$ are positive numbers) in order for the state $|\psi(\theta)\rangle$ to be entangled. The physical picture is that each pair aA and bB oscillates inside duplets $|10\rangle$ and $|01\rangle$. The situation is analogous to two classical harmonic oscillators with distinct frequencies starting from a common extremal point. The implied relation is necessary for them to meet within an odd number of half oscillations, which is the condition for a complete transfer of state. To insist on this point, note that for $g_{aA} = 2g_{bB}$, at time $t = \frac{\pi}{g_{aA}}$, the pair aA (or, more precisely, its analogous oscillator) has suffered a full oscillation, but the pair bB (respectively its analogous oscillator) has undergone half an oscillation, so one could find entanglement between a and B , but no entanglement can be found between any other pair of qubits, including the studied pair AB .

For other values of θ we obtain similar pictures, with the only important difference in the entanglement scale, since no maximally entangled pair will be formed.

B. Entanglement versus energy

In this section we use time as a parameter to draw graphics on an entanglement vs energy diagram. As we will show, the paths followed in this phase diagram exhibit interesting patterns.

The graphics for $\theta = \pi/4$ (Fig. 2) and $\theta = \pi/3$ (Fig. 3) are qualitatively different. However, it can be seen through Fig. 4 that, independent of the available initial entanglement (given by the value of θ), the accessible region in the parameter space $N \times U$ is bounded by an upper curve. This bound can be explained in the following way: using the fact that $U = -a$ in Eq. (14) we have

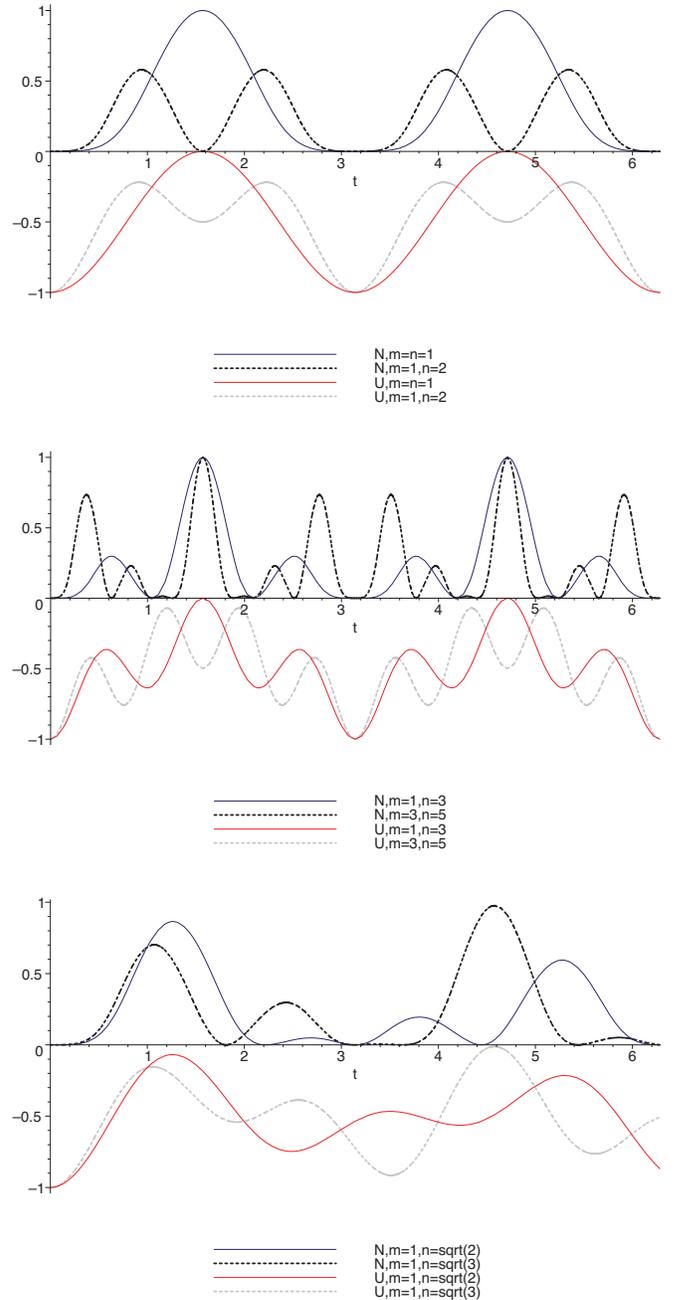


FIG. 1. (Color online) Energy U and negativity N of the state ρ_{AB} given by Eq. (10) vs time. The initial state is given by Eq. (4) with $\theta = \pi/4$, which means a maximally entangled pair ab is initially present, and its entanglement can be transferred to the pair AB . Several values of m and n were used in the relation $mg_{aA} = ng_{bB}$.

