Einstein-Podolsky-Rosen correlations of Dirac particles: Quantum field theory approach

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We calculate correlation function in the Einstein-Podolsky-Rosen type of experiment with massive relativistic Dirac particles in the framework of the quantum field theory formalism. We perform our calculations for states which are physically interesting and transform covariantly under the full Lorentz group action—i.e., for pseudoscalar and vector states.

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I. INTRODUCTION

One of the most puzzling aspects of quantum mechanics, its nonlocality, is illustrated by the Einstein-Podolsky-Rosen (EPR) paradox [1]. Quantum mechanics predicts that for a pair of entangled particles, flying apart from each other, measurements give results incompatible with our intuitive conceptions about reality and locality. Quantum mechanical predictions have been confirmed in many EPR-type experiments. Most of these experiments have been performed with photon pairs. However, all EPR experiments with photons are subject to the so-called detection loophole—low efficiency of photon detection allows the possibility that the subensemble of detected events agrees with quantum mechanics even though the entire ensemble satisfies the requirements of local realism. Therefore the fair-sampling hypothesis, stating that the detected events fairly represent the entire ensemble, must be assumed. The detection loophole has been closed recently in an experiment with massive particles [2]. This experiment in turn does not overcome the so-called locality loophole in which the correlations of apparently separate events could result from unknown subluminal signals propagating between different particles. (The locality loophole has been closed in an experiment with photons [3].) One can hope that future experiments with relativistic massive particles could close both of the above-mentioned loopholes. Therefore such experiments seem to be very interesting for the basics of quantum mechanics.

On the other hand, in the last decade, starting from Czachor's papers [4,5], EPR correlations in the relativistic context have been widely discussed [6–43]. Unfortunately the incompleteness of the relativistic quantum mechanics formalism[1] (e.g., lack of the covariant notion of localization) causes our understanding of relativistic aspects of quantum information theory to be far from satisfactory. Moreover, it is unclear which spin operator should be used in the relativistic context (spin is not a self-contained, irreducible geometrical object in the relativistic quantum mechanics). There are several different operators which have the proper nonrelativistic limit (for a discussion see Sec. IV). Measurement of quantum spin correlations in EPR experiments could help us to decide which spin operator is more appropriate. In the present paper we use quantum field theory methods to calculate the correlation function for a massive Dirac particle-antiparticle pair in an EPR-type experiment using a particular, in our opinion the most adequate, spin operator. Our results can be useful for a discussion of EPR experiments in which the spin correlation function of elementary particles is measured. Such an experiment was performed in the 1970s [44] but with the nonrelativistic particles (protons).

Entangled pairs of massive particles are usually created in the decay processes of elementary particles (e.g., $\pi^0 \to e^+e^-$, $Z^0 \to e^+e^-$). A decaying particle has of course a well-defined Poincaré-covariant state (e.g., π^0 is a pseudoscalar particle, Z^0 four-vector one). The dynamics of the decay process is Poincaré invariant. Therefore the two-particle entangled state of decay products also possesses well-defined transformation properties with respect to the full Poincaré group (compare [18]). In our paper we classify such states and calculate correlation functions in the pseudoscalar and four-vector states which correspond to the states of the pair created in the π^0 and Z^0 decay, respectively. Our results are valid for any Dirac particle-antiparticle pair but we concentrate our discussion on the $\pi^0 \to e^+e^-$ and $Z^0 \to e^+e^-$ decays because particles produced in these decays are ultrarelativistic.

In Sec. II we establish the notation and briefly recall basic facts concerning free quantum Dirac fields. In Sec. III we consider two-particle states and classify them according to transformation properties with respect to the full Lorentz group. The next section we devote to a discussion of the spin operator. Finally, in Sec. V we calculate explicitly the correlation function for the particle-antiparticle pair in the pseudoscalar and vector states. The last section contains our concluding remarks.

II. SETTING

Let the field operator $\hat{\Psi}(x)$ fulfill the Dirac equation

$$(i\gamma^\mu \partial_\mu - m)\hat{\Psi}(x) = 0, \tag{1}$$

where γ^μ are Dirac matrices (their explicit form used in the present paper and related conventions can be

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[1]At least if we accept standard clock synchronization convention in special relativity. See in this context [51] where consistent relativistic quantum mechanics in the framework of nonstandard synchronization scheme for clocks was formulated and [33] where quantum correlations in this framework were discussed

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found in Appendix A). The field transforms under Lorentz transformations according to

$$U(\Lambda)\hat{\Psi}(x)U(\Lambda^{-1}) = \mathsf{D}(\Lambda^{-1})\hat{\Psi}(\Lambda x), \tag{2}$$

where $U(\Lambda)$ belongs to the unitary irreducible representation of the Poincaré group and $\mathsf{D}(\Lambda)$ is the bispinor representation of the Lorentz group (see Appendix B). The field operator has the standard momentum expansion

$$\hat{\Psi}_\alpha(x) = (2\pi)^{-3/2}\sum_{\sigma=\pm 1/2}\int \frac{d^3\mathbf{k}}{2k^0}[e^{-ikx}u_{\alpha\sigma}(k)a_\sigma(k) + e^{ikx}v_{\alpha\sigma}(k)b_\sigma^\dagger(k)], \tag{3}$$

where $a_\sigma^\dagger(k)$ $[b_\sigma^\dagger(k)]$ are creation operators of the particle [antiparticle] with four-momentum k and spin component along the z axis equal to σ and $k^0=\sqrt{m^2+\mathbf{k}}$. These operators fulfill the standard canonical anticommutation relations

$$\{a_{\sigma'}(k'), a_\sigma^\dagger(k)\} = 2k^0\delta^3(\mathbf{k}-\mathbf{k}')\delta_{\sigma\sigma'}, \tag{4}$$

$$\{b_{\sigma'}(k'), b_\sigma^\dagger(k)\} = 2k^0\delta^3(\mathbf{k}-\mathbf{k}')\delta_{\sigma\sigma'} \tag{5}$$

and all other anticommutators vanish. The one-particle and antiparticle states with momentum k and spin component σ are defined as

$$|k,\sigma\rangle_a \equiv a_\sigma^\dagger(k)|0\rangle, \quad |k,\sigma\rangle_b \equiv b_\sigma^\dagger(k)|0\rangle, \tag{6}$$

respectively. Here $|0\rangle$ denotes Lorentz-invariant vacuum, $\langle 0|0\rangle=1$, $a_\sigma(k)|0\rangle=0$. The states (6) span the carrier space of the irreducible unitary representation $U(\Lambda)$ of the Poincaré group:

$$U(\Lambda)|k,\sigma\rangle_{a/b} = \mathcal{D}(R(\Lambda,k))_{\lambda\sigma}|\Lambda k,\lambda\rangle_{a/b}, \tag{7}$$

where $\mathcal{D}\in \mathrm{SU}(2)$ is the matrix spin-1/2 representation of the SO(3) group, $R(\Lambda,k)=L_{\Lambda k}^{-1}\Lambda L_k$ is the Wigner rotation, and L_k denotes the standard Lorentz boost defined by the relations $L_k\tilde{k}=k$, $L_{\tilde{k}}=I$, and $\tilde{k}=(m,\mathbf{0})$. As follows from Eqs. (4) and (5), states (6) are normalized covariantly:

$${}_{a/b}\langle k,\sigma|k',\sigma'\rangle_{a/b} = 2k^0\delta^3(\mathbf{k}-\mathbf{k}')\delta_{\sigma\sigma'}. \tag{8}$$

