# Optical bistability and multistability via atomic coherence in an *N*-type atomic medium

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We analyze hybrid absorptive-dispersive optical bistability (OB) and multistability (OM) behavior in a generic *N*-type atomic system driven by a degenerate probe field and a coherent coupling field by means of a unidirectional ring cavity. We show that the OB can be controlled by adjusting the intensity and the detuning of the coupling field, and the OM can also be observed under the appropriate detuning. The influence of the atomic cooperation parameter on atomic OB behavior is also discussed.

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Optical bistability (OB) has been extensively studied both experimentally and theoretically in two-level atomic systems due to its wide applications such as optical transistors, memory elements, and all optical switches [1,2]. The OB in three-level atomic systems confined optical ring cavity has also been studied theoretically [3] and experimentally [4]. It has been shown that the field-induced transparency and quantum interference effects could significantly decrease the OB threshold [5]. The phase fluctuation effects [6,7] and the effects of squeezed state fields [8–10] on the optical bistability have subsequently studied. It has been found that the OB could appear for small cooperation parameters due to the present of squeezed vacuum field [10].

In recent years there has been much interest in the effect of spontaneously generated coherence (SGC) on the dynamics [11], the amplification without population inversion [12], the disappearance of the dark state due to SGC in $\Lambda$-type atomic systems [13] and the enhanced index of refraction without absorption [14–18]. It also affects the optical bistability behavior in three-level atomic systems such as its threshold [19–21] and the shape of the bistable hysteresis cycle [22]. However, the existence of SGC or vacuum-induced coherence (VIC) requires that two close-lying levels be near degenerate and that the atomic dipole moments be nonorthogonal for the atoms in free space. Unfortunately, it is very difficult, if not impossible, to find a real atomic system with SGC or VIC because the rigorous conditions of near-degenerate levels and nonorthogonal dipole matrix elements are hard to simultaneously satisfy. As a result, few experiments have been performed to observe these interesting phenomena based on SGC or VIC. It is thus desirable to put forward new schemes *without* the SGC effect to realize the OB and/or OM to overcome the above-mentioned difficulties.

Here we present such a scheme of an *N*-type atomic system in a unidirectional ring cavity for OB and OM without need to resort the SGC effect. The nature of OB and OM in our scheme is a hybrid type that combines both absorptive and dispersive types. Interestingly, the OM in our scheme can be achieved only by tuning the frequency of the coupling field. On the other hand, our scheme only requires a degenerate probe beam, which seems better to achieve the enhanced nonlinearity due to multiple atomic coherence, and is simpler in experimental and theoretical arrangements than nondegenerate beams do because less laser beams are required in the former. Of course, nondegenerate probe beams still work although a bit worse.

We consider a generic *N*-type four-level atomic medium as shown in Fig. 1, with two upper excited states $|3\rangle$ and $|4\rangle$ and two lower ground states $|1\rangle$ and $|2\rangle$. The transition $|3\rangle \leftrightarrow |2\rangle$ of frequency $\omega_{32}$ is driven by a coherent coupling field with amplitude $E_c$ and angular frequency $\omega_c$. A degenerate probe field with amplitude $E_p$ and angular frequency $\omega_p$ is applied to couple simultaneously the transitions $|3\rangle \leftrightarrow |1\rangle$ of frequency $\omega_{31}$ and $|4\rangle \leftrightarrow |2\rangle$ of frequency $\omega_{42}$. The decay rates from the excited state $|3\rangle$ to the ground states $|1\rangle$ and $|2\rangle$ and from the excited state $|4\rangle$ to the ground state $|2\rangle$ are $\gamma_{31}$, $\gamma_{32}$, $\gamma_{32}$, and $\gamma_{42}$, respectively. The relaxation rates of coherence between the ground states are negligible and thus can be safely neglected. In the interaction picture and under the rotating-wave approximation, the semiclassical Hamiltonian describing the atom-field interaction for the system under study can be written as (taking $\hbar = 1$) [23]

$$H_{\text{int}} = -(\Delta_{p1} - \Delta_c)|2\rangle\langle 2| - \Delta_{p1}|3\rangle\langle 3| - (\Delta_{p1} + \Delta_{p2} - \Delta_c)|4\rangle\langle 4| - \frac{1}{2}(\Omega_{p1}|3\rangle\langle 1| + \Omega_c|3\rangle\langle 2| + \Omega_{p2}|4\rangle\langle 2| + \text{H.c.}), \qquad (1)$$

