

## Superfluidity in a three-flavor Fermi gas with SU(3) symmetry

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We investigate the superfluidity and the associated Nambu-Goldstone modes in a three-flavor atomic Fermi gas with SU(3) global symmetry. The  $s$ -wave pairing occurs in flavor antitriplet channel due to the Pauli principle, and the superfluid state contains both gapped and gapless fermionic excitations. Corresponding to the spontaneous breaking of the SU(3) symmetry to a SU(2) symmetry with five broken generators, there are only three Nambu-Goldstone modes, one is with linear dispersion law and two are with quadratic dispersion law. The other two expected Nambu-Goldstone modes become massive with a mass gap of the order of the fermion energy gap in a wide coupling range. The abnormal number of Nambu-Goldstone modes, the quadratic dispersion law, and the mass gap have significant effect on the low-temperature thermodynamics of the matter.

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### I. INTRODUCTION

The superfluidity in strongly interacting atomic Fermi gas and the associated BCS-BEC (Bose-Einstein Condensation) crossover phenomena [1–3] have been observed in experiments [4–7] via the method of Feshbach resonance. The experimental study of superfluidity in atomic Fermi gas may be important for us to understand the solid-state phenomena such as high-temperature superconductivity, and may give some clue to search for the ground state of the dense quark matter and nuclear matter. In the past years, most theoretical and experimental studies concentrated on the two-flavor systems such as a  ${}^6\text{Li}$  gas with the two lowest hyperfine states. (In this paper, we use the word “flavor” in particle physics to denote the internal degrees of freedom of the fermionic atoms.) Compared to electrons in solids, atomic systems offer more internal degrees of freedom. For alkali atoms, nuclear spin  $I$  and electron spin  $S$  are combined in a hyperfine state with total angular momentum  $F$ . While typical electronic systems are constrained to a SU(2) spin rotational symmetry, the total angular momentum  $F$  can be larger than  $1/2$ , resulting in  $2F+1$  hyperfine states differing by their azimuthal quantum number  $m_F$ . Therefore, the atomic Fermi gas can provide us a way to study the superfluidity with broken symmetry higher than the U(1) one. In this paper, we will focus on a three-flavor system with a SU(3) global symmetry. Such a system has been investigated in some works [8–10].

It is well-known that, associated with the spontaneous breaking of a global symmetry, there should be corresponding Nambu-Goldstone (NG) bosons. Such NG bosons dominate the low-temperature thermodynamics of the system. According to the Goldstone theorem [11,12], if an internal continuous symmetry group is spontaneously broken down to a subgroup with  $N$  broken generators,  $N$  NG bosons appear in Lorentz-invariant systems, i.e., the number of NG bosons is equal to the number of broken generators. However, from the Nielsen-Chadha (NC) theorem [13], for systems without Lorentz invariance the number of NG bosons can be less than the number of broken generators. Let  $N_1$  and  $N_2$  be the numbers of gapless excitations which have, respectively, the dispersion laws  $\omega \sim |\vec{p}|$  and  $\omega \sim |\vec{p}|^2$  in the limit of long wavelength, the number of broken generators satisfies

the relation  $N < N_1 + 2N_2$ . For the equality between the number of NG bosons and the number of the broken generators, there is an important criterion: If  $\langle [Q_i, Q_j] \rangle = 0$  for any two broken generators  $Q_i$  and  $Q_j$ ,  $i, j = 1, 2, \dots, N$ , the number of NG bosons is equal to the number of the broken generators [14].

For the three-flavor Fermi gas with SU(3) symmetry we will consider in this paper, the ground state of the system contains both gapped and gapless fermionic excitations. When the SU(3) symmetry is spontaneously broken to a SU(2) subgroup with five broken generators, we will show with an explicit calculation that there are only three NG modes. Among them, one has linear dispersion law and the other two have quadratic dispersion law. The reason for the abnormal number of NG modes and the appearance of quadratic dispersion law is found to be the fact that the condition  $\langle [Q_i, Q_j] \rangle = 0$  is not satisfied due to the density imbalance between the gapped and gapless fermions.