Equations (2), (6), and (7) imply standard consistency (Weinberg) conditions for the amplitudes:

$$v(\Lambda k) = \mathsf{D}(\Lambda)v(k)\mathcal{D}^T(R(\Lambda,k)), \tag{9}$$

$$u(\Lambda k) = \mathsf{D}(\Lambda)u(k)\mathcal{D}^\dagger(R(\Lambda,k)), \tag{10}$$

where $v(k)$ and $u(k)$ denote matrices $[v_{\mu\alpha}(k)]$ and $[u_{\mu\alpha}(k)]$, respectively. The explicit form of the amplitudes depends on the chosen representation of the γ matrices. Under the choice given in Appendix A the amplitudes can be written as

$$u(k) = i\mathsf{v}(k)\sigma_2, \tag{11}$$

$$v(k) = \gamma^5\mathsf{v}(k), \tag{12}$$

where, for the sake of convenience, we have introduced the matrix

$$\mathsf{v}(k) = \frac{1}{2\sqrt{1+\frac{k^0}{m}}}\begin{pmatrix}\left(I+\frac{1}{m}k^\mu\sigma_\mu\right)\sigma_2\\ \left(I+\frac{1}{m}k^{\pi\mu}\sigma_\mu\right)\sigma_2\end{pmatrix}, \tag{13}$$

with $k^\pi=(k^0,-\mathbf{k})$, and σ_i, $i=1,2,3$, designate the standard Pauli matrices, $\sigma_0=I$.[2]

The action of the charge conjugation C and space inversion P on the Dirac field has the form

$$\mathsf{C}\hat{\Psi}(x)\mathsf{C}^\dagger = \eta_C\mathcal{C}\hat{\bar{\Psi}}^T(x), \tag{14}$$

$$\mathsf{P}\hat{\Psi}(x)\mathsf{P}^\dagger = \eta_P\gamma^0\hat{\Psi}(x^\pi), \tag{15}$$

where $\hat{\bar{\Psi}}=\hat{\Psi}^\dagger\gamma^0$, $|\eta_P|=|\eta_C|=1$, and $\mathcal{C}$ is the charge conjugation matrix (A2).

The operators P and C act on the creation operators as follows:

$$\mathsf{P}a_\sigma^\dagger(k)\mathsf{P}^\dagger = \eta_P^* a_\sigma^\dagger(k^\pi), \quad \mathsf{P}b_\sigma^\dagger(k)\mathsf{P}^\dagger = -\eta_P b_\sigma^\dagger(k^\pi), \tag{16}$$

$$\mathsf{C}a_\sigma(k)\mathsf{C}^\dagger = \eta_C b_\sigma(k), \quad \mathsf{C}b_\sigma^\dagger(k)\mathsf{C}^\dagger = \eta_C a_\sigma^\dagger(k). \tag{17}$$

It is convenient to introduce covariant one-particle and antiparticle states

$$|\alpha,k\rangle_{a/b} = \mathsf{v}_{\alpha\sigma}(k)|k,\sigma\rangle_{a/b}. \tag{18}$$

Such states have the transformation properties

$$U(\Lambda)|\alpha,k\rangle_{a/b} = \mathsf{D}(\Lambda^{-1})_{\alpha\beta}|\beta,k\rangle_{a/b}, \tag{19}$$

implied by Eqs. (7) and (C6) and

$$\mathsf{P}|\alpha,k\rangle_a = \eta_P^*\gamma^0_{\alpha\beta}|\beta,k^\pi\rangle_a, \quad \mathsf{C}|\alpha,k\rangle_a = \eta_C^*|\alpha,k\rangle_b, \tag{20}$$

$$\mathsf{P}|\alpha,k\rangle_b = -\eta_P\gamma^0_{\alpha\beta}|\beta,k^\pi\rangle_b, \quad \mathsf{C}|\alpha,k\rangle_b = \eta_C|\alpha,k\rangle_a. \tag{21}$$

III. TWO-PARTICLE STATES

The main goal of our paper is to calculate the spin correlation function of two Dirac particles. We calculate correlations in the states consisting of one particle and one antiparticle. The space of such two-particle states is spanned by the vectors

$$|(k,\sigma)_b,(p,\lambda)_a\rangle \equiv b_\sigma^\dagger(k)a_\lambda^\dagger(p)|0\rangle. \tag{22}$$

To analyze the transformation properties of the states it is convenient to introduce the covariant basis, analogous to Eq. (18), defined by

$$|(\alpha,k),(\beta,p)\rangle = \mathsf{v}(k)_{\alpha\sigma}\mathsf{v}(p)_{\beta\lambda}|(k,\sigma)_b,(p,\lambda)_a\rangle. \tag{23}$$

Notice that for simplicity we have omitted the indices a and b in the state vector on the left-hand side of Eq. (23).

[2]Note that in our paper [12] the matrix $\mathsf{v}(k)$ was denoted by $v(k)$. In the present paper we use sans serif font, since we want to preserve the standard notation for amplitudes in Dirac field expansion (3).

Hereafter we use the convention that in the two-particle state vector the left pair of indexes refers to antiparticle, the right pair to particle, respectively. Using Eq. (C6) we have

$$U(\Lambda)|(\alpha,k),(\beta,p)\rangle = \mathsf{D}(\Lambda^{-1})_{\alpha\alpha'}\mathsf{D}(\Lambda^{-1})_{\beta\beta'}|(\alpha',\Lambda k),(\beta',\Lambda p)\rangle. \tag{24}$$

Moreover, we can easily determine the action of the discrete operations P and C on the states (23):

$$\mathsf{P}|(\alpha,k),(\beta,p)\rangle = -\gamma^0_{\alpha\alpha'}\gamma^0_{\beta\beta'}|(\alpha',k^\pi),(\beta',p^\pi)\rangle, \tag{25}$$

$$\mathsf{C}|(\alpha,k),(\beta,p)\rangle = -|(\beta,p),(\alpha,k)\rangle. \tag{26}$$