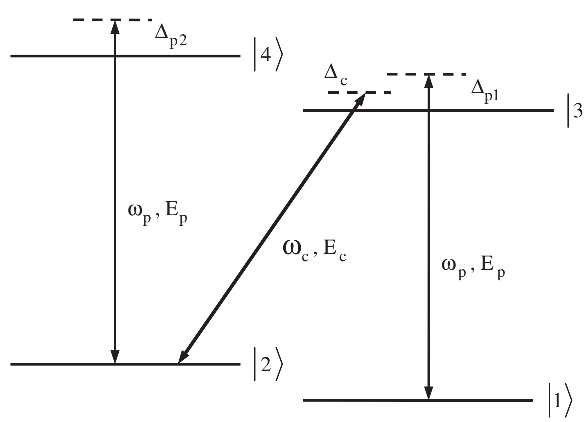


FIG. 1. Schematic diagram of generic *N*-type four-level atoms in a coherent medium interacting with a degenerate probe field $E_p$ and a coherent coupling field $E_c$. The atomic states are labeled as $|1\rangle$, $|2\rangle$, $|3\rangle$, and $|4\rangle$, respectively. $\Delta_{p1}$, $\Delta_{p2}$, and $\Delta_c$ are the frequency detunings of the corresponding optical fields, see text for details.

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where H.c. means Hermitian conjugation. In the above derivation process, we have taken the ground state $|1\rangle$ as the energy origin for the sake of simplicity. $\Omega_n(n=c,p1,p2)$ stands for Rabi frequencies for the respective transitions, i.e., $\Omega_{p1}=\mu_{31}E_p/\hbar$, $\Omega_c=\mu_{32}E_c/\hbar$, and $\Omega_{p2}=\mu_{42}E_p/\hbar$, where $\mu_{ij}=\boldsymbol{\mu}_{ij}\cdot\hat{\boldsymbol{e}}_L$ ($\hat{\boldsymbol{e}}_L$ is the polarization unit vector of the laser field) denotes the dipole moment for the transition between levels $|i\rangle$ and $|j\rangle$. $\Delta_{p1}=\omega_p-\omega_{31}$, $\Delta_c=\omega_c-\omega_{32}$, and $\Delta_{p2}=\omega_p-\omega_{42}$ are the detunings of the probe and coupling fields from the corresponding two-level transitions (see Fig. 1).

Using the density-matrix formalism, we begin to describe the atomic dynamics of the resonant coherent medium under study. By adopting the standard approach [24], we can easily obtain the time-dependent density matrix equations of motion as follows:

$$\dot{\rho}_{11}=\gamma_{31}\rho_{33}+\frac{i}{2}(\Omega^*_{p1}\rho_{31}-\Omega_{p1}\rho_{13}),$$

$$\dot{\rho}_{22}=\gamma_{32}\rho_{33}+\gamma_{42}\rho_{44}+\frac{i}{2}(\Omega^*_{c}\rho_{32}+\Omega^*_{p2}\rho_{42}-\Omega_{c}\rho_{23}-\Omega_{p2}\rho_{24}),$$

$$\dot{\rho}_{33}=-(\gamma_{31}+\gamma_{32})\rho_{33}+\frac{i}{2}(\Omega_{p1}\rho_{13}+\Omega_{c}\rho_{23}-\Omega^*_{p1}\rho_{31}-\Omega^*_{c}\rho_{32}),$$

$$\dot{\rho}_{44}=-\gamma_{42}\rho_{44}+\frac{i}{2}(\Omega_{p2}\rho_{24}-\Omega^*_{p2}\rho_{42}),$$

$$\dot{\rho}_{21}=i(\Delta_{p1}-\Delta_c)\rho_{21}+\frac{i}{2}(\Omega^*_{c}\rho_{31}+\Omega^*_{p2}\rho_{41}-\Omega_{p1}\rho_{23}),$$

$$\dot{\rho}_{31}=\left[i\Delta_{p1}-\frac{1}{2}(\gamma_{31}+\gamma_{32})\right]\rho_{31}+\frac{i}{2}[\Omega_{p1}(\rho_{11}-\rho_{33})+\Omega_c\rho_{21}],$$

$$\dot{\rho}_{41}=\left[i(\Delta_{p1}+\Delta_{p2}-\Delta_c)-\frac{1}{2}\gamma_{42}\right]\rho_{41}+\frac{i}{2}(\Omega_{p2}\rho_{21}-\Omega_{p1}\rho_{43}),$$

$$\dot{\rho}_{32}=\left[i\Delta_{c}-\frac{1}{2}(\gamma_{31}+\gamma_{32})\right]\rho_{32}+\frac{i}{2}[\Omega_{p1}\rho_{12}+\Omega_c(\rho_{22}-\rho_{33})-\Omega_{p2}\rho_{34}],$$