The abnormal number of NG modes and the nonlinear dispersion law were widely discussed in relativistic field theory at finite density [14–16]. They were also found in the study of two-flavor color superconductivity in the Nambu-Jona-Lasinio model [17] where the condition  $\langle [Q_i, Q_j] \rangle = 0$  is not satisfied due to the lack of color neutrality. However, the abnormal number of NG bosons cannot be realized in superfluid quark matter and has no observable effect, since the color neutrality should be imposed via some mechanism such as gluon condensation and the NG bosons should be eaten up by the gluons via the Higgs mechanism. In atomic Fermi gas, there is no constraint like the color neutrality, and the NG modes are physical degrees of freedom which dominate the low temperature thermodynamics of the system. The theoretic prediction of the NG modes may be tested in future experiments via the measurement of the thermodynamic quantities. In addition, the mass gap of the two massive collective modes found in [17] is very small compared with the quark energy gap, while the corresponding mass gap in the three-flavor Fermi gas is of the order of the fermion energy gap, which makes remarkable effect on the low-temperature thermodynamics.

The paper is organized as follows. In Sec. II, we set up the model for the three-flavor Fermi gas with SU(3) global sym-

metry. In Sec. III, we investigate the ground state of the system and the symmetry breaking. In Sec. IV, we investigate the pair fluctuation around the superfluid ground state, identify the NG modes and find their dispersion laws. In Sec. V we discuss whether we can recover the five NG modes. We summarize in Sec. VI. The natural unit of  $c = \hbar = k_B = 1$  is used through the paper.

## II. THE MODEL

The physical system we are interested in this paper is an idea system composed of three flavors of fermions with attractive interaction. Such a system can be realized in cold atomic Fermi gas such as a  $^6\text{Li}$  or  $^{40}\text{K}$  gas where the three flavors come from three degenerate hyperfine states [18]. Generally, the system can be modeled by the Lagrangian density

$$\mathcal{L} = \psi^\dagger \left( i\partial_t + \frac{\nabla^2}{2m} + \mu \right) \psi + \mathcal{L}_{int}, \quad (1)$$

where  $\psi \equiv (\psi_1, \psi_2, \psi_3)^T$  and  $\psi^\dagger \equiv (\psi_1^*, \psi_2^*, \psi_3^*)$  are the three-component fermion fields,  $\mu$  is their chemical potential, and  $m$  is their mass. We have assumed that the chemical potentials of three flavors are the same due to the chemical equilibrium.

It is generally believed that the  $s$ -wave channel is the dominant pairing channel. According to the Pauli principle, the total wave function of the Cooper pair must be antisymmetric. Using the decomposition  $3 \otimes 3 = \bar{3} \oplus 6$  for the SU(3) group, the  $s$ -wave pairing must be associated with the anti-triplet channel in flavor space. In this paper we consider only the  $s$ -wave pairing channel, the interaction can be modeled by

$$\mathcal{L}_{int} = g/4 (\psi_{\alpha'}^\dagger i\epsilon_{\alpha\beta\gamma} \psi_{\beta'}^*) (\psi_{\alpha'} i\epsilon_{\alpha'\beta'\gamma'} \psi_{\beta'}), \quad (2)$$

where  $g$  is the bare coupling related to the  $s$ -wave scattering length and  $\epsilon_{ijk}$  is the total antisymmetric tensor. Throughout, summation is implicit over repeated flavor index. Note that the interaction Lagrangian can also be written as

$$\mathcal{L}_{int} = g/4 \sum_{a=2,5,7} (\psi^\dagger \lambda_a \psi^*) (\psi^T \lambda_a \psi), \quad (3)$$

where  $\lambda_a (a=1, 2, \dots, 8)$  are the Gell-Mann matrices. The model Lagrangian has the symmetry SU(3)  $\otimes$  U(1), i.e., it is invariant under the transformation

$$\psi \rightarrow e^{-iT_a \theta_a} \psi, \quad a = 0, 1, 2, \dots, 8, \quad (4)$$

where  $T_0 = I_3$  is the generator of the U(1) group and  $T_a = \frac{\lambda_a}{2} (a=1, 2, \dots, 8)$  are the generators of the SU(3) group. Due to the above symmetry, the system possesses nine conserved charges or generators  $Q_a (a=0, 1, 2, \dots, 8)$  given by

$$Q_a = \int d^3\mathbf{x} \psi^\dagger T_a \psi. \quad (5)$$

Like the two flavor or U(1) system, for attractive coupling  $g$  we can perform an exact Stratonovich-Hubbard transformation to introduce the pair fields