Particle-antiparticle pairs are usually created in a state in which the total four-momentum is determined such as, for example, in the decay $\pi^0\to e^-e^+$ where the total four-momentum of the electron-positron pair is equal to the four-momentum of the decaying π^0. (This decay channel has very small but nonzero width [45].) The most general particle-antiparticle state with total four-momentum q has the form

$$|\varphi\rangle_q = \int \frac{d^3\mathbf{k}}{2k^0}\frac{d^3\mathbf{p}}{2p^0}\delta^4(q-(k+p)) \times \left[\sum_A \varphi^A(k,p)\mathcal{C}\Gamma_A\right]_{\alpha\beta}|(\alpha,k),(\beta,p)\rangle, \tag{27}$$

where $\varphi^A(k,p)$ are numerical functions, $\mathcal{C}$ is given by Eq. (A2), and Γ_A forms a subset of matrices

$$I,\gamma^5,\gamma^\mu,\gamma^\mu\gamma^5,[\gamma^\mu,\gamma^\nu], \tag{28}$$

which transform covariantly under Lorentz transformations and form a basis of the Clifford algebra generated by Dirac γ matrices. The matrices Γ_A transform with respect to the index A according to a certain representation $\mathbb{D}(\Lambda)$ of the Lorentz group:

$$\mathsf{D}(\Lambda)\Gamma_A\mathsf{D}(\Lambda^{-1}) = \mathbb{D}_{AB}(\Lambda^{-1})\Gamma_B. \tag{29}$$

Note that we have inserted the matrix $\mathcal{C}$ into Eq. (27) because from Eq. (A3) we have $\mathsf{D}^T(\Lambda)\mathcal{C}=\mathcal{C}\mathsf{D}^{-1}(\Lambda)$.

Identification of the singlet or vector state is based on the transformation properties of the state (27) under Lorentz transformations and parity P. Using Eq. (24) we have

$$U(\Lambda)|\varphi\rangle_q = \int \frac{d^3\mathbf{k}}{2k^0}\frac{d^3\mathbf{p}}{2p^0}\delta^4(\Lambda q-(k+p)) \times\left[\sum_A \varphi'^A(k,p)\mathcal{C}\Gamma_A\right]_{\alpha\beta}|(\alpha,k),(\beta,p)\rangle, \tag{30}$$

where from Eq. (29) we obtain

$$\varphi'^A(k,p) = \mathbb{D}_{BA}(\Lambda^{-1})\varphi^B(\Lambda^{-1}k,\Lambda^{-1}p). \tag{31}$$

Under parity the elements of the Clifford algebra transform as follows:

$$\gamma^0\Gamma_A\gamma^0 = \mathbb{P}_{AB}\Gamma_B, \tag{32}$$

where $\mathbb{P}$ represents parity in the carrier space of the representation $\mathbb{D}$; namely, the state transforming according to Eqs. (29) and (32) will be called: scalar if $\mathbb{D}(\Lambda)=I$, $\mathbb{P}=I$, $\{\Gamma_A\}=\{I\}$; pseudoscalar if $\mathbb{D}(\Lambda)=I$, $\mathbb{P}=-I$, $\{\Gamma_A\}=\{\gamma^5\}$; four-vector if $\mathbb{D}(\Lambda)=\Lambda$, $\mathbb{P}=\eta=\text{diag}(1,-1,-1,-1)$, $\{\Gamma_A\}=\{\gamma^\mu\}$; and pseudo-four-vector if $\mathbb{D}(\Lambda)=\Lambda$, $\mathbb{P}=-\eta$, $\{\Gamma_A\}=\{\gamma^\mu\gamma^5\}$.

In this paper we restrict ourselves only to the pseudoscalar and four-vector case since such states can be identified with real particles decaying into an e^+e^- pair (compare, for example, π^0 and Z^0 and their corresponding decay channels [45]).

A. Pseudoscalar

The general pseudoscalar state reads

$$\begin{aligned}|\varphi\rangle_q^{\text{ps}} &= \int \frac{d^3\mathbf{k}}{2k^0}\frac{d^3\mathbf{p}}{2p^0}\delta^4(q-k-p)\varphi(k,p) \times (\mathcal{C}\gamma^5)_{\alpha\beta}|(\alpha,k),(\beta,p)\rangle \\ &= \int \frac{d^3\mathbf{k}}{2k^0}\frac{d^3\mathbf{p}}{2p^0}\delta^4(q-k-p)\varphi(k,p) \times (\mathsf{v}^T(k)\mathcal{C}\gamma^5\mathsf{v}(p))_{\sigma\lambda}|(k,\sigma),(p,\lambda)\rangle,\end{aligned} \tag{33}$$

where, according to Eq. (31), the density function transforms under Lorentz transformations as follows:

$$\varphi'(\Lambda k,\Lambda p) = \varphi(k,p). \tag{34}$$

B. Four-vector

The most general four-vector state has the following form:

$$\begin{aligned}|\varphi\rangle_q^{\text{vec}} &= \int \frac{d^3\mathbf{k}}{2k^0}\frac{d^3\mathbf{p}}{2p^0}\delta^4(q-k-p)\varphi^\mu(k,p) \times (\mathcal{C}\gamma_\mu)_{\alpha\beta}|(\alpha,k),(\beta,p)\rangle \\ &= \int \frac{d^3\mathbf{k}}{2k^0}\frac{d^3\mathbf{p}}{2p^0}\delta^4(q-k-p)\varphi^\mu(k,p) \times (\mathsf{v}^T(k)\mathcal{C}\gamma_\mu\mathsf{v}(p))_{\sigma\lambda}|(k,\sigma),(p,\lambda)\rangle.\end{aligned} \tag{35}$$

One can easily check that

$$(p_\mu+k_\mu)\mathsf{v}^T(k)\mathcal{C}\gamma^\mu\mathsf{v}(p) = 0. \tag{36}$$

Therefore, in the integral (35) the nonzero contribution has only such $\varphi^\mu(k,p)$, which fulfills the transversality condition

$$q_\mu\varphi^\mu(k,p) = (p_\mu+k_\mu)\varphi^\mu(k,p) = 0. \tag{37}$$

Notice that in the decay cases the functions $\varphi(k,p)$ and $\varphi^\mu(k,p)$ are related to the dynamics of the decay.

IV. RELATIVISTIC SPIN OPERATOR

To calculate the correlation function we have to introduce the spin operator for a relativistic massive particle. In the discussion of the relativistic EPR experiment various authors use different spin operators [4,8,22,23,33,35,36]. However, it seems [12,37,46] that the best candidate for the relativistic spin operator is

$$\hat{\mathbf{S}} = \frac{1}{m}\left(\hat{\mathbf{W}} - \hat{W}^0 \frac{\hat{\mathbf{P}}}{\hat{P}^0 + m}\right), \tag{38}$$

where $\hat{W}^\mu$ is the Pauli-Lubanski four-vector:

$$\hat{W}^\mu = \tfrac{1}{2}\varepsilon^{\mu\nu\sigma\lambda}\hat{P}_\nu \hat{J}_{\sigma\lambda}. \tag{39}$$

Here $\hat{P}_\nu$ is a four-momentum operator and $\hat{J}_{\sigma\lambda}$ denotes generators of the Lorentz group—i.e., $U(\Lambda)=\exp(i\omega^{\mu\nu}\hat{J}_{\mu\nu})$. One can show [46] that the operator (38) is the only operator which is a linear function of W^μ and fulfills the relations

$$[\hat{J}^i, \hat{S}^j] = i\varepsilon_{ijk}\hat{S}^k, \tag{40a}$$

$$[\hat{S}^i, \hat{S}^j] = i\varepsilon_{ijk}\hat{S}^k, \tag{40b}$$

$$[\hat{P}^\mu, \hat{S}^j] = 0, \tag{40c}$$

and is a pseudovector—i.e., $\mathsf{P}\hat{S}^i\mathsf{P}=\hat{S}^i$. Here $\hat{J}^i=\frac{1}{2}\varepsilon_{ijk}\hat{J}^{jk}$.