$$\dot{\rho}_{42}=\left(i\Delta_{p2}-\frac{1}{2}\gamma_{42}\right)\rho_{42}+\frac{i}{2}[\Omega_{p2}(\rho_{22}-\rho_{44})-\Omega_c\rho_{43}],$$

$$\dot{\rho}_{43}=\left[i(\Delta_{p2}-\Delta_c)-\frac{1}{2}(\gamma_{31}+\gamma_{32}+\gamma_{42})\right]\rho_{43}+\frac{i}{2}(\Omega_{p2}\rho_{23}-\Omega^*_{p1}\rho_{41}-\Omega^*_{c}\rho_{42}), \tag{2}$$

where the overdots represent the derivative with respect to time $t$ and $\rho_{ij}=\rho^*_{ji}$.

In what follows, we assume that all Rabi frequencies are real without loss of generality. For simplicity of discussion, we assume uniform decay rates and uniform electric dipole matrix elements, i.e., $\gamma_{31}=\gamma_{32}=\gamma_{42}=\gamma$ and $\mu_{31}=\mu_{42}=\mu$. Now, we put the ensemble of $N$ homogeneously broadened four-level atoms in a unidirectional ring cavity as shown in Fig. 1 of Ref. [5] but with the notation change of $E_1\rightarrow E_c$ and $E_2\rightarrow E_p$. For simplicity, we assume that mirror 3 and 4 have 100% reflectivity, and the intensity reflection and transmission coefficient of mirrors 1 and 2 are $R$ and $T$ (with $R+T=1$), respectively.

The total electromagnetic field can be written as $E=E_pe^{-i\omega_pt}+E_ce^{-i\omega_ct}+\text{c.c.}$, where the probe field $E_p$ circulates in the ring cavity and the coupling field $E_c$ does not circulate in the cavity. Then under slowly varying envelope approximation, the dynamic response of the probe field is governed by Maxwell's equation

$$\frac{\partial E_p}{\partial t}+c\frac{\partial E_p}{\partial z}=i\frac{\omega_p}{2\varepsilon_0}P(\omega_p), \tag{3}$$

where $c$ and $\varepsilon_0$ is the light speed and permittivity of free space respectively. $P(\omega_p)$ is the slowly oscillating term of the induced polarization in both the transitions $|1\rangle\leftrightarrow|3\rangle$ and $|2\rangle\leftrightarrow|4\rangle$, and is given by $P(\omega_p)=N\mu(\rho_{31}+\rho_{42})$, where $N$ is the number density of the atoms in the sample.

We consider the field equation (3) in the steady-state case. Setting the time derivative in Eq. (3) equal to zero for the steady state, we can obtain the field amplitude as follows:

$$\frac{\partial E_p}{\partial z}=i\frac{N\omega_p\mu}{2c\varepsilon_0}(\rho_{31}+\rho_{42}). \tag{4}$$

For a perfectly tuned ring cavity, in the steady state limit, the boundary conditions impose the following conditions between the incident field $E_p^I$ and the transmitted field $E_p^T$

$$E_p(L)=E_p^T/\sqrt{T}, \tag{5a}$$

$$E_p(0)=\sqrt{T}E_p^I+RE_p(L), \tag{5b}$$

where $L$ is the length of the atomic sample, and the second term on the right-hand side of Eq. (5b) describes a feedback mechanism due to the mirror, which is essential to give rise to bistability, that is to say, no bistability can occur if $R=0$.

In the mean-field limit [25], using the boundary conditions Eq. (5) and normalizing the fields by letting $y=\frac{\mu E_p^I}{\hbar\sqrt{T}}$ and $x=\frac{\mu E_p^T}{\hbar\sqrt{T}}$, we can get input-output relationship:

$$y=x-iC\gamma[\rho_{31}(x)+\rho_{42}(x)], \tag{6}$$

where $C=\frac{N\omega_pL\mu^2}{2\hbar\varepsilon_0cT\gamma}$ is the usual cooperation parameter. It is worth pointing out that the second term on the right-hand side of Eq. (6) is vital for optical bistability and multistability to take place.