$$\Phi_I \sim g/2 (\psi_{\alpha'} i\epsilon_{\alpha\beta\gamma} \psi_{\beta'}), \quad \Phi_I^* \sim g/2 (\psi_{\alpha'}^* i\epsilon_{\alpha\beta\gamma} \psi_{\beta'}^*) \quad (6)$$

for  $I=1, 2, 3$ . With the Nambu-Gorkov fields defined as

$$\Psi = \begin{pmatrix} \psi \\ \psi^* \end{pmatrix}, \quad \Psi^\dagger = (\psi^\dagger \quad \psi^{\dagger T}), \quad (7)$$

the partition function  $Z$  of the system can be expressed as

$$Z = \int [d\Psi^\dagger][d\Psi][d\Phi_I^*][d\Phi_I] e^{\int_x \left( \frac{1}{2} \Psi^\dagger \mathcal{K} \Psi - \frac{\Phi_I^* \Phi_I}{g} \right)} \quad (8)$$

with the kernel  $\mathcal{K}[\Phi_I^*, \Phi_I]$  defined as

$$\mathcal{K}[\Phi_I^*, \Phi_I] = \begin{pmatrix} -\partial_\tau + \frac{\nabla^2}{2m} + \mu & i\epsilon_{\alpha\beta\gamma} \Phi_I \\ i\epsilon_{\alpha\beta\gamma} \Phi_I^* & -\partial_\tau - \frac{\nabla^2}{2m} - \mu \end{pmatrix} \quad (9)$$

in the imaginary time ( $\tau=it$ ) formalism of finite temperature field theory with  $\int_x = \int_0^\beta d\tau \int d^3\mathbf{x}$ , where  $\beta$  is the inverse of temperature,  $\beta=1/T$ . Integrating out the fermionic degrees of freedom, we obtain

$$Z = \int [d\Phi_I^*][d\Phi_I] e^{-S_{eff}[\Phi_I^*, \Phi_I]} \quad (10)$$

with the effective action

$$S_{eff}[\Phi_I^*, \Phi_I] = \int_x \frac{\Phi_I^* \Phi_I}{g} - \frac{1}{2} \text{Tr} \ln \mathcal{K}[\Phi_I^*, \Phi_I]. \quad (11)$$

## III. THE GROUND STATE

At some critical temperature  $T_c$  the system should undergo the phase transition from the normal phase with the SU(3) symmetry to the superfluid phase where the SU(3) symmetry is spontaneously broken. Since we focus on the low-temperature region where  $T \ll T_c$ , the mean field approximation or saddle point approximation is believed to be a good treatment for the ground state. The order parameters which characterize the superfluid phase or symmetry broken phase are defined as the expectation values of the pair fields

$$\Delta_I = \langle \Phi_I \rangle, \quad \Delta_I^* = \langle \Phi_I^* \rangle, \quad I = 1, 2, 3. \quad (12)$$

Let us consider the homogeneous and isotropic superfluid state where the order parameters are independent of the coordinates. The thermodynamic potential  $\Omega$  in mean field approximation can be expressed as

$$\begin{aligned} \Omega &= \frac{1}{\beta V} S_{eff}[\Phi_I^* = \Delta_I^*, \Phi_I = \Delta_I] \\ &= \frac{\Delta_I^* \Delta_I}{g} - \frac{1}{2\beta} \sum_n \int \frac{d^3\mathbf{p}}{(2\pi)^3} \text{Tr} \ln \mathcal{G}^{-1}(i\omega_n, \mathbf{p}), \end{aligned} \quad (13)$$

where  $V$  is the volume of the system and  $\mathcal{G}^{-1}$  is the inverse of the fermion propagator in momentum space,

$$\mathcal{G}^{-1}(i\omega_n, \mathbf{p}) = \begin{pmatrix} i\omega_n - \xi_p & i\epsilon_{\alpha\beta l}\Delta_l \\ i\epsilon_{\alpha\beta l}\Delta_l^* & i\omega_n + \xi_p \end{pmatrix} \quad (14)$$