In the representation (A1) of γ matrices we have

$$\hat{W}^0|(\alpha,k)\rangle = W^0_{\alpha\beta}|(\beta,k)\rangle, \tag{41}$$

$$\hat{\mathbf{W}}|(\alpha,k)\rangle = \mathbf{W}_{\alpha\beta}|(\beta,k)\rangle, \tag{42}$$

where

$$W^0 = -\frac{1}{2}\begin{pmatrix} \mathbf{k}\cdot\boldsymbol{\sigma} & 0 \\ 0 & \mathbf{k}\cdot\boldsymbol{\sigma}\end{pmatrix}, \tag{43}$$

$$\mathbf{W} = -\frac{1}{2}\left[k^0\boldsymbol{\sigma}\begin{pmatrix} I & 0 \\ 0 & I\end{pmatrix} - i(\mathbf{k}\times\boldsymbol{\sigma})\begin{pmatrix} I & 0 \\ 0 & -I\end{pmatrix}\right]. \tag{44}$$

Therefore for the spin operator we easily find

$$\hat{\mathbf{S}}|k,\sigma\rangle = \frac{1}{2}\boldsymbol{\sigma}_{\lambda\sigma}|k,\lambda\rangle, \tag{45}$$

$$\hat{\mathbf{S}}|\alpha,k\rangle = \left(\mathsf{v}(k)\frac{\boldsymbol{\sigma}^T}{2}\bar{\mathsf{v}}(k)\right)_{\alpha\beta}|\beta,k\rangle. \tag{46}$$

Real detectors register only particles the momenta of which belong to some definite region in momentum space. Therefore, taking into account Eqs. (40c) and (45), in one-particle subspace of Fock space, the operator measuring spin of the particle with four-momentum from the region $\mathcal{A}$ of momentum space has the form

$$\hat{\mathbf{S}}^a_{\mathcal{A}} = \int_{\mathcal{A}} \frac{d^3\mathbf{k}}{2k^0} a^\dagger_\sigma(k)\frac{\boldsymbol{\sigma}_{\sigma\lambda}}{2}a_\lambda(k). \tag{47}$$

This operator gives zero when acting on the antiparticle state or state of the particle with four-momentum outside the region $\mathcal{A}$. An analogous operator measuring spin of the antiparticle with four-momentum belonging to the region $\mathcal{B}$ of the momentum space can be written as

$$\hat{\mathbf{S}}^b_{\mathcal{B}} = \int_{\mathcal{B}} \frac{d^3\mathbf{k}}{2k^0} b^\dagger_\sigma(k)\frac{\boldsymbol{\sigma}_{\sigma\lambda}}{2}b_\lambda(k). \tag{48}$$

We have

$$[\hat{S}^{a,i}_{\mathcal{A}}, \hat{S}^{b,j}_{\mathcal{B}}] = 0. \tag{49}$$

V. CORRELATION FUNCTIONS

In this section we calculate correlation function in the EPR-Bohm-type experiment. In such an experiment we have two distant observers—say, Alice and Bob. We assume that both observers are at rest with respect to the same inertial reference frame. A particle-anti particle pair is produced in an entangled state $|\varphi\rangle$; the particle is registered by Alice while the antiparticle by Bob. Alice measures the spin component of the particle along direction $\mathbf{a}$, Bob the spin component of the antiparticle along direction $\mathbf{b}$. Therefore Alice uses the observable $\mathbf{a}\cdot\hat{S}^a_{\mathcal{A}}$ and Bob the observable $\mathbf{b}\cdot\hat{S}^b_{\mathcal{B}}$ [see Eqs. (47) and (48)]. So by virtue of Eq. (49), the normalized correlation function in the state $|\varphi\rangle$ has the following form:

$$C^{\mathcal{AB}}_\varphi(\mathbf{a},\mathbf{b}) = 4\frac{\langle\varphi|(\mathbf{a}\cdot\hat{\mathbf{S}}^a_{\mathcal{A}})(\mathbf{b}\cdot\hat{\mathbf{S}}^b_{\mathcal{B}})|\varphi\rangle}{\langle\varphi|\varphi\rangle}. \tag{50}$$

We calculate correlation function in two important cases when an EPR pair is produced in the pseudoscalar or four-vector state.

A. Pseudoscalar state

The pseudoscalar state is given by Eq. (33). Therefore from Eq. (50) we find

$$\begin{aligned} C^{\mathcal{AB}}_{\mathsf{ps}}(\mathbf{a},\mathbf{b}) = &\left\{\int \frac{d^3\mathbf{k}}{2k^0}\frac{d^3\mathbf{p}}{2p^0}\chi_{\mathcal{A}}(p)\chi_{\mathcal{B}}(k)[\delta^4(q-k-p)]^2|\varphi(k,p)|^2\right. \\ &\left.\times \mathrm{Tr}\{\mathbf{b}\cdot\boldsymbol{\sigma}[\mathsf{v}^T(k)\mathcal{C}\gamma^5\mathsf{v}(p)]\mathbf{a}\cdot\boldsymbol{\sigma}^T[\mathsf{v}^T(k)\mathcal{C}\gamma^5\mathsf{v}(p)]^\dagger\}\right\} \\ &\times\left\{\int \frac{d^3\mathbf{k}}{2k^0}\frac{d^3\mathbf{p}}{2p^0}\chi_{\mathcal{A}}(p)\chi_{\mathcal{B}}(k)[\delta^4(q-k-p)]^2|\varphi(k,p)|^2\right. \\ &\left.\times \mathrm{Tr}\{[\mathsf{v}^T(k)\mathcal{C}\gamma^5\mathsf{v}(p)][\mathsf{v}^T(k)\mathcal{C}\gamma^5\mathsf{v}(p)]^\dagger\}\right\}^{-1}, \end{aligned} \tag{51}$$

where $\chi_{\mathcal{A}}(p)$ and $\chi_{\mathcal{B}}(k)$ are characteristic functions of regions $\mathcal{A}$ and $\mathcal{B}$ in the corresponding momentum spaces. Using Eqs. (13), (A2), and (A1) we arrive after little algebra at

$$\mathrm{Tr}\{\mathbf{b}\cdot\boldsymbol{\sigma}[\mathsf{v}^T(k)\mathcal{C}\gamma^5\mathsf{v}(p)]\mathbf{a}\cdot\boldsymbol{\sigma}^T[\mathsf{v}^T(k)\mathcal{C}\gamma^5\mathsf{v}(p)]^\dagger\} = \frac{-1}{m^2}\Bigg\{\mathbf{a}\cdot\mathbf{b}(m^2+kp) - (\mathbf{k}\times\mathbf{p})\Bigg[\mathbf{a}\times\mathbf{b} + \frac{(\mathbf{b}\cdot\mathbf{p})(\mathbf{a}\times\mathbf{k}) - (\mathbf{a}\cdot\mathbf{k})(\mathbf{b}\times\mathbf{p})}{(m+k^0)(m+p^0)}\Bigg]\Bigg\}, \quad (52)$$

$$\mathrm{Tr}\{[\mathsf{v}^T(k)\mathcal{C}\gamma^5\mathsf{v}(p)][\mathsf{v}^T(k)\mathcal{C}\gamma^5\mathsf{v}(p)]^\dagger\} = \frac{m^2+kp}{m^2}. \quad (53)$$