We set the time derivatives $\partial\rho_{ij}/\partial t=0$ $(i,j=1,2,3,4)$ in the above density matrix equation (2) for the steady state, and solve the corresponding density matrix equation together with the coupled field equation (6) via simple Matlab codes, then we can arrive at the steady-state solutions. In the following numerical calculations, all the parameters used are scaled with $\gamma$, which should be in the order of MHz for rubidium or sodium atoms. Figure 2 demonstrates the depen-

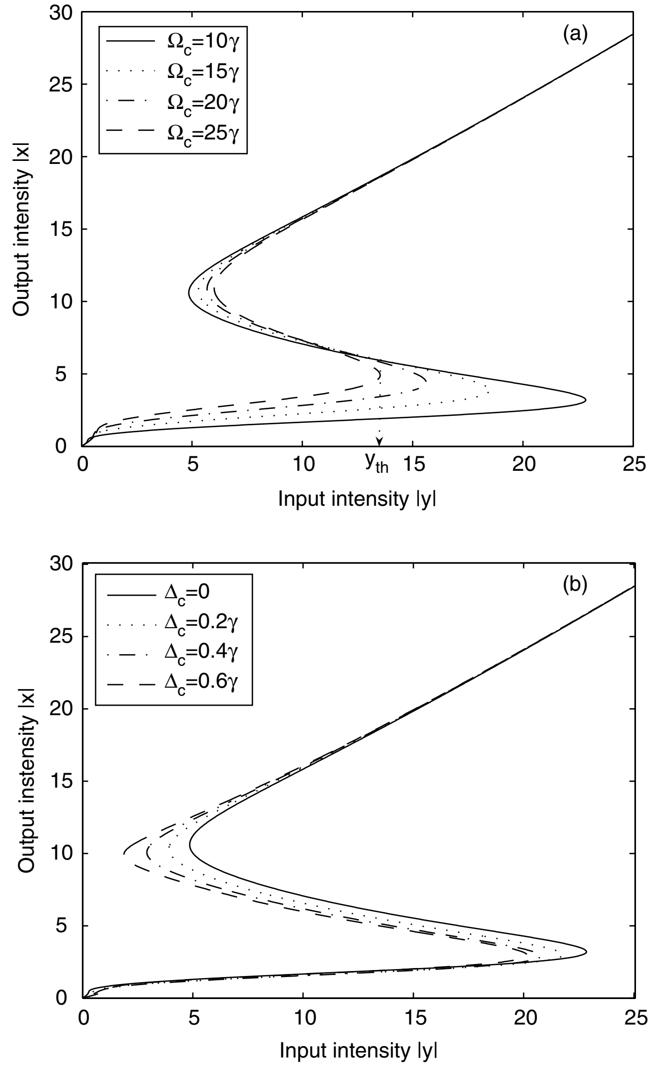


FIG. 2. Output intensity $|x|$ versus input intensity $|y|$ for different values $\Omega_c$ (panel a, $\Delta_c=0$) and for different detunings $\Delta_c$ (panel b, $\Omega_c=10\gamma$), respectively. The other parameters are $C=200$, $\Delta_{p1}=0$, $\Delta_{p2}=0.4\gamma$, and $\gamma_{31}=\gamma_{32}=\gamma_{42}=\gamma$.