with  $\omega_n$  the Matsubara frequency for fermions and  $\xi_p = p^2/(2m) - \mu$ . A straightforward algebra shows that the determinate of  $\mathcal{G}^{-1}$  in the Nambu-Gorkov  $\otimes$  flavor space reads

$$\det \mathcal{G}^{-1} = [(i\omega_n)^2 - \xi_p^2][(i\omega_n)^2 - \xi_p^2 - \Delta^2]^2, \quad (15)$$

which indicates that the thermodynamic potential depends only on the quantity  $\Delta^2$  defined as

$$\Delta^2 = |\Delta_1|^2 + |\Delta_2|^2 + |\Delta_3|^2. \quad (16)$$

The fermionic excitation spectra can be determined by  $\det \mathcal{G}^{-1} = 0$ . For positive chemical potential, the superfluid phase contains both gapped and gapless fermionic excitations,

$$\omega_{1,2}(\mathbf{p}) = \pm \sqrt{\xi_p^2 + \Delta^2}, \quad \omega_3(\mathbf{p}) = \xi_p. \quad (17)$$

With the quasiparticle dispersions, the thermodynamic potential can be evaluated as

$$\begin{aligned} \Omega = & -\frac{m\Delta^2}{4\pi a_s} - \Delta^2 \int \frac{d^3\mathbf{p}}{(2\pi)^3} \left( \frac{1}{E_p + \xi_p} - \frac{1}{2\epsilon_p} \right) \\ & - \frac{1}{\beta} \int \frac{d^3\mathbf{p}}{(2\pi)^3} [2 \ln(1 + e^{\beta E_p}) + \ln(1 + e^{\beta \xi_p})], \end{aligned} \quad (18)$$

where we have defined the notation  $E_p = \sqrt{\xi_p^2 + \Delta^2}$  and replaced the bare coupling  $g$  by the low-energy limit of the two-body  $T$  matrix

$$\frac{m}{4\pi a_s} = -\frac{1}{g} + \int \frac{d^3\mathbf{p}}{(2\pi)^3} \frac{1}{2\epsilon_p} \quad (19)$$

with  $a_s$  the  $s$ -wave scattering length and  $\epsilon_p = p^2/(2m)$ .

Now we discuss the symmetry breaking pattern. Similar to the two-flavor color superconductivity, the symmetry breaking pattern in the current case is

$$\text{SU}(3) \otimes \text{U}(1) \rightarrow \text{SU}(2) \otimes \tilde{\text{U}}(1). \quad (20)$$

To see the broken and unbroken symmetry groups explicitly, it is convenient for us to choose

$$\Delta_1 = \Delta_2 = 0, \quad \Delta_3 \equiv \Delta \neq 0 \quad (21)$$

without loss of generality. In this case, only flavors 1 and 2 participate in the Cooper pairing and flavor 3 remains unpaired, a  $\text{SU}(2)$  subgroup with generators  $T_1, T_2, T_3$ , and a  $\tilde{\text{U}}(1)$  subgroup with generator  $\tilde{T}_0 = (\sqrt{3}T_0 - T_8)/2$  remain unbroken, and the broken generators are  $T_4, T_5, T_6, T_7$ , and  $\tilde{T}_8 = (T_0 + \sqrt{3}T_8)/2$ . We should emphasize that all the physical results do not depend on the specific choice of symmetry breaking direction due to the fact that the Lagrangian is invariant under the  $\text{SU}(3)$  transformation.

To determine physical quantities in the superfluid state, we should solve the gap equation together with the number equation. Assuming the total number density  $n$  is fixed, we can introduce the Fermi momentum  $p_F$  and Fermi energy  $\epsilon_F$  through the definitions  $n = 3 \times p_F^3/(6\pi^2)$  and  $\epsilon_F = p_F^2/(2m)$ . At

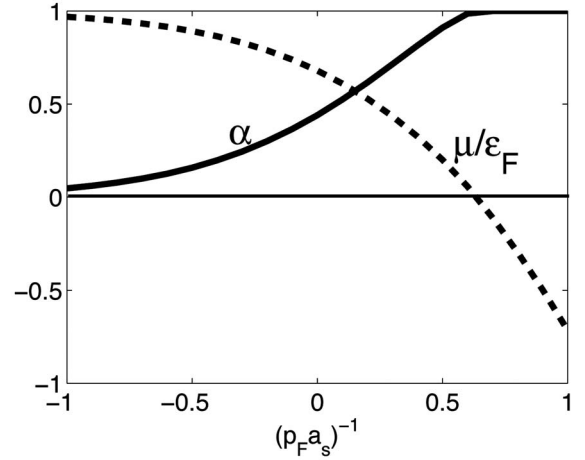


FIG. 1. The chemical potential  $\mu$  scaled by the Fermi energy  $\epsilon_F$  and the ratio  $\alpha$  as functions of  $(p_F a_s)^{-1}$  at  $T=0$ .