$$N^2 - 2NU = 4|d|^2. \quad (15)$$

However, when $g_{aA} = g_{bB}$ and $\theta = \pi/4$ we have $b = c = d$, in which case $N^2 - 2NU = 4b^2$. Using the normalization condition $a + b + c = 1$, we get

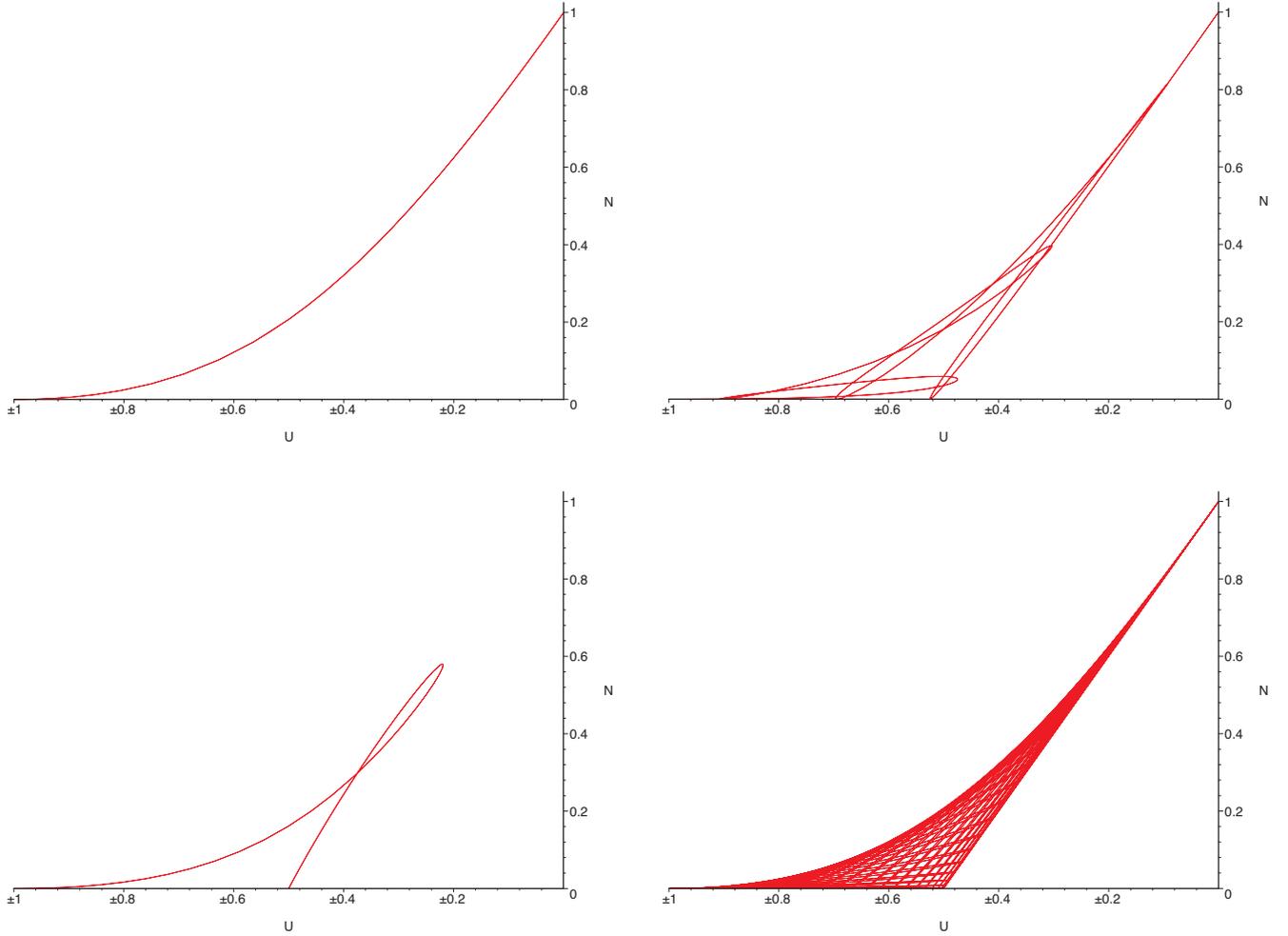


FIG. 2. (Color online) Negativity (N) vs energy (U) for ρ_{AB} given by Eq. (10) with $\theta = \pi/4$, i.e., the initial ab state is maximally entangled. Parameter relation $g_{aA} = ng_{bB}$ with $n=1$ (left upper panel), $n=2$ (right upper panel), $n=7$ (left lower panel), and $n=53$ (right lower panel).

$$(1 + U)^2 = 4d^2. \quad (16)$$

Therefore in the ideal situation of equal coupling $g_{aA} = g_{bB}$ and maximally entangled initial ab state ($\theta = \pi/4$), we can write

$$N^2 - 2NU = (1 + U)^2. \quad (17)$$

As we will see now this equation is exactly the one that limits the phase-space for quantum states in this problem. The normalization condition yields $b+c=1+U$ that allows us to obtain

$$4bc = (U+1)^2 - (b-c)^2,$$

which implies

$$4bc \leq (U+1)^2. \quad (18)$$

At the same time, the condition for matrix (10) to be considered a true density matrix is that its eigenvalues are all positive, which is reached if and only if $|d|^2 \leq bc$. Therefore we can conclude that

$$4|d|^2 \leq 4bc \leq (U+1)^2, \quad (19)$$

and, from Eq. (15), we find

$$N^2 - 2NU \leq (U+1)^2. \quad (20)$$

As we saw in Eq. (17) the equality is reached for $g_{aA} = g_{bB}$ and $\theta = \pi/4$. So Eq. (17) bounds the region that density matrices of the form (10) can occupy in the diagram $N \times U$.

IV. OPEN SYSTEM

Up to this moment we have considered any two pairs of qubits. Now we will adhere to one specific physical realization, namely: two atoms resonantly coupled to two independent cavity modes. If the cavities are initially in the ground state (vacuum, no photon) and the usual approximations are valid [7], the Jaynes-Cummings Hamiltonian

$$H_{JC} = \frac{\hbar\nu}{2}\sigma_z + \hbar\omega\left(a^\dagger a + \frac{1}{2}\right) + \hbar\Omega(a^\dagger\sigma_- + a\sigma_+) \quad (21)$$

essentially reduces to the form (2), with the lower case qubit representing the two-level atom and the capital one, the first