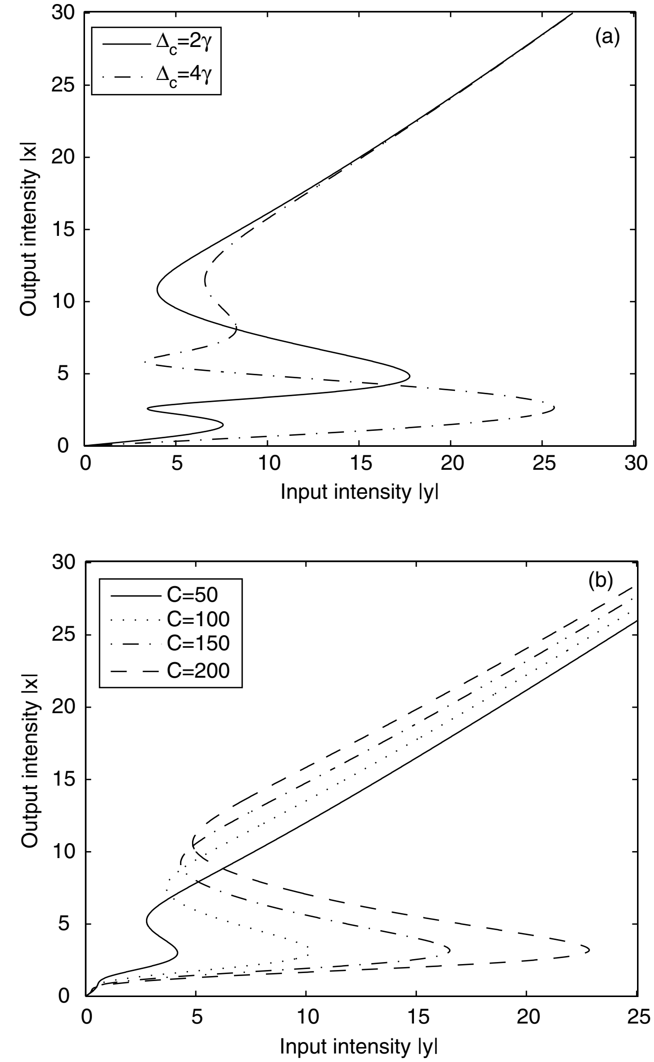


FIG. 3. Output intensity $|x|$ versus input intensity $|y|$ for different detunings $\Delta_c$ (panel a, $C=200$) and for different values $C$ (panel b, $\Delta_c=0$), respectively. The other parameters are $\Omega_c=10\gamma$, $\Delta_{p1}=0$, $\Delta_{p2}=0.4\gamma$, and $\gamma_{31}=\gamma_{32}=\gamma_{42}=\gamma$.

dence of the optical bistability on the coupling field intensities $\Omega_c$ [panel (a)] and on the small coupling-field detunings $\Delta_c$ [panel (b)], respectively. While Fig. 3 shows the dependence of the optical multistability on the large coupling-field detunings $\Delta_c$ [panel (a)] and on the cooperation parameter $C$ [panel (b)], respectively.

It can be easily seen from Fig. 2(a) that increasing the intensity of the coupling field leads to a significant decreasing of the bistable threshold $y_{\mathrm{th}}$. The reason can be qualitatively explained as follows. By applying a strong coupling field between the states $|2\rangle$ and $|3\rangle$, we can dramatically reduce the absorption for the probe field on the transitions $|1\rangle \leftrightarrow |3\rangle$ and $|2\rangle \leftrightarrow |4\rangle$ and enhance the Kerr nonlinearity of the atomic medium, which makes the cavity field easier to reach saturation. This might be useful to control the threshold value and the hysteresis cycle width of the bistable curve simply by adjusting the intensity of the coupling field. It should be noted that very high values of $\Omega_c$ are good for observing OB due to the decreasing of the threshold value in one hand. On the other hand, too strong a coupling field, i.e., very large $\Omega_c$, can cause some detrimental effects. For instance, too strong a coupling field leads to nonnegligible ac Stark effects that change the detunings. As a result, there is a trade off in choosing properly the intensity of the coupling field (not too strong and not too weak) in order to observe OB or OM.

The effects of the frequency detuning of the coupling laser on the OB and OM can be clearly seen from Figs. 2(b) and 3(a), respectively. For a small detuning $\Delta_c$, say $\Delta_c \sim 0.6\gamma$ as shown in Fig. 2(b), only bistable behavior can occur with its threshold value decreasing slowly with respect to $\Delta_c$, while increasing $\Delta_c$ further to large values, say $\Delta_c \sim 2\gamma$ as shown in Fig. 3(a), gives rise to the appearance of OM patterns and the shape of the OM patterns dramatically changes with respect to $\Delta_c$. The reason for the multistability existence is that $y$ in Eq. (6) is not a cubic polynomial of the variable $x$ in certain parameter regimes. OM has advantages over OB in some applications where more than two states are required. We have thus found a novel phenomenon of the transition from OB to OM, and this transition can take place only through tuning the frequency of the coupling field.

It is seen from Fig. 3(b) that the threshold of OB is reduced drastically when the atomic cooperation parameter $C = N\omega_p L\mu^2/2\hbar\varepsilon_0 cT\gamma \propto N$ or the number density $N$ of atoms inside the cavity becomes small, and OB tends to disappear for $C \leq 25$ (the corresponding results are not shown here). Obviously the enhancement in the absorption of the sample as the number density of atoms increases could account for the raise of the threshold intensity with respect to the cooperation parameter $C$.

In conclusion, we have illustrated the OB and OM behaviors in the $N$-configuration four-level atomic system driven by the degenerate probe field and the coupling field inside a unidirectional ring cavity. We find that the intensity and the frequency detuning of the coupling field as well as the cooperation parameters can affect the OB behavior dramatically, which can be used to control the bistable threshold intensity and the hysteresis loop. Interestingly, OM can be observed for an appropriate choice of the frequency detuning of the coupling field in such a system. Our results provide clues for achieving optimally the desired OB or OM behaviors and the transition from OB to OM and vice versa. Lastly, we point out that the OB comes essentially from the Kerr nonlinearity and hence solitons could form in those systems of demonstrating OB behaviors [26–28].

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