Therefore in the simple situation when momenta of both particles in the state $|\varphi\rangle_q^{\mathrm{ps}}$ are sharp [i.e., the characteristic functions in Eq. (51) are replaced by δ functions] we obtain the following correlation function:

$$C_{\mathrm{ps}}^{pk}(\mathbf{a},\mathbf{b}) = -\mathbf{a}\cdot\mathbf{b} + \frac{\mathbf{k}\times\mathbf{p}}{m^2+kp}\Bigg[\mathbf{a}\times\mathbf{b} + \frac{(\mathbf{b}\cdot\mathbf{p})(\mathbf{a}\times\mathbf{k}) - (\mathbf{a}\cdot\mathbf{k})(\mathbf{b}\times\mathbf{p})}{(m+k^0)(m+p^0)}\Bigg]. \quad (54)$$

In this special case of sharp momenta we obtained the same correlation function (54) in our previous paper [12] where we discussed the Lorentz-covariant spin density matrix in the framework of relativistic quantum mechanics. Notice that in the case when the laboratory frame (observers) coincides with the center-of-mass frame, or even when $\mathbf{p}\|\mathbf{k}$, from Eq. (54) we get the same correlation function as for the singlet in the nonrelativistic case:

$$C_{\mathrm{ps}}^{\mathrm{CMF}}(\mathbf{a},\mathbf{b}) = -\mathbf{a}\cdot\mathbf{b} = C_{\mathrm{ps}}^{\mathbf{k}\|\mathbf{p}}(\mathbf{a},\mathbf{b}). \quad (55)$$

It is interesting that correlation function calculated by Czachor in [5], in the center-of-mass frame, still depends on momentum and has the following form:

$$C_{\mathrm{Czachor}}^{\mathrm{CMF}}(\mathbf{a},\mathbf{b}) = -\frac{\mathbf{a}\cdot\mathbf{b} - \beta^2\mathbf{a}_\perp\cdot\mathbf{b}_\perp}{\sqrt{1+\beta^2[(\mathbf{n}\cdot\mathbf{a})^2-1]}\sqrt{1+\beta^2[(\mathbf{n}\cdot\mathbf{b})^2-1]}}, \quad (56)$$

where $\mathbf{n}=\mathbf{k}/|\mathbf{k}|$, $\beta=|\mathbf{k}|/k^0$, $\mathbf{a}_\perp=\mathbf{a}-(\mathbf{n}\cdot\mathbf{a})\mathbf{n}$, and $\mathbf{b}_\perp=\mathbf{b}-(\mathbf{n}\cdot\mathbf{b})\mathbf{n}$.

Formulas (55) and (56) are different because Czachor uses a different spin operator. Therefore experimental measurement of the correlation function in the center-of-mass frame could show which spin operator is more adequate in relativistic quantum mechanics. One of the possible sources of the electron-positron pairs is the π^0 decay into the channel $\pi^0\to e^+e^-$. Since $m_{\pi^0}=134.98\ \mathrm{MeV}/c^2$ and $m_e=0.51\ \mathrm{MeV}/c^2$ [45], electrons and positrons produced in this decay in the center-of-mass frame are ultrarelativistic. Therefore let us find the limit $\beta\to 1$ of the formulas (55) and (56). We get

$$C_{\mathrm{ps}}^{\mathrm{CMF}}(\mathbf{a},\mathbf{b})|_{\beta\to 1} = -\mathbf{a}\cdot\mathbf{b}, \quad (57)$$

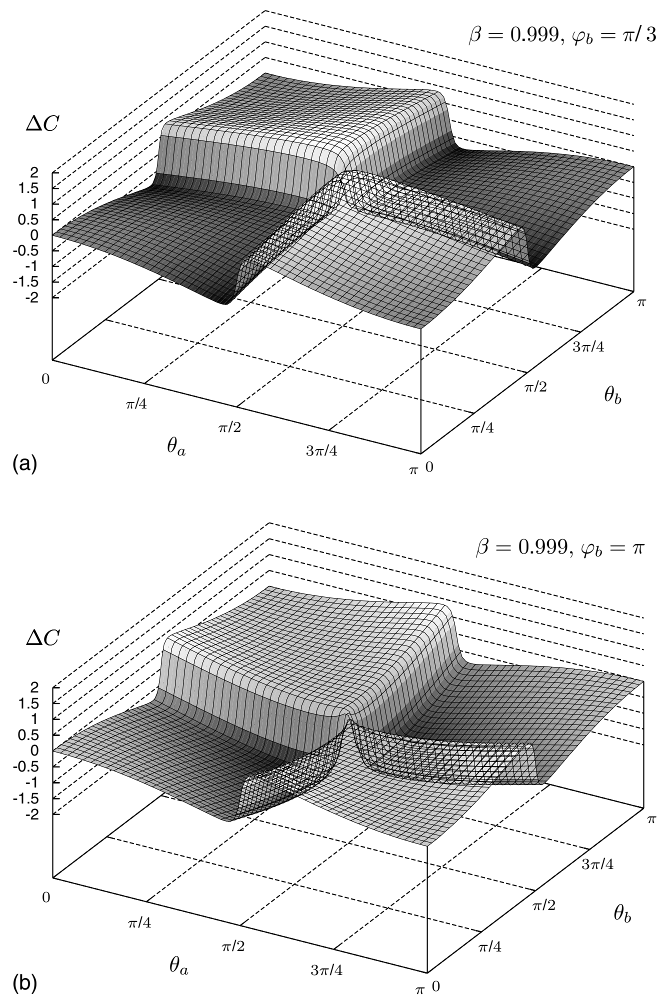

FIG. 1. Function ΔC [Eq. (59)] in the parametrization $\mathbf{n}=(0,0,1)$, $\mathbf{a}=(\sin\theta_a,0,\cos\theta_a)$, $\mathbf{b}=(\cos\varphi_b\sin\theta_b,\sin\varphi_b\sin\theta_b,\cos\theta_b)$. We have plotted the graph of the function ΔC for two arbitrarily chosen values of φ_b and for $\beta=0.999$, which is the value for an electron-positron pair created in the π^0 decay.

$$C_{\mathrm{Czachor}}^{\mathrm{CMF}}(\mathbf{a},\mathbf{b})|_{\beta\to 1} = -\frac{(\mathbf{n}\cdot\mathbf{a})(\mathbf{n}\cdot\mathbf{b})}{|(\mathbf{n}\cdot\mathbf{a})(\mathbf{n}\cdot\mathbf{b})|} = \pm 1, \quad (58)$$

respectively. We point out the discontinuity in Czachor's correlation function in the ultrarelativistic limit (58). It is also interesting to notice that the function

$$\Delta C = C_{\mathrm{Czachor}}^{\mathrm{CMF}}(\mathbf{a},\mathbf{b}) - C_{\mathrm{ps}}^{\mathrm{CMF}}(\mathbf{a},\mathbf{b}) \quad (59)$$

can take quite large values for β close to 1. We show this function in Fig. 1 for the value $\beta=0.999$ which corresponds to e^+e^- created in the π^0 decay at rest.