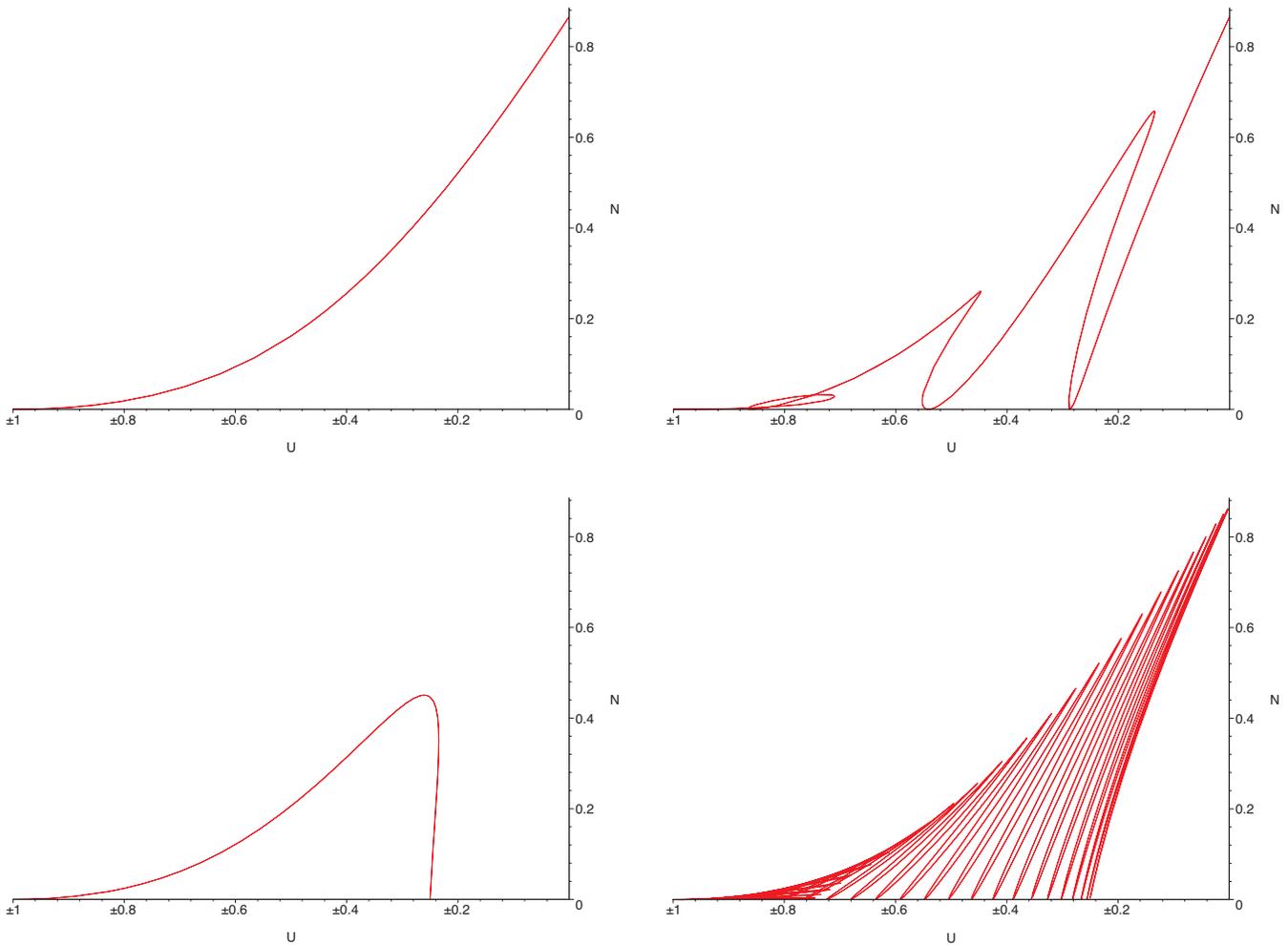


FIG. 3. (Color online) Negativity (N) vs energy (U) for ρ_{AB} given by Eq. (10) with $\theta = \pi/3$, i.e., the initial ab is *partially* entangled. Parameter relation $g_{aA} = n g_{bB}$ with $n = 1$ (left upper panel), $n = 2$ (right upper panel), $n = 7$ (left lower panel), and $n = 53$ (right lower panel).

two energy levels of the field mode. Hence the situation here studied models an experiment where previously entangled atoms transfer such entanglement to independent cavity modes. One nice point when considering this particular situation of atoms transferring entanglement and energy to resonant cavities is that those systems can have very different dissipation times. In fact, atomic levels can be selected so that their dissipation time scale is much larger than those of typical resonant cavities. In this case, one can ask what happens if the system that receives the energy and the entanglement dissipates it to an external reservoir. In order to answer this question, the unitary analysis considered up to now has to be abandoned and we must change from a Hamiltonian approach to a master equation one.

We will consider that the qubits AB , now represented by the cavity field modes, are in contact with independent reservoirs and interact with them. Since atomic lifetimes (for the atomic transitions used in cavity QED experiments) are usually much greater than cavity decay times, we will not couple the lower case qubits to any external device.