mean field level, the gap equation which determines the energy gap  $\Delta$  can be derived via  $\partial\Omega/\partial\Delta=0$ , namely,

$$-\frac{m\Delta}{4\pi a_s} = \Delta \int \frac{d^3\mathbf{p}}{(2\pi)^3} \left[ \frac{1 - 2f(E_p)}{2E_p} - \frac{1}{2\epsilon_p} \right], \quad (22)$$

and the number equation can be derived via  $n = -\partial\Omega/\partial\mu$ , namely,

$$n = \int \frac{d^3\mathbf{p}}{(2\pi)^3} \left[ \left( 1 - \frac{\xi_p}{E_p} \right) + 2 \frac{\xi_p}{E_p} f(E_p) + f(\xi_p) \right], \quad (23)$$

where  $f(x)$  is the Fermi-Dirac distribution function. For the specific choice of symmetry breaking direction (21), the number densities  $n_1, n_2$  for the paired flavors and  $n_3$  for the unpaired flavor can be evaluated as

$$n_1 = n_2 = \int \frac{d^3\mathbf{p}}{(2\pi)^3} \left[ \frac{1}{2} \left( 1 - \frac{\xi_p}{E_p} \right) + \frac{\xi_p}{E_p} f(E_p) \right],$$

$$n_3 = \int \frac{d^3\mathbf{p}}{(2\pi)^3} f(\xi_p), \quad (24)$$

which satisfy  $n = n_1 + n_2 + n_3$ . Once the Cooper pairing occurs, the number of the paired fermions becomes different from the number of the unpaired fermions,  $n_1 = n_2 \neq n_3$ . This difference can be parametrized by the ratio  $\alpha$  defined as

$$\alpha = \frac{n_1 + n_2 - 2n_3}{n}. \quad (25)$$

Solving the coupled set of gap equation and number equation, we can obtain the gap  $\Delta$ , the chemical potential  $\mu$ , and the ratio  $\alpha$  as functions of the coupling  $(p_F a_s)^{-1}$ . Before the detailed numerical calculation, we give a qualitative estimate at  $T=0$ . In the BCS limit, the energy gap  $\Delta$  is very small and the chemical potential is approximately the Fermi energy, the ratio  $\alpha$  will be very small. In the BEC limit, the chemical potential becomes negative and the unpaired fermions disappear, the ratio approaches to the limit  $\alpha=1$ . In Fig. 1, we

show the numerical results of the chemical potential  $\mu$  and the ratio  $\alpha$  at  $T=0$ . The ratio  $\alpha$  is always positive which means  $n_1=n_2>n_3$ .

What does the nonzero ratio  $\alpha$  mean? To answer this question, we calculate the expectation values of the generators  $Q_a$  ( $a=1,2,\dots,8$ ). They can be calculated via the formula

$$\langle Q_a \rangle = \frac{V}{2\beta} \sum_n \int \frac{d^3\mathbf{p}}{(2\pi)^3} \text{Tr}[\tau_3 \otimes T_a \mathcal{G}(i\omega_n, \mathbf{p})], \quad (26)$$

where  $\tau_3$  is the third Pauli matrix in the Nambu-Gorkov space. The explicit form of the fermion propagator in the Nambu-Gorkov $\otimes$ flavor space can be evaluated as

$$\mathcal{G}(i\omega_n, \mathbf{p}) = \begin{pmatrix} \mathcal{G}_-^\Delta & 0 & 0 & 0 & -i\mathcal{F} & 0 \\ 0 & \mathcal{G}_-^\Delta & 0 & i\mathcal{F} & 0 & 0 \\ 0 & 0 & \mathcal{G}_-^0 & 0 & 0 & 0 \\ 0 & -i\mathcal{F} & 0 & \mathcal{G}_+^\Delta & 0 & 0 \\ i\mathcal{F} & 0 & 0 & 0 & \mathcal{G}_+^\Delta & 0 \\ 0 & 0 & 0 & 0 & 0 & \mathcal{G}_+^0 \end{pmatrix} \quad (27)$$