B. Vector state

The general vector state is given by Eq. (35). So in this case from Eq. (50) we get

$$C_{\mathrm{vec}}^{\mathcal{AB}}(\mathbf{a},\mathbf{b}) = \left\{ \int \frac{d^3\mathbf{k}}{2k^0}\frac{d^3\mathbf{p}}{2p^0}\chi_{\mathcal{A}}(p)\chi_{\mathcal{B}}(k)[\delta^4(q-k-p)]^2\varphi_\mu(k,p) \times \varphi_\nu^*(k,p)\mathrm{Tr}\{\mathbf{b}\cdot\boldsymbol{\sigma}(\mathsf{v}^T(k)\mathcal{C}\gamma^\mu\mathsf{v}(p)]\mathbf{a}\cdot\boldsymbol{\sigma}^T[\mathsf{v}^T(k)\mathcal{C}\gamma^\nu\mathsf{v}(p)]^\dagger\} \right\} \times \left\{ \int \frac{d^3\mathbf{k}}{2k^0}\frac{d^3\mathbf{p}}{2p^0}\chi_{\mathcal{A}}(p)\chi_{\mathcal{B}}(k)[\delta^4(q-k-p)]^2\varphi_\mu(k,p)\varphi_\nu^*(k,p)\mathrm{Tr}\{[\mathsf{v}^T(k)\mathcal{C}\gamma^\mu\mathsf{v}(p)][\mathsf{v}^T(k)\mathcal{C}\gamma^\nu\mathsf{v}(p)]^\dagger\} \right\}^{-1}, \qquad (60)$$

where, as before, $\chi_{\mathcal{A}}(p)$ and $\chi_{\mathcal{B}}(k)$ are characteristic functions of regions $\mathcal{A}$ and $\mathcal{B}$ in momentum space. If the observer's frame coincides with the center-of-mass reference frame [in which $p=k^\pi=(k^0,-\mathbf{k})$], then by means of the transversality condition (37), we have

$$\varphi_\mu(k,k^\pi)\varphi_\nu^*(k,k^\pi)\mathrm{Tr}\{\mathbf{b}\cdot\boldsymbol{\sigma}[\mathsf{v}^T(k)\mathcal{C}\gamma^\mu\mathsf{v}(k^\pi)]\mathbf{a}\cdot\boldsymbol{\sigma}^T[\mathsf{v}^T(k)\mathcal{C}\gamma^\nu\mathsf{v}(k^\pi)]^\dagger\} = \frac{2}{m^2}\Bigg\{(\mathbf{a}\cdot\mathbf{b})[k^{0^2}|\boldsymbol{\varphi}|^2-|\mathbf{k}\cdot\boldsymbol{\varphi}|^2] - k^{0^2}[(\mathbf{a}\cdot\boldsymbol{\varphi}^*)(\mathbf{b}\cdot\boldsymbol{\varphi})+(\mathbf{a}\cdot\boldsymbol{\varphi})(\mathbf{b}\cdot\boldsymbol{\varphi}^*)] - \frac{2(\mathbf{a}\cdot\mathbf{k})(\mathbf{b}\cdot\mathbf{k})|\mathbf{k}\cdot\boldsymbol{\varphi}|^2}{(m+k^0)^2} + \frac{k^0}{m+k^0}\{(\mathbf{a}\cdot\mathbf{k})[(\mathbf{b}\cdot\boldsymbol{\varphi})(\boldsymbol{\varphi}^*\cdot\mathbf{k}) + (\mathbf{b}\cdot\boldsymbol{\varphi}^*)(\boldsymbol{\varphi}\cdot\mathbf{k})] + (\mathbf{b}\cdot\mathbf{k})[(\mathbf{a}\cdot\boldsymbol{\varphi})(\boldsymbol{\varphi}^*\cdot\mathbf{k}) + (\mathbf{a}\cdot\boldsymbol{\varphi}^*)(\boldsymbol{\varphi}\cdot\mathbf{k})]\}\Bigg\}, \qquad (61)$$

$$\varphi_\mu(k,k^\pi)\varphi_\nu^*(k,k^\pi)\mathrm{Tr}\{[\mathsf{v}^T(k)\mathcal{C}\gamma^\mu\mathsf{v}(k^\pi)][\mathsf{v}^T(k)\mathcal{C}\gamma^\nu\mathsf{v}(k^\pi)]^\dagger\} = \frac{2}{m^2}[k^{0^2}|\boldsymbol{\varphi}|^2-|\mathbf{k}\cdot\boldsymbol{\varphi}|^2]. \qquad (62)$$

Therefore, if we assume that the momenta of the particles in the state $|\varphi\rangle_q^{\mathrm{vec}}$ are sharp, the correlation function in the center-of-mass frame is given by (the antiparticle has momentum k, particle k^π)

$$C_{\mathrm{vec}}^{\mathrm{CMF}}(\mathbf{a},\mathbf{b}) = \mathbf{a}\cdot\mathbf{b} - \frac{k^{0^2}}{k^{0^2}|\varphi|^2-|\mathbf{k}\cdot\varphi|^2}[(\mathbf{a}\cdot\boldsymbol{\varphi}^*)(\mathbf{b}\cdot\boldsymbol{\varphi})+(\mathbf{a}\cdot\boldsymbol{\varphi})(\mathbf{b}\cdot\boldsymbol{\varphi}^*)] - \frac{2(\mathbf{a}\cdot\mathbf{k})(\mathbf{b}\cdot\mathbf{k})|\mathbf{k}\cdot\boldsymbol{\varphi}|^2}{(m+k^0)^2(k^{0^2}|\varphi|^2-|\mathbf{k}\cdot\varphi|^2)} + \frac{k^0\{(\mathbf{a}\cdot\mathbf{k})[(\mathbf{b}\cdot\boldsymbol{\varphi})(\mathbf{k}\cdot\boldsymbol{\varphi}^*)+(\mathbf{b}\cdot\boldsymbol{\varphi}^*)(\mathbf{k}\cdot\boldsymbol{\varphi})]+(\mathbf{b}\cdot\mathbf{k})[(\mathbf{a}\cdot\boldsymbol{\varphi})(\mathbf{k}\cdot\boldsymbol{\varphi}^*)+(\mathbf{a}\cdot\boldsymbol{\varphi}^*)(\mathbf{k}\cdot\boldsymbol{\varphi})]\}}{(m+k^0)(k^{0^2}|\boldsymbol{\varphi}|^2-|\mathbf{k}\cdot\boldsymbol{\varphi}|^2)}. \qquad (63)$$

Notice that in this case, in opposition to the one considered previously, in the center-of-mass frame the correlation function depends explicitly on the momentum. Moreover, if $\mathbf{k}\perp\boldsymbol{\varphi}$ from Eq. (63), we get the same result as for the nonrelativistic triplet state (D5). Also in the nonrelativistic limit ($k^{0^2}\gg|\mathbf{k}|^2$) from Eq. (63) we get the correlation function for the nonrelativistic triplet state (D5).

