# Decay and storage of multiparticle entangled states of atoms in collective thermostat

A. M. Basharov,* V. N. Gorbachev, and A. A. Rodichkina†

*Laboratory for Quantum Information & Computation, Aerospace University, St.-Petersburg 190000 Bolshaia Morskaia 67, Russia*

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We derive a master equation describing the collective decay of two-level atoms inside a single mode cavity in the dispersive limit. By considering atomic decay in the collective thermostat, we found a decoherence-free subspace of the multiparticle entangled states of the *W*-like class. We present a scheme for writing and storing these states in collective thermostat.

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## I. INTRODUCTION

When information is encoded in a quantum state of a physical system, the robustness of the state is an important factor for successful communication. Due to decoherence, i.e., interaction with the environment, the state of the system can degrade and lose its quantum correlations. One of the possible solutions of the decoherence problem is to use decoherence-free subspaces (DFSs); these include wave functions immune to decoherence [1].

The first DFS has been introduced by Zanardi *et al.* [2] for two-level atoms interacting with an electromagnetic field playing the role of environment. The wave functions belonging to the DFS are annihilated by the interaction Hamiltonian and, therefore, are left invariant during evolution. Examples of DFSs for various physical systems, in particular, for light, have been proposed by several authors (see, for example, [3–5]). Weinfurter *et al.* have demonstrated experimentally decoherence-free quantum communication based on four-photon polarized states [6].

Simple observations show that quantum correlations between particles can be produced and maintained in collective processes. These are interesting for DFSs and a large number of physical systems with collective interactions can be found. For the Dicke model with a single resonant mode, Bonifatio *et al.* have found a master equation describing a collective atomic decay when illuminated with a resonant mode [7]. Palma and Knight [8] have shown that two-atom decay can result in pure entangled states in a collective squeezed thermostat. However, entanglement of two atoms can be achieved in collective decay with vacuum thermostat, when atoms are placed inside a cavity, as it has been shown by Basharov [9]. There is a simple reason for collective decay in the cavity scheme. When atoms interact with a single cavity mode, atomic relaxation arises because radiation leaves the cavity. This can be modeled as an interaction between the mode and an external broadband field, which plays role of a thermostat. Therefore, the atoms, being coupled with the single mode, have a collective decay.

The aim of this paper is to investigate the collective decay of atoms in the entangled state. To achieve this we consider a simple model of two-level atoms inside a single mode cavity and a broadband field. For this model Klimov *et al.* [10] derived master equations for atoms and a collective relaxation operator in the dispersive limit, assuming a vacuum bath. We discuss a more general squeezed thermostat, for which a master equation is derived using a formalism of unitary transformations. There are some differences between relaxation operators due to methods of derivation of master equations. From the physical point of view, in the dispersive limit there is only an exchange of phase between atoms and cavity mode. This is described by the effective Hamiltonian we found, which is diagonal over atomic and field variables. However, the behavior of atoms becomes more complicated due to an interaction between the cavity mode and broadband field, which plays the role of a thermostat. Then one finds a coupling of atoms with the thermostat, providing atomic collective relaxation. In this aspect our results differ from that of Ref. [10].

In contrast to [10] we use the idea developed in Ref. [11]; we first find the effective Hamiltonian of the total system including the thermostat and then derive the master equation. Using the master equation we consider the dynamics of a class of multiparticle entangled states, which is a slightly generalized *W* class introduced by Cirac *et al.* [12]. Some of the optical and atomic implementations of the presented states have been demonstrated experimentally by Weinfurter *et al.* [13] and Schmidt-Kaler [14]. Some properties of these states, different schemes to generate them, and several applications have been considered in Ref. [15]. We also find that in the case when these entangled states are reduced to the Dicke states, they belong to DFS and are immune to collective decay. To explain this feature we use symmetry arguments. In fact, the total space of the Dicke states is represented by irreducible subspaces distinguished by their symmetry type. The collective interaction we consider does not mix the wave functions from different subspaces due to symmetry conservation. Using these properties, we present a model of a quantum memory for writing, storing, and reading information encoded in these entangled states.

The paper is organized as follows. We first derive the master equation in the dispersive limit assuming a general model of thermostat. Then we introduce a set of multiparticle entangled states which can be reduced to the Dicke family and we consider their decay in squeezed and vacuum thermostats. Finally we present a scheme for reading and storing entangled states in the collective thermostat.