with the nonzero matrix elements defined as

$$\mathcal{G}_\pm^\Delta = \frac{i\omega_n \mp \xi_p}{(i\omega_n)^2 - E_p^2}, \quad \mathcal{G}_\pm^0 = \frac{1}{i\omega_n \pm \xi_p}, \quad \mathcal{F} = \frac{\Delta}{(i\omega_n)^2 - E_p^2}. \quad (28)$$

After a straightforward matrix algebra, we find

$$\langle Q_a \rangle = 0, \quad a = 1, 2, \dots, 7, \quad \langle Q_8 \rangle = \frac{n_1 + n_2 - 2n_3}{\sqrt{3}} = \frac{\alpha n}{\sqrt{3}}. \quad (29)$$

According to the commutation relation of SU(3) group, we have

$$\langle [Q_4, Q_5] \rangle = \langle [Q_6, Q_7] \rangle = i\sqrt{3} \langle Q_8 \rangle = i\alpha n V \neq 0. \quad (30)$$

Therefore, the nonzero ratio  $\alpha$  in the superfluid phase means that the condition  $\langle [Q_i, Q_j] \rangle = 0$  which is sufficient for the equality between the number of NG bosons and the number of the broken generators is not satisfied in such a system. However, we now cannot conclude that the number of NG bosons is not equal to the number of broken generators.

#### IV. THE NAMBU-GOLDSTONE MODES

We investigate now the pair fluctuations around the superfluid state and examine whether there are five NG modes. If there exist five NG modes, they must be the collective modes associated with the pair fluctuations. After the field shift  $\Phi_3 \rightarrow \Phi_3 + \Delta$ , the effective action reads

$$S_{eff} = \int_x \frac{\Phi_I^* \Phi_I}{g} - \frac{1}{2} \text{Tr} \ln[\mathcal{G}^{-1} + \Sigma(\Phi_I^*, \Phi_I)] \quad (31)$$

with the matrix  $\Sigma$  defined as

$$\Sigma = \begin{pmatrix} 0 & 0 & 0 & 0 & i\Phi_3 & -i\Phi_2 \\ 0 & 0 & 0 & -i\Phi_3 & 0 & i\Phi_1 \\ 0 & 0 & 0 & i\Phi_2 & -i\Phi_1 & 0 \\ 0 & i\Phi_3^* & -i\Phi_2^* & 0 & 0 & 0 \\ -i\Phi_3^* & 0 & i\Phi_1^* & 0 & 0 & 0 \\ i\Phi_2^* & -i\Phi_1^* & 0 & 0 & 0 & 0 \end{pmatrix}. \quad (32)$$

To identify the existence of NG modes and study their dispersion laws, we need to evaluate the effective action only to the quadratic terms of the pair fields. Using the derivative expansion, the effective action up to the quadratic terms can be expressed as

$$S_{eff} = \int_x \frac{\Phi_I^* \Phi_I}{g} + \frac{1}{4} \text{Tr}[\mathcal{G}\Sigma(\Phi_I^*, \Phi_I)\mathcal{G}\Sigma(\Phi_I^*, \Phi_I)]. \quad (33)$$

To do calculations in momentum space, we define the Fourier transformations

$$\Phi_I(x) = \frac{1}{\sqrt{\beta V}} \sum_q e^{-i\nu_n \tau + i\mathbf{q}\cdot\mathbf{x}} \Phi_I(q), \quad \Phi_I^*(x) = \frac{1}{\sqrt{\beta V}} \sum_q e^{-i\nu_n \tau + i\mathbf{q}\cdot\mathbf{x}} \Phi_I^*(q) \quad (34)$$

for the pair fields, where the four momentum in the imaginary time formalism is defined as  $q=(i\nu_n, \mathbf{q})$  with  $\nu_n$  the Matsubara frequency for bosons. We have  $\Phi_I^*(-q) = [\Phi_I(q)]^*$  due to the above definitions. After a straightforward matrix algebra, the effective action of the pair fields can be decomposed into three parts:

$$S_{eff}[\Phi_I^*, \Phi_I] = S_1[\Phi_1^*, \Phi_1] + S_2[\Phi_2, \Phi_2] + S_3[\Phi_3, \Phi_3]. \quad (35)$$

The pair fields for  $I=1,2,3$  do not mix with each other, which makes the calculation of NG modes quite easy. Each part of the effective action  $S_I$  takes the form

$$S_I = \frac{1}{2} \sum_q [\Pi_I^{11}(q) \Phi_I^*(-q) \Phi_I(q) + \Pi_I^{22}(q) \Phi_I^*(q) \Phi_I(-q) + \Pi_I^{12}(q) \Phi_I^*(-q) \Phi_I^*(q) + \Pi_I^{21}(q) \Phi_I(q) \Phi_I(-q)], \quad (36)$$

where the functions  $\Pi_I^{ij}(q)$  for  $I=1,2$  take the same form due to the residue SU(2) symmetry,

$$\begin{aligned} \Pi_I^{11}(q) = \Pi_I^{22}(-q) &= \frac{1}{g} + \frac{1}{2\beta V} \sum_p [\mathcal{G}_+^0(p) \mathcal{G}_-^\Delta(p+q) \\ &+ \mathcal{G}_+^\Delta(p) \mathcal{G}_-^0(p+q)], \\ \Pi_I^{12}(q) = \Pi_I^{21}(q) &= 0, \end{aligned} \quad (37)$$

and the functions  $\Pi_3^{ij}(q)$  are given by

$$\Pi_3^{11}(q) = \Pi_3^{22}(-q) = \frac{1}{g} + \frac{1}{\beta V} \sum_p [\mathcal{G}_+^\Delta(p) \mathcal{G}_-^\Delta(p+q)],$$

$$\Pi_3^{12}(q) = \Pi_3^{21}(q) = \frac{1}{\beta V} \sum_p [\mathcal{F}(p)\mathcal{F}(p+q)]. \quad (38)$$

The functions  $\Pi_I^{ij}$  for  $I=1,2,3$  are evaluated in Appendixes A and B. To identify the existence of the NG modes and determine their dispersion laws, it is convenient to use the real and imaginary parts of the complex pair fields defined as

$$\begin{aligned} \Phi_I(x) &= [\varphi_I(x) + i\phi_I(x)]/\sqrt{2}, \\ \Phi_I^*(x) &= [\varphi_I(x) - i\phi_I(x)]/\sqrt{2} \end{aligned} \quad (39)$$

for  $I=1,2,3$ . The dispersion laws are determined by the zeros of the determinate of the matrix  $\Pi_I^{ij}$ ,

$$\Pi_I^{11}(q)\Pi_I^{22}(q) - \Pi_I^{12}(q)\Pi_I^{21}(q) = 0. \quad (40)$$

#### A. The $I=1,2$ or $T_4, T_5, T_6, T_7$ sector

For  $I=1,2$ , after the analytical continuation  $i\nu_n \rightarrow q_0 + i\varepsilon$ , the function  $\Pi_I^{11}$  can be expressed as

$$\Pi_I^{11}(q_0, \mathbf{q}) = q_0 H(q_0, \mathbf{q}) + J(q_0, \mathbf{q}), \quad (41)$$

where the functions  $H$  and  $J$  are even function of  $\mathbf{q}$  and hence depend only on  $\mathbf{q}^2$ , see Appendix A. First, let us examine whether there are four gapless NG modes corresponding to the broken generators  $T_4, T_5, T_6, T_7$ . To this end, we take  $\mathbf{q}^2=0$  to calculate the mass gaps of the collective modes. The mass gaps of the collective modes are given by the roots of the equation

$$q_0^2 H(q_0, \mathbf{0}) H(-q_0, \mathbf{0}) = 0, \quad (42)$$

where we have used the fact  $J(q_0, \mathbf{0})=0$ . Obviously,  $q_0^2=0$  is a root which gives two gapless NG modes. To examine whether there exist other two gapless modes, we need to check whether the equation

$$H(0, \mathbf{0}) = 0 \quad (43)$$

is satisfied in the superfluid phase. It is easy to find the interesting relation between  $H(0, \mathbf{0})$  and  $\langle Q_8 \rangle$ ,

$$H(0, \mathbf{0}) = -\frac{n_1 + n_2 - 2n_3}{\Delta^2} = -\frac{\sqrt{3}\langle Q_8 \rangle}{\Delta^2 V} = -\frac{\alpha n}{\Delta^2}. \quad (44)$$

Since  $\alpha$  cannot be zero once BCS pairing occurs, as we have shown in the last section, we conclude that there are only two gapless NG modes, and the other two expected NG modes become massive.

We now calculate the dispersion law of the gapless NG modes and the mass gap of the massive modes. In the low-energy limit  $q_0 \rightarrow 0$ ,  $\mathbf{q} \rightarrow 0$ , we can expand the functions  $H$  and  $J$  as Taylor series of  $(q_0, \mathbf{q})$  at the point  $(0, \mathbf{0})$  and keep only the leading terms. To find the dispersion law of gapless NG modes, we take the expansion

$$q_0 H(0, \mathbf{0}) + \frac{\mathbf{q}^2}{2} \frac{\partial^2 J(q_0, \mathbf{q})}{\partial \mathbf{q}^2} \Big|_{(0,0)} = 0. \quad (45)$$

Due to the relation obtained in Appendix A,

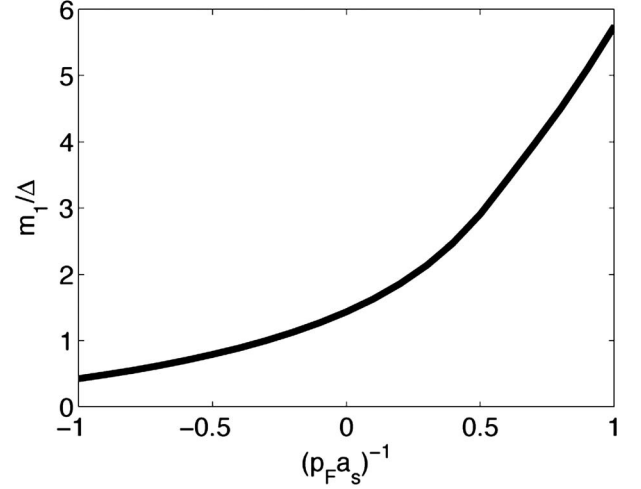


FIG. 2. The ratio between the mass gap  $m_1$  of the massive collective modes and the energy gap  $\Delta$  as a function of  $(p_F a_s)^{-1}$  at  $T=0$ .

$$\frac{\partial^2 J(q_0, \mathbf{q})}{\partial \mathbf{q}^2} \Big|_{(0,0)} = -\frac{1}{m} H(0, \mathbf{0}), \quad (46)$$

the NG modes have a quadratic dispersion law near  $\mathbf{q}^2=0$ ,

$$q_0 = \frac{\mathbf{q}^2}{2m}. \quad (47)$$

It is very interesting that here the quantity  $m$  is just the fermion mass. For the massive modes, we try to find the zero of the function  $H(q_0, \mathbf{q})$ . In the low-energy limit, we take the expansion

$$H(0, \mathbf{0}) + q_0 \frac{\partial H(q_0, \mathbf{q})}{\partial q_0} \Big|_{(0,0)} + \frac{\mathbf{q}^2}{2} \frac{\partial^2 H(q_0, \mathbf{q})}{\partial \mathbf{q}^2} \Big|_{(0,0)} = 0, \quad (48)$$

which leads to the dispersion law

$$q_0 = m_1 + \frac{\mathbf{q}^2}{2m_2}, \quad (49)$$

where the mass gap  $m_1$  is given by

$$m_1 = -H(0, \mathbf{0}) \left\{ \frac{\partial H(q_0, \mathbf{q})}{\partial q_0} \Big|_{(0,0)} \right\}^{-1}, \quad (50)$$

and the quantity  $m_2$  is defined as

$$m_2 = -\frac{\partial H(q_0, \mathbf{q})}{\partial q_0} \Big|_{(0,0)} \left\{ \frac{\partial^2 H(q_0, \mathbf{q})}{\partial \mathbf{q}^2} \Big|_{(0,0)} \right\}^{-1}. \quad (51)$$

In the above calculations, we have employed the trick of Taylor expansion which is valid in the low-energy limit. In Fig. 