When we consider electron-positron pairs produced in Z^0 decay two remarks are in order. First, in some experiments it is possible to produce polarized Z^0 (SLAC, Stanford [47]), so in such experiments the vector $\boldsymbol{\varphi}$ is determined. Second, electrons and positrons produced in the channel $Z^0\to e^+e^-$ are highly ultrarelativistic. In the limit $\beta\to 1$ the formula (63) takes the form

$$C_{\mathrm{vec}}^{\mathrm{CMF}}(\mathbf{a},\mathbf{b})|_{\beta\to 1} = \mathbf{a}\cdot\mathbf{b} - \frac{1}{|\boldsymbol{\varphi}|^2-|\mathbf{n}\cdot\boldsymbol{\varphi}|^2}\{(\mathbf{a}\cdot\boldsymbol{\varphi}^*)(\mathbf{b}\cdot\boldsymbol{\varphi}) + (\mathbf{a}\cdot\boldsymbol{\varphi})(\mathbf{b}\cdot\boldsymbol{\varphi}^*) + 2(\mathbf{a}\cdot\mathbf{n})(\mathbf{b}\cdot\mathbf{n})|\mathbf{n}\cdot\boldsymbol{\varphi}|^2 - (\mathbf{a}\cdot\mathbf{n})[(\mathbf{b}\cdot\boldsymbol{\varphi})(\mathbf{n}\cdot\boldsymbol{\varphi}^*)+(\mathbf{b}\cdot\boldsymbol{\varphi}^*)(\mathbf{n}\cdot\boldsymbol{\varphi})] - (\mathbf{b}\cdot\mathbf{n})[(\mathbf{a}\cdot\boldsymbol{\varphi})(\mathbf{n}\cdot\boldsymbol{\varphi}^*)+(\mathbf{a}\cdot\boldsymbol{\varphi}^*)(\mathbf{n}\cdot\boldsymbol{\varphi})]\}, \qquad (64)$$

where $\mathbf{n}=\mathbf{k}/|\mathbf{k}|$. Equation (64) takes a very simple form in the configuration $\mathbf{a}\perp\boldsymbol{\varphi}$, $\mathbf{b}\perp\boldsymbol{\varphi}$:

$$C_{\mathrm{vec}}^{\mathrm{CMF}}(\mathbf{a}\perp\boldsymbol{\varphi},\mathbf{b}\perp\boldsymbol{\varphi})|_{\beta\to 1} = \mathbf{a}\cdot\mathbf{b} - \frac{2(\mathbf{a}\cdot\mathbf{n})(\mathbf{b}\cdot\mathbf{n})|\mathbf{n}\cdot\boldsymbol{\varphi}|^2}{|\boldsymbol{\varphi}|^2-|\mathbf{n}\cdot\boldsymbol{\varphi}|^2}. \qquad (65)$$

It is interesting to notice that the function

$$\Delta C_{\mathrm{vec}} = C_{\mathrm{vec}}^{\mathrm{CMF}}(\mathbf{a}\perp\boldsymbol{\varphi},\mathbf{b}\perp\boldsymbol{\varphi})|_{\beta\to 1} - C_{\mathrm{nonrelativ}} = -\frac{2(\mathbf{a}\cdot\mathbf{n})(\mathbf{b}\cdot\mathbf{n})|\mathbf{n}\cdot\boldsymbol{\varphi}|^2}{|\boldsymbol{\varphi}|^2-|\mathbf{n}\cdot\boldsymbol{\varphi}|^2}, \qquad (66)$$

where $C_{\mathrm{nonrelativ}}$ denotes the correlation function for the nonrelativistic triplet state (D5), can take an arbitrary value from the interval $[-2,2]$. We show this function in Fig. 2.

VI. CONCLUSIONS

We have discussed the correlations of two relativistic massive particles in EPR-type experiments in the context of quantum field theory formalism. Choosing the most appropriate spin operator (38), we have calculated the correlation

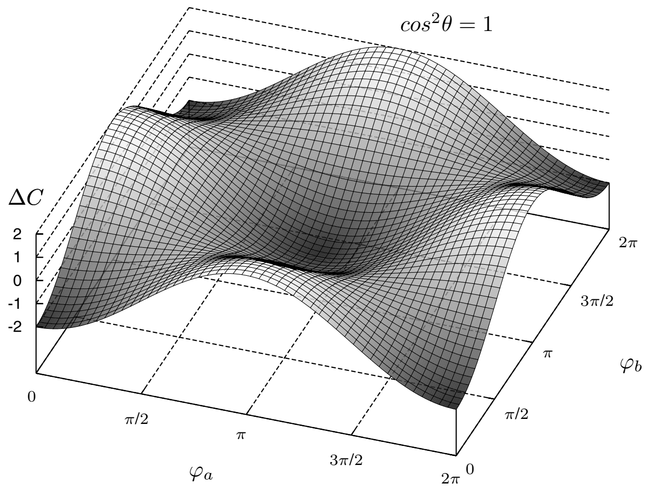

FIG. 2. Function ΔC [Eq. (66)] in the parametrization $\boldsymbol{\varphi}=(0,0,1)$, $\mathbf{n}=(\sin\theta,0,\cos\theta)$, $\mathbf{a}=(\cos\varphi_a,\sin\varphi_a,0)$, $\mathbf{b}=(\cos\varphi_b,\sin\varphi_b,0)$. In this parametrization $\Delta C=-2\cos\varphi_a\cos\varphi_b\cos^2\theta$. We have plotted the graph for the value θ such that $\cos^2\theta=1$. We see that ΔC is of the same order of magnitude as the correlation function.

function for the particle-antiparticle pair in the pseudoscalar and vector states. We have found general formulas and discussed them in some details for the sharp momentum states in the specific configurations. Relativistic correlations in the vector state have been never discussed before. It should also be noted that all these functions possess a proper nonrelativistic limit.

It is interesting that for the pseudoscalar (singlet) state the correlation function in the center-of-mass frame (CMF) is the same as in the nonrelativistic case. Therefore, in opposition to Czachor's results [4], the degree of violation of the Bell inequality for the pseudoscalar state in CMF is independent of the particle momentum. The correlation function for the vector state depends on the configuration and for same configurations gives the same result as for the nonrelativistic triplet state.

It seems that EPR-type experiments with relativistic elementary particles are feasible with the present technology [48].