To address this problem we consider the time evolution of the global system described by the master equation in the Lindblad form [8]:

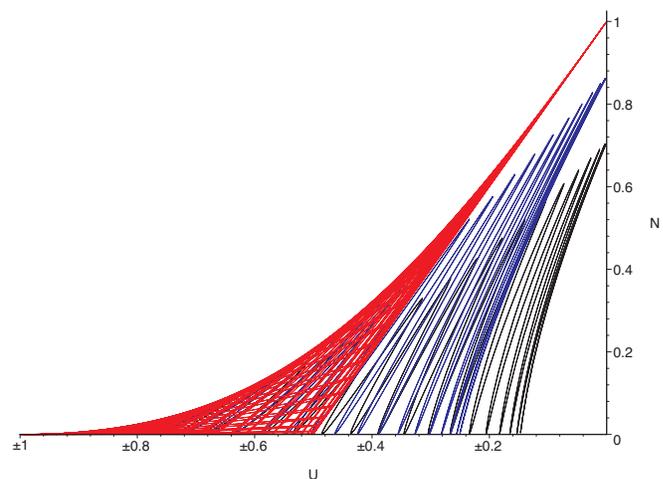


FIG. 4. (Color online) Negativity (N) vs energy (U) for ρ_{AB} given by Eq. (10) with $\theta = \pi/4$ [red (left)], $\theta = \pi/3$ [blue (middle)], and $\theta = 3\pi/8$ [black (right)]. Parameter relation $g_{aA} = 53 g_{bB}$.

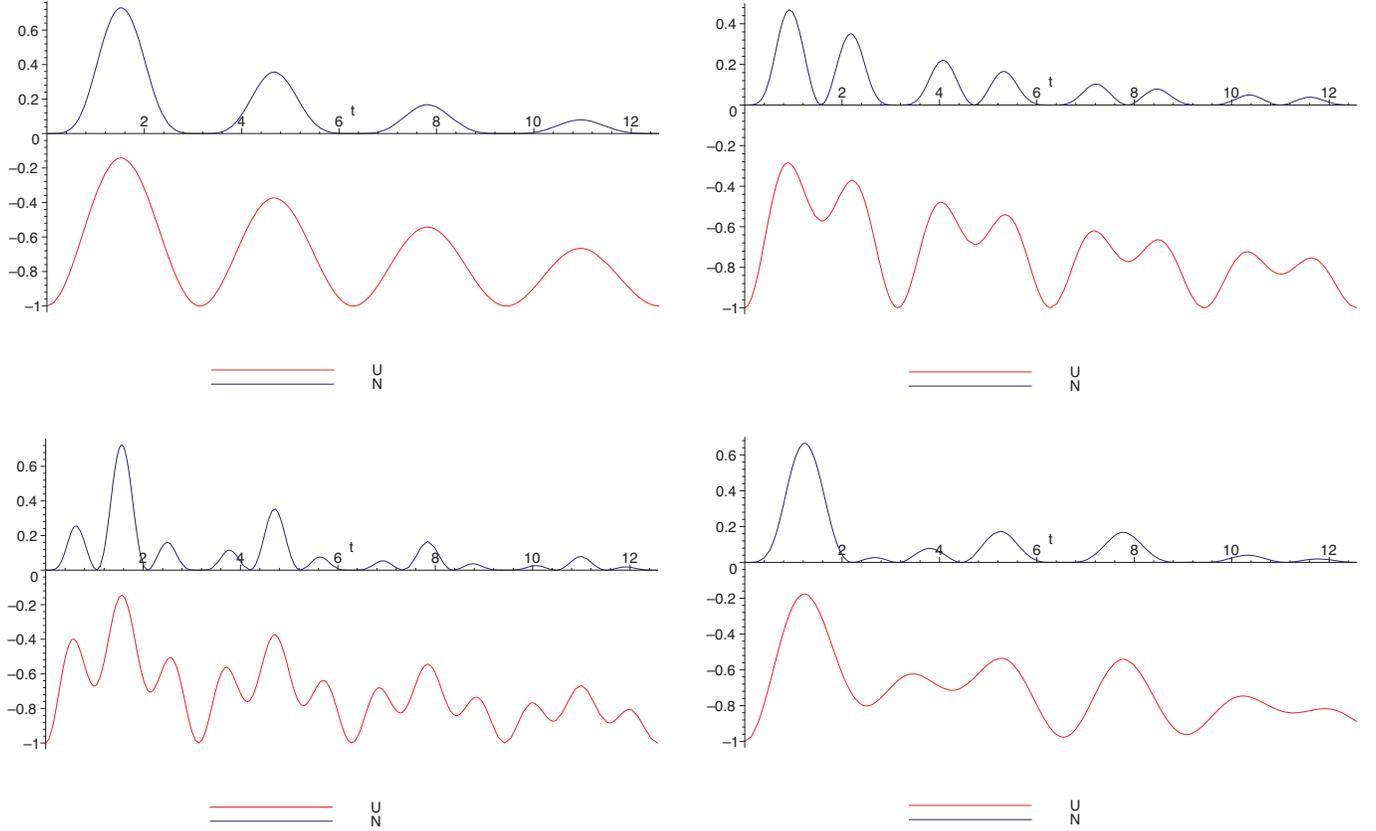


FIG. 5. (Color online) Energy U [red (bottom curve)] and negativity N [blue (top curve)] of the cavity modes state (10) with matrix elements given by Eq. (24) vs time. The initial state of the system is given by Eq. (4) with $\theta = \pi/4$, i.e., a maximally entangled ab state. Parameter relations: $\kappa_{aA} = \kappa_{bB} = 0.1g_{aA}$ and $g_{bB}/g_{aA} = n$. Left-above: $n=1$. Right-above: $n=2$. Left-below: $n=3$. Right-below: $n=\sqrt{2}$.

$$\frac{d}{dt}\rho_{aAbB}(t) = \frac{1}{i\hbar}[H, \rho_{aAbB}(t)] + \frac{1}{2\hbar} \sum_i ([\hat{V}_i \rho_{aAbB}(t), \hat{V}_i^\dagger] + [\hat{V}_i^\dagger, \rho_{aAbB}(t) \hat{V}_i]), \quad (22)$$

where H is given by Eq. (1) and the operators \hat{V}_i and \hat{V}_i^\dagger describe the effects of the coupling to the reservoirs. For simplicity, we will model only the dissipation of energy in the cavities coupled to null temperature reservoirs, which can be done using

$$\hat{V}_1 = \sqrt{2\hbar\kappa_A} \sigma_-^A \otimes I_{abB}, \quad (23a)$$

$$\hat{V}_2 = \sqrt{2\hbar\kappa_B} I_{aAb} \otimes \sigma_-^B, \quad (23b)$$

$$\hat{V}_i = 0, \quad \forall i > 2. \quad (23c)$$

The constants κ_A and κ_B are directly given by the decay rates of each cavity mode.

Starting from the special case of the initial state (4), given by $\theta = \frac{\pi}{4}$ (a Bell state for the donor pair of qubits), after a somewhat lengthy calculation we find that the state of both cavities can still be written in the form of Eq. (10), but now with the matrix elements given by

$$a = 1 - \left\{ \left[\frac{\sqrt{2}g_{aA}}{\Omega_{aA}} \sin\left(\frac{\Omega_{aA}t}{2}\right) e^{-\kappa_A t/2} \right]^2 + \left[\frac{\sqrt{2}g_{bB}}{\Omega_{bB}} \sin\left(\frac{\Omega_{bB}t}{2}\right) e^{-\kappa_B t/2} \right]^2 \right\}, \quad (24a)$$

$$b = \left[\frac{\sqrt{2}g_{bB}}{\Omega_{bB}} \sin\left(\frac{\Omega_{bB}t}{2}\right) e^{-\kappa_B t/2} \right]^2 \quad (24b)$$

$$c = \left[\frac{\sqrt{2}g_{aA}}{\Omega_{aA}} \sin\left(\frac{\Omega_{aA}t}{2}\right) e^{-\kappa_A t/2} \right]^2, \quad (24c)$$

$$d = \left[\frac{\sqrt{2}g_{aA}}{\Omega_{aA}} \sin\left(\frac{\Omega_{aA}t}{2}\right) e^{-\kappa_A t/2} \right] \left[\frac{\sqrt{2}g_{bB}}{\Omega_{bB}} \sin\left(\frac{\Omega_{bB}t}{2}\right) e^{-\kappa_B t/2} \right], \quad (24d)$$

with the definitions

$$\Omega_{aA} = \sqrt{4g_{aA}^2 - \kappa_A^2}, \quad (25a)$$

$$\Omega_{bB} = \sqrt{4g_{bB}^2 - \kappa_B^2}. \quad (25b)$$

Since the state of the system AB is still described by density matrices of the form (10) we expect the existence of the same bounds in the energy-time diagram (17). Also note that energy and negativity are still, respectively, described by

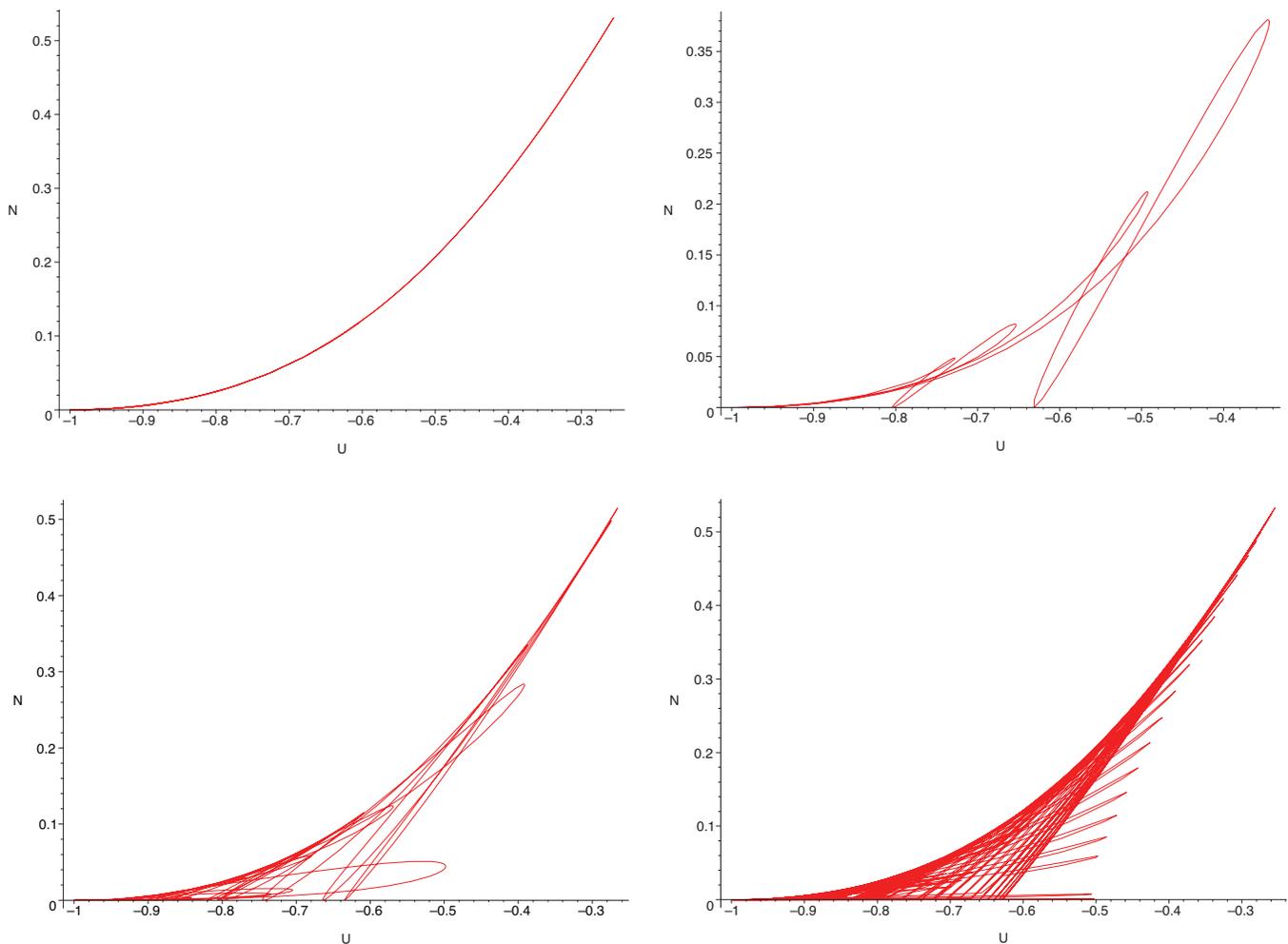


FIG. 6. (Color online) Negativity (N) vs energy (U) of the cavity modes state (10) with matrix elements given by Eq. (24) for fixed decay rates and different couplings. The initial state of the system ab is given by Eq. (4) with $\theta = \pi/4$. Parameter relations: $\kappa_{aA} = \kappa_{bB} = 0.1g_{aA}$ and $g_{aA} = ng_{bB}$ with $n=1$ (left upper panel), $n=2$ (right upper panel), $n=7$ (left lower panel), and $n=53$ (right lower panel).

equations of the forms (12) and (14). In Fig. 5 we have plotted some curves of negativity and energy vs time, for the system AB considering the temporal evolution of the matrix elements given by Eq. (24). Note that the graphics are qualitatively similar to the ones displayed in Fig. 1 for unitary evolutions. However, as expected, both entanglement and energy decay exponentially to their lowest values as a function of time. This can be understood from the fact that the environment drives the system exponentially to the state $|00\rangle$ [see the matrix elements in Eq. (24)—the element (24a) goes to unity while all the others go to zero], which has no entanglement and has the minimum energy value $U = -1$.

We have also plotted the negativity vs energy for the non-unitary case for different values of g_{aA} , g_{bB} , κ_{aA} , and κ_{bB} . This is displayed in Fig. 6. As commented before the followed paths are still bounded by the same limits of the unitary case. However, as the dissipative mechanisms get stronger (i.e., coefficients κ_A and κ_B get closer to the coupling constants g_{aA} and g_{bB}), less and less entanglement is transferred to subsystem ρ_{AB} .

A different phenomenology can be anticipated for the case of non-null temperature. Since thermal photons can now be

captured by both cavities, the state $|11\rangle$ will be populated and the form (10) will not be valid anymore. One consequence is that one can expect the phenomenon known as entanglement sudden death [9], since it will not be necessary to nullify the elements d in order to have a positive partial transpose, and hence no entanglement.

V. DISCUSSIONS

In this paper we have addressed the problem of entanglement and energy transfer between pairs of qubits. We considered the particular example of two atoms interacting with two cavities in the Jaynes-Cummings model. This evolution can be seen as a state transferring process and, for specific coupling constants, a dynamical entanglement swapping. If the atoms are initially in an entangled state, this entanglement is fully or partially transferred to the cavities depending on coupling constants and time. This entanglement swapping process is accompanied by an energy transfer as well, and we have shown that entanglement and energy in the cavities system are strictly related.

To clarify this relation we studied these quantities in various scenarios. First we considered the whole system as isolated, and investigated its time evolution for several coupling constants and different initial atomic entanglement. In each case, we traced out the atoms (as we now refer to the lower case qubits), we drew an entanglement vs energy phase-diagram for the cavity modes, and we found an upper-limit for all the possible paths in these diagrams. This bound corresponds to maximally entangled atoms transferring its entanglement and energy to independent cavities at exactly the same rates.

We also considered the possibility of dissipation in the cavities. In this case, while the atoms are transferring excitations and entanglement to the cavities, some energy is lost to the environment. The cavities state goes asymptotically to state $|00\rangle$. In the dissipative regime, the evolution of entanglement and energy of the cavities state exhibits the same characteristics pointed out for the unitary case, i.e., the paths followed in the entanglement-energy diagram are limited to a restricted region whose frontier is identified by the trajectory described when the couplings are identical and the initial entanglement of the atomic state is maximum. However, as

expected, neither entanglement nor energy can be fully transferred unless the dissipative times are much larger than the inverse Rabi frequencies involved. Fortunately, this regime is usually achieved in cavity QED experiments.

We only analyzed the entanglement between qubits AB (the modes, in the physical realization proposed). However, most of the time the whole system presents multipartite entanglement which may provide interesting new results if further studied. Other important continuations of this work include the treatment of noncompletely resonant systems (e.g, $\omega_a = \omega_A \neq \omega_b = \omega_B$, which corresponds to two distinct atoms resonantly coupled to cavity modes, as well as the case of dispersive coupling) and also other couplings to reservoirs, like including temperature in the scenario here presented and also considering spontaneous decay for the atoms.

ACKNOWLEDGMENTS

The authors recognize fruitful discussions with M.C. Nemes. Financial support from CNPq and PRPq-UFMG is acknowledged. This work is part of the Millenium Institute for Quantum Information project (CNPq).

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