## II. INITIAL EQUATIONS

By considering the interaction between atoms and a field, one can obtain a master equation for one of the systems. This

*Permanent address: Laboratory for nonlinear optics, RRC "Kurchatov Institute," Moscow 123182, Russia.

†Electronic address: an@qic.org.ru

 

equation is also known as kinetic and often has a Lindblad form. It describes irreversible processes including atomic relaxation, absorption or amplification of light, and other phenomena which can be reduced to the Lindblad equations.

### A. Hamiltonian

We consider $n$ two-level atoms inside a high-finesse optical cavity, a single cavity mode, and a broadband field outside the cavity. We assume the Hamiltonian of the system has the form

$$H = H_a + H_c + H_b + V_1 + V_2, \tag{1}$$

where the Hamiltonians of free atoms, cavity mode, and broadband field are, respectively, $H_a = \hbar\omega_0 R_3$, $H_c = \hbar\omega_c c^\dagger c$, $H_b = \Sigma_\omega \hbar\omega b_\omega^\dagger b_\omega$; here $R_3 = \Sigma_j(|1\rangle_j\langle 1| - |0\rangle_j\langle 0|)$, 0, 1 label the lower and upper levels of atom, and $c, c^\dagger, b_\omega, b_\omega^\dagger$ are creation and annihilation operators for photons of the cavity mode and broadband field, respectively. The interaction between atoms and cavity mode $V_1$ has the form

$$V_1 = g(c^\dagger R_- + cR_+), \tag{2}$$

where collective atomic operators are given by $R_\pm = \Sigma_j R_\pm^{(j)}, R_+^{(j)} = [R_-^{(j)}]^\dagger = |1\rangle_j\langle 0|$. The term $V_2$ describes two processes: (1) an interaction between the broadband field and the cavity mode due to nonzero transmittance of the output mirror; (2) an interaction between atoms and the broadband field due to nonideal sidewalls of the cavity. It reads

$$V_2 = \sum_\omega b_\omega \left[\Gamma_\omega c^\dagger + \sum_j K_{\omega j} R_+^{(j)}\right] + \text{H.c.} \tag{3}$$

From the physical point of view the broadband field plays the role of a thermostat and causes the relaxation of atoms and cavity mode. Relaxation terms can be achieved by switching on this field. It can be done in different ways using various approximations.

### B. Dispersive limit

We assume that detuning $\Delta = |\omega_c - \omega_0|$ is large and consider the dispersive limit, which can be justified when [17]

$$|\Delta| \gg ng\sqrt{\langle c^\dagger c\rangle + 1}. \tag{4}$$

To derive the master equation let us introduce a transformation of Hamiltonian $H$ given by a time independent unitary operator $S$

$$H' = e^{-iS} H e^{iS} = -i[S;H] - \left(\tfrac{1}{2}\right)[S;[S;H]] + \cdots . \tag{5}$$

Using perturbation theory over interactions $V_1$ and $V_2$ one finds the operator $S$, from which an effective Hamiltonian describing the interaction between atoms and cavity mode can be obtained. This Hamiltonian is diagonal over the field and atomic variables and has the form

$$H_e = g^2 \frac{R_- R_+ + cc^\dagger 2R_3}{\hbar\Delta}. \tag{6}$$

Under this approximation there is another effective Hamiltonian $H_g$ which describes the interaction between atoms and broadband field

$$H_g = -\frac{g}{\hbar\Delta}\sum_\omega \Gamma_\omega (R_+ b_\omega + R_- b_\omega^\dagger). \tag{7}$$

In contrast to the second term in $V_2$ the obtained Hamiltonian $H_g$ describes the collective interaction of atoms. Indeed, in the usual case of the dispersive limit there is no energy exchange between atoms and light, and this is in accordance with the effective Hamiltonian $H_e$, which is similar to Ref. [10]. In the same time the effective Hamiltonian $H_g$ shows that atoms and the thermostat field exchange excitations. This is a particular feature of the dispersive limit due to the initial interaction (3). A close analogy is parametric down conversion in transparent nonlinear media, in which the virtual transitions result in an interaction between photons.

Now we have a problem specified by $H' = H_a + H_b + H_c + H_e + H_g + V_2$, where broadband field can be considered as a thermostat in a given state. Assume the thermostat is $\delta$-correlated and its state is squeezed with a center frequency $\Omega$:

$$\langle b_\omega^\dagger b_{\omega'}\rangle = N(\omega)\,\delta_{\omega,\omega'},$$

$$\langle b_\omega b_{\omega'}^\dagger\rangle = (N(\omega)+1)\,\delta_{\omega,\omega'},$$

$$\langle b_\omega b_{\omega'}\rangle = M(\omega)\,\delta_{2\Omega,\omega+\omega'},$$

$$\langle b_\omega^\dagger b_{\omega'}^\dagger\rangle = M^*(\omega)\,\delta_{2\Omega,\omega+\omega'}, \tag{8}$$

where the photon numbers $N(\omega)$ and $M$ are related as $|M(\omega)| \leq \sqrt{N(\omega)[N(\omega)+1]}$. A physical model of this thermostat can be represented by a light generated in parametric down conversion process. Its simple nondegenerate version is described by the Hamiltonian $H = \Sigma_\omega(k_\omega b_{\Omega+\omega}^\dagger b_{\Omega-\omega}^\dagger + \text{H.c.})$, where $2\,\Omega$ is the pump frequency and $\omega$ belongs to a frequency band $h$ given by phase matching conditions. The photon numbers $N$ and $M$ have the form $N(\omega) = \sinh^2 r_\omega$, $M(\omega) = \exp(i \arg k_\omega)\cosh r_\omega \sinh r_\omega$, where $r_\omega \sim |k_\omega|$ is a squeezing parameter. For a squeezed vacuum $r \ll 1$ and $N \approx 0$, while $M \approx \exp(i \arg k_\omega) r_\omega$. The generated light is broadband if $h$ is much bigger than all representative frequencies of the problem, like the atomic and cavity mode decay rates. More precisely, assume that the width of the squeezed broadband field given by Eqs. (8) is much bigger than the detuning $\Delta$ as in Eq. (4). Then following the standard procedure of replacing a finite bandwidth system with white noise [16] we can make all parameters of squeezed light to be independent from the frequency: $N(\omega) = N, M(\omega) = M$. Assume the squeezed thermostat is modeled by a parametric down conversion source. Then its bandwidth is determined by the phase matching conditions and can be experimentally varied on a wide range.

The next step is switching on the broadband field. This can be achieved by several methods based on projection operator techniques, stochastic differential equations, and others. In any case we need a Markovian approximation to obtain a closed equation. In our case this means that the evolution of the broadband light is given by the free Hamiltonian $H_b$ only and the thermostat parameters $N$ and $M$ are frequency independent. As a result we find a master equation for the density matrix $\rho$ of atoms and cavity mode. The equation includes an effective Hamiltonian $H_e$ and relaxation terms. In the dispersive limit and in the interaction picture the master equation has the form

$$\dot{\rho} = -(i/\hbar)[H_e;\rho] - T\rho, \tag{9}$$

where the relaxation operator $T$ includes three terms of the Lindblad form: $T=\Sigma_j\mathcal{L}_j+\mathcal{L}_c+\mathcal{L}_a$. The first term describes the independent decay of atoms in the squeezed thermostat. When the atoms have the same coupling constant $K_{\omega j}=K_\omega$ it reads

$$\mathcal{L}_j\rho = (\gamma_\downarrow/2)(R_+^{(j)}R_-^{(j)}\rho - 2R_-^{(j)}\rho R_+^{(j)} + \rho R_+^{(j)}R_-^{(j)}) + (\gamma_\uparrow/2) \times (R_-^{(j)}R_+^{(j)}\rho - 2R_+^{(j)}\rho R_-^{(j)} + \rho R_-^{(j)}R_+^{(j)}) - 2MK^2R_+^{(j)}\rho R_+^{(j)} - 2M^*K^2R_-^{(j)}\rho R_-^{(j)}, \tag{10}$$

where the decay rates of atomic levels are denoted by $\gamma_\downarrow=|K|^2(N+1)$, $\gamma_\uparrow=|K|^2N$, and $|K|^2=\hbar^{-2}\Sigma_\omega|K_\omega|^2\delta(\omega_0-\omega)$. In free space one finds that $|K|^2$ reduces to the well-known formula for the spontaneous decay rate $4\omega_0^2d^2/3\hbar c^3$. Equation (10) describes spontaneous decay of independent atoms in the squeezed thermostat, for which the transversal decay rate becomes slow because of squeezing: $\gamma_\perp=(\gamma_\downarrow+\gamma_\uparrow)/2-\mathrm{Re}\{MK^2\}$. The second term of $T$ is the relaxation of the cavity mode due to photons leaving the cavity, and has the form

$$\mathcal{L}_c\rho = |\Gamma|^2[(N+1)(c^\dagger c\rho - 2c\rho c^\dagger + \rho c^\dagger c) + N(cc^\dagger\rho - 2c^\dagger\rho c + \rho cc^\dagger) + M(cc\rho - 2c\rho c + \rho cc)M^*(c^\dagger c^\dagger\rho - 2c^\dagger\rho c^\dagger + \rho c^\dagger c^\dagger)], \tag{11}$$

where $|\Gamma|^2=\hbar^{-2}\Sigma_\omega|\Gamma_\omega|^2\delta(\omega_c-\omega)$. If $R$ is the reflectance of the output cavity mirror, then $|\Gamma|^2\to c(1-R)/2L$, where $L$ is length of the cavity. The collective decay of atoms is represented by the operator $\mathcal{L}_a$:

$$\mathcal{L}_a\rho = |\chi|^2[(N+1)(R_+R_-\rho - 2R_-\rho R_+ + \rho R_+R_-) + N(R_-R_+\rho - 2R_+\rho R_- + \rho R_-R_+) + M(R_+R_+\rho - 2R_+\rho R_+ + \rho R_+R_+) + M^*(R_-R_-\rho - 2R_-\rho R_- + \rho R_-R_-)], \tag{12}$$

where $|\chi|^2=|g\Gamma/\hbar^2\Delta|^2\tau, \tau=L/c$. As a result, in the dispersive limit there are three relaxation operators describing single-particle and collective decays. They have a straightforward physical meaning and they differ from the relaxation operator in Ref. [10], which has cross terms including products of the collective atomic operators by operators of the cavity mode.

In order to consider the collective decay of atoms let us introduce the interaction picture $\rho'=\exp(-i\hbar^{-1}H_et)\rho\exp(i\hbar^{-1}H_et)$ and assume the following approximations: Let the first term in $H_e$ and single-particle relaxation be small, $g^2R_-R_+/\hbar\Delta$, $\Sigma_j\mathcal{L}_j \ll cc^\dagger R_3/\hbar\Delta$, $\mathcal{L}_a$, $\mathcal{L}_c$. This is true if $g^2/\hbar|\Delta|$, $\gamma_{\downarrow,\uparrow}\ll\langle cc^\dagger\rangle g^2/|\Delta|$, $|\chi|^2n$, and $|\Gamma|^2\langle c^\dagger c\rangle/n$. Then we can neglect the difference between $H_e$ and $g^22c^\dagger cR_3/\hbar\Delta$ so that the master equation for the atomic density matrix $f=\mathrm{Tr}_c\rho'$ is

$$\dot{f} = -\mathcal{L}_af. \tag{13}$$

## III. COLLECTIVE DECAY AND STORAGE OF ENTANGLED STATES

Considering decay of atoms in collective thermostats one finds that quantum correlations between particles can be supported, and their final or steady state depends on the initial one.

### A. Entangled Dicke states

We introduce the multiparticle entangled states, the slight modification of the $W$ states discovered by Cirac [12]

$$\eta_n(1) = q_1|10\ldots0\rangle + q_2|01\ldots0\rangle + \cdots q_n|00\ldots1\rangle, \tag{14}$$

where $\Sigma_k|q_k|^2=1$. Some of these states belong to the Dicke states $|jma\rangle$ [18], specified by three quantum numbers $j,m,a$, where $|m|\leq j=0,\ldots,n/2-1,n/2$, $n$ is a particle number, and parameter $a$ describes the degeneracy and takes $n_j=C_n^{n/2+j}-C_n^{n/2+j+1}$ values. The numbers $j$ and $m$ are eigenvalues of two commuting collective operators $J_3$ and $J^2=J_1^2+J_2^2+J_3^2$

$$J_3|jma\rangle = m|jma\rangle, \quad J^2|jma\rangle = j(j+1)|jma\rangle, \tag{15}$$

where $J_b$ obeys the commutation relations of the momentum operators $[J_b;J_c]=i\varepsilon_{bcd}J_d$, $b,c,d=1,2,3$. In the considered case $J_1=(\frac{1}{2})(R_-+R_+)$, $J_2=(i/2)(R_+-R_-)$. When

$$\sum_k q_k = 0, \tag{16}$$

we have a set of the zero sum amplitude states discovered by Pati [19]. However, the wave functions $\eta_n(1)$ under condition (16) belong to the Dicke family with $j=m=n/2-1$ [20]. The states have the next representation

$$\eta_n(1) = \sqrt{2}\sum_{k=2}^{n} q_k|\Psi^-\rangle_{1k}\otimes|0\rangle_{(1k)}, \tag{17}$$

where $|0\rangle_{(1k)}$ denotes a $|0\rangle$ state of $n-2$ particles (without first and $k$th), $\Psi^-=(1/\sqrt{2})(|01\rangle-|10\rangle)$. Equation (17) gives the structure of entanglement of $\eta_n(1)$; it tells that one of the particles, say 1, forms EPR pairs with all other particles $2,\ldots,n$. This feature is invariant under particle permutations. Due to antisymmetric vectors $\Psi^-$ the collective evolution of $n$ particles in the state (17) involves only $n-2$ particles. This has a simple reason. Two-particle collective operators $R_\pm$ and $R_3$ annihilate $\Psi^-$, this causes that for any evolution operator $U$ depending on $R_\pm,R_3$ we have

$$U|\eta_n\rangle\langle\eta_n| = 2\sum_{ks}|\Psi_{1k}^-\rangle\langle\Psi_{1s}^-|U(1k;1s)|0\rangle_{(1k)(1s)}\langle0|, \tag{18}$$

where $U(1k;1s)|0\rangle_{(1k)(1s)}\langle0|$ acts on all particles except 1, $k$ and 1, $s$. These features allow us to get simple exact solutions for several collective decay problems of $\eta_n(1)$.

### B. Collective squeezed thermostat

In squeezed thermostat there is an interesting feature. It can produce or store quantum correlations between particles for several initial states. Considering a two-atom collective decay with initial density matrix $f(0)=A|00\rangle\langle 00| + B|\Psi^{+}\rangle\langle\Psi^{+}| + C|11\rangle\langle 00| + C^{*}|00\rangle\langle 11| + D|11\rangle\langle 11|$, where $\Psi^{+}=(1/\sqrt{2})(|01\rangle+|10\rangle)$, $A+B+D=1$, one finds the next pure steady state [8]:

$$s = (\sqrt{N+1}|00\rangle + \sqrt{N}|11\rangle)/(\sqrt{2N+1}). \tag{19}$$

This state is entangled. Note that this solution is correct for the initially symmetric state $f(0)$.

Consider the collective decay of the two entangled states $\eta_3$ and $\eta_4$ described by (13). Under conditions (16) the wave functions read

$$\eta_3 = q_2\Psi^{-}_{12}|0\rangle_3 + q_3\Psi^{-}_{13}|0\rangle_2,$$

$$\eta_4 = q_2\Psi^{-}_{12}|0\rangle_{23} + q_3\Psi^{-}_{13}|0\rangle_{24} + q_4\Psi^{-}_{14}|0\rangle_{23}. \tag{20}$$

According to Eq. (18) the evolution of the density matrix $|\eta_3\rangle\langle\eta_3|$ reduces to the dynamics of the single-particle state $|0\rangle\ \langle 0|$ for which there is a simple solution $|0\rangle\langle 0| \to \lambda|0\rangle\langle 0| + (1-\lambda)|1\rangle\langle 1|$. The $\lambda = N/(2N+1)$ is the occupation number of the lower atomic level. One finds that $\eta_3$ decays into a mixed state with complex structure. This fact can be explained using a symmetry argument, which tells that under the single-particle decay the symmetry of the initial state is not conserved. In contrast to dynamics of $\eta_3$, the dynamics of $\eta_4$ has other features. To obtain the solution we use (19) and find that the final state is obtained by replacing $|00\rangle \to s$

$$\eta_4 \to q_2\Psi^{-}_{12}|s\rangle_{23} + q_3\Psi^{-}_{13}|s\rangle_{24} + q_4\Psi^{-}_{14}|s\rangle_{23}. \tag{21}$$

From this equation it follows that the state is pure, has a more complicated entanglement structure, but as before one of the atoms forms EPR pairs with all other atoms.

### C. Vacuum thermostat

Assuming a simpler thermostat model for which $M=N=0$ the master equation (13) reduces to

$$\dot{f} = -\kappa(R_+R_-f - R_-fR_+ + fR_+R_-), \tag{22}$$

where $\kappa=|\chi|^2$. This equation describes a collective decay in the vacuum thermostat conserving quantum correlations. The simplest example is the two-particle antisymmetric function $\Psi^{-}$ belonging to DFS and immune to decay. The more interesting examples, introduced by Zanardi [2], are DFSs of multiatom states, products of $\Psi^{-}$.

Suppose the atoms inside the cavity are prepared in the state $\eta_n(1)$, then they evolve according to Eq. (22) which can be solved exactly. It is easy to verify that the Lindblad operator $\mathcal{L}_0 f = R_+R_-f - R_-FR_+ + \text{H.c.}$ in Eq. (22) has the following properties:

$$\mathcal{L}_0|\eta_n\rangle\langle\eta_n| = Q|1;n\rangle\langle\eta_n| - |Q|^2|0\rangle\langle 0| + Q^{*}|\eta_n\rangle\langle 1;n|,$$

$$\mathcal{L}_0|\eta_n\rangle\langle 1;n| = Q|1;n\rangle\langle 1;n| - 2Qn|0\rangle\langle 0| + n|1;n\rangle\langle\eta_n|,$$

$$\mathcal{L}_0|1;n\rangle\langle\eta_n| = [\mathcal{L}_0|\eta_n\rangle\langle 1;n|]^{\dagger}, \tag{23}$$

where $Q=\Sigma_k q_q$, $|0\rangle=|00\ldots 0\rangle$ is a ground state of atoms and $|1;n\rangle$ is a fully symmetric state, the normalized version of which, $W_n=(1/\sqrt{n})|1;n\rangle$, is known as $W$ state

$$W_n = (1/\sqrt{n})(|10\ldots 0\rangle + |01\ldots 0\rangle + \cdots |00\ldots 1\rangle). \tag{24}$$

It follows from Eqs. (23) that the Lindblad operator $\mathcal{L}_0$ maps the set of states $\{|\eta_n\rangle\langle\eta_n|, |0\rangle\langle 0|, |1;n\rangle\langle 1;n|, |1;n\rangle\langle\eta_n|, \times|\eta_n\rangle\langle 1;n|\}$ into itself. This observation allows us to get an exact solutions for density matrix

$$f(t) = A(t)|1;n\rangle\langle\eta_n| + A^{*}(t)|\eta_n\rangle\langle 1;n| + B(t)|1;n\rangle\langle 1;n| + S(t) \times|0\rangle\langle 0| + D|\eta_n\rangle\langle\eta_n|, \tag{25}$$

where the normalization condition reads $A(t)Q^{*}+A(t)^{*}Q + B(t)n + S(t) + D(t) = 1$ and coefficients obey equations

$$\dot{A} = -\kappa(An + DQ),$$

$$\dot{B} = -\kappa(AQ^{*} + A^{*}Q) - \gamma nB,$$

$$\dot{S} = -2\kappa[nS + n(1-D) + |Q|^2 D],$$

$$\dot{D} = 0.$$

Similarly to the squeezed thermostat there is a steady state solution, if $t\to\infty$

$$f_{ss} = D[-(Q/n)|1;n\rangle + |\eta_n\rangle][-(Q^{*}/n)\langle 1;n| + \langle\eta_n|] + [(1-D) + |Q|^2 D/n]|0\rangle\langle 0| \tag{26}$$

which depends on the initial state through parameter $D$. If $D=0$, then $f_{ss}=|0\rangle\langle 0|$. If $D=1$ one finds evolution of $\eta_n(1)$

$$|\eta_n\rangle\langle\eta_n| \to (Q/\sqrt{n})(e^{-n\kappa t}-1)|W_n\rangle\langle\eta_n| + \text{H.c.} + (|Q|^2/n) \times(1-e^{-n\kappa t})^2|W_n\rangle\langle W_n| + (|Q|^2/n)(1-e^{-2n\kappa t}) \times|0\rangle\langle 0| + |\eta_n\rangle\langle\eta_n|. \tag{27}$$

It follows from (27) that under condition (16) $\eta_n(1)$ is a Dicke state and has immunity to the collective decay. However, this result can be obtained without any calculations due to its annihilation by the Lindblad operator $\mathcal{L}_0$. In contrast to $\eta_n(1)$, the fully symmetric $W_n$ state degrades: $W_n \to |0\rangle$.

The robustness of $\eta_n$ entangled states can be clear from the symmetry argument. In the considered collective processes the particles permutation operator is an integral of motion, so that state symmetry is conserved. Therefore the antisymmetric wave function $\Psi^{-}$ is robust to decay because the transition $\Psi^{-}\to|0\rangle$ is forbidden, but the fully symmetric $W$ state can transform into $|0\rangle$. In the case of $\eta_n(1)$ the situation is more complicated, nevertheless symmetry plays a principal role here also. As it is known the space of Dicke states is represented by irreducible subspaces distinguished by their symmetry type over particles permutations. Under the condition (16) the wave functions $\eta_n(1)$ belong to $[n,n-1]$ irreducible representation of the Dicke states in contrast to $W$ and ground state $|0\rangle$, which belong to the $[n,0]$

one. Due to symmetry conservation the subspaces of different symmetry type do not mix. This point is in accordance with the fact that in the dynamics of the wave functions the final states at $t\to\infty$ are not usual steady states but depend on their subspace and initial conditions.

Robustness of $\eta_n(1)$ is a natural basis for a quantum memory. Memory includes writing, storing, and reading of information encoded by a quantum state. By choosing $\eta_n(1)$ states to encode information, a model of quantum memory can be designed. Writing and reading are achieved by swapping: $a\otimes b\to b\otimes a$. A particular case of swapping of two mode light in a Fock state into atomic ensemble has been considered in Ref. [21]. Here we introduce a scheme for writing and storing multiparticle states.

Assume that an interaction between atoms inside cavity and light, represented by its spacial modes with wave vectors $j$, has the form

$$V = i\hbar\sum_j f[R_+^j a_j \exp(ijr_j) - \text{H.c.}], \tag{28}$$

where $r_j$ is a position of $j$th atom. This Hamiltonian describes an exchange of excitation between a single atom and a single mode. Assume that initially atoms and field are independent: $|0\rangle_a\otimes|\eta_n\rangle_b$, where $|0\rangle_a$ is the ground state of atoms and $|\eta_n\rangle_b$ is the light state given by Eq. (14), where $|0\rangle$, $|1\rangle$ are the Fock states with 0 and 1 photons, respectively. The multimode light can be prepared, i.e., by a set of beamsplitters, distributing a single photon to different paths. For simplicity assume $\exp(ijr_j)\approx 1$, then the evolution is given by

$$\exp(-i\hbar^{-1}Vt)|0\rangle_a\otimes|\eta_n\rangle_b = \cos(ft)|0\rangle_a\otimes|\eta_n\rangle_b + \sin(ft)|\eta_n\rangle_a \otimes|0\rangle_b. \tag{29}$$

If $\sin(ft)=1$, the state of light $|\eta_n\rangle_b$ is swapped into atoms. Under condition (16) it can be stored in the collective thermostat. Due to the unitarity of transformation (29), we can achieve a reading of the atomic state.

## IV. CONCLUSIONS

Being collective properties of a physical system, quantum correlations between particles and entanglement can be produced and stored in the collective processes. These processes can describe interaction between the physical system and its environment, which often plays a role of thermostat. In contrast to the usual thermostat the collective thermostat supports quantum correlations and it is possible to find a DFS, which is a natural basis for quantum memory. For the considered example of collective decay of atoms inside a cavity, we found that a set of entangled states of the $W$-like class is decoherence free and therefore is suitable to encode quantum information for storing it in the collective thermostat.

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