ACKNOWLEDGMENTS

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APPENDIX A: DIRAC MATRICES

In this paper we use the following conventions. Dirac matrices fulfill the relation $\gamma^\mu\gamma^\nu+\gamma^\nu\gamma^\mu=2g^{\mu\nu}$ where the Minkowski metric tensor $g^{\mu\nu}=\text{diag}(1,-1,-1,-1)$; moreover, we adopt the convention $\varepsilon^{0123}=1$. We use the following explicit representation of gamma matrices:

$$\gamma^0=\begin{pmatrix}0 & I\\ I & 0\end{pmatrix},\quad \gamma=\begin{pmatrix}0 & -\boldsymbol{\sigma}\\ \boldsymbol{\sigma} & 0\end{pmatrix},\quad \gamma^5=\begin{pmatrix}I & 0\\ 0 & -I\end{pmatrix}, \tag{A1}$$

where $\boldsymbol{\sigma}=(\sigma_1,\sigma_2,\sigma_3)$ and σ_i are standard Pauli matrices. The charge conjugation matrix has the form

$$\mathcal{C}=-i\gamma^2\gamma^0=i\begin{pmatrix}\sigma_2 & 0\\ 0 & -\sigma_2\end{pmatrix}, \tag{A2}$$

so

$$\mathcal{C}\gamma^{T\mu}\mathcal{C}^{-1}=-\gamma^\mu. \tag{A3}$$

APPENDIX B: BISPINOR REPRESENTATION

By the bispinor representation of the Lorentz group we mean the representation $D^{(1/2,0)}\oplus D^{(0,1/2)}$. Explicitly, if $A \in \mathrm{SL}(2,\mathbb{C})$ and $\Lambda(A)$ is an image of A in the canonical homomorphism of the $\mathrm{SL}(2,\mathbb{C})$ group onto the Lorentz group, we take the chiral form of $D^{(1/2,0)}\oplus D^{(0,1/2)}$: namely,

$$\mathsf{D}(\Lambda(A))=\begin{pmatrix}A & 0\\ 0 & (A^\dagger)^{-1}\end{pmatrix}. \tag{B1}$$

The canonical homomorphism between the group $\mathrm{SL}(2,\mathbb{C})$ (universal covering of the proper ortochronous Lorentz group $L_+^\uparrow$) and the Lorentz group $L_+^\uparrow \sim \mathrm{SO}(1,3)_0$ [49] is defined as follows: With every four-vector k^μ we associate a two-dimensional Hermitian matrix $k^\mu\sigma_\mu$, where σ_i, $i=1,2,3$, are the standard Pauli matrices and $\sigma_0=I$. In the space of two-dimensional Hermitian matrices the Lorentz group action is given by $\mathsf{k}'^\mu\sigma_\mu=A(\mathsf{k}^\mu\sigma_\mu)A^\dagger$, where A denotes the element of the $\mathrm{SL}(2,\mathbb{C})$ group corresponding to the Lorentz transformation $\Lambda(A)$ which converts the four-vector k to k' (i.e., $k'^\mu=\Lambda^\nu{}_\nu k^\nu$).

APPENDIX C: USEFUL FORMULAS

The following relations hold:

$$(k\gamma)\mathsf{v}(k)=m\mathsf{v}(k), \tag{C1}$$

$$\bar{\mathsf{v}}(k)\mathsf{v}(k)=I, \tag{C2}$$

$$\gamma^0\mathsf{v}(k)=\mathsf{v}(k^\pi), \tag{C3}$$

$$\bar{\mathsf{v}}(k)\gamma^\mu\mathsf{v}(k)=\frac{k^\mu}{m}I, \tag{C4}$$

$$\mathsf{v}(k)\bar{\mathsf{v}}(k)=\frac{1}{2m}(k\gamma+mI), \tag{C5}$$

and

$$\mathsf{v}(\Lambda k)=\mathsf{D}(\Lambda)\mathsf{v}(k)\mathcal{D}^T(R(\Lambda,k)), \tag{C6}$$

$$\mathsf{v}(\Lambda k)\sigma_2=\mathsf{D}(\Lambda)\mathsf{v}(k)\sigma_2\mathcal{D}^\dagger(R(\Lambda,k)). \tag{C7}$$

Notice that when the operator $\hat{A}$ acts on standard basis vectors in the following way,

$$\hat{A}|k,\sigma\rangle = A_{\sigma\sigma'}|k,\sigma'\rangle, \tag{C8}$$

then its action on the covariant basis (18) is of the form

$$\hat{A}|\alpha,k\rangle = (\mathsf{v}(k)A\bar{\mathsf{v}}(k))_{\alpha\alpha'}|\alpha',k\rangle. \tag{C9}$$

Analogously for two-particle states the relation

$$(\hat{A}\otimes\hat{B})|(k,\sigma)_b,(p,\lambda)_a\rangle = A_{\sigma\sigma'}B_{\lambda\lambda'}|(k,\sigma')_b,(p,\lambda')_a\rangle \tag{C10}$$

implies

$$(\hat{A}\otimes\hat{B})|(\alpha,k),(\beta,p)\rangle = (\mathsf{v}(k)\hat{A}\bar{\mathsf{v}}(k))_{\alpha\alpha'}(\mathsf{v}(p)\hat{B}\bar{\mathsf{v}}(p))_{\beta\beta'}|(\alpha',k),(\beta',p)\rangle. \tag{C11}$$

APPENDIX D: CORRELATIONS IN A NONRELATIVISTIC TRIPLET STATE

We recall in this appendix the correlation function in a nonrelativistic triplet state (see, e.g., [50]). Let $|\sigma\rangle$, $\sigma=\pm\frac{1}{2}$, denote the eigenvector of the spin component along the z axis corresponding to a value of the spin-z component equal to σ. The general triplet state has the form

$$|\varphi\rangle = \sum_{\sigma,\lambda=\pm 1/2}\varphi_{\sigma\lambda}|\sigma\rangle\otimes|\lambda\rangle, \tag{D1}$$

with the symmetry condition

$$\varphi_{\sigma\lambda} = \varphi_{\lambda\sigma}. \tag{D2}$$

It is convenient to parametrize the matrix $\varphi=[\varphi_{\sigma\lambda}]$ in the following way:

$$\varphi = \frac{i}{\sqrt{2}}(\boldsymbol{\varphi}\cdot\boldsymbol{\sigma})\sigma_2. \tag{D3}$$

So finally the normalized correlation function in the triplet state is given by

$$C(\mathbf{a},\mathbf{b}) = \langle\varphi|\mathbf{a}\cdot\boldsymbol{\sigma}\otimes\mathbf{b}\cdot\boldsymbol{\sigma}|\varphi\rangle \tag{D4}$$

$$= \mathbf{a}\cdot\mathbf{b} - \frac{1}{|\boldsymbol{\varphi}|^2}[(\mathbf{a}\cdot\boldsymbol{\varphi})(\mathbf{b}\cdot\boldsymbol{\varphi}^*) + (\mathbf{b}\cdot\boldsymbol{\varphi})(\mathbf{a}\cdot\boldsymbol{\varphi}^*)]. \tag{D5}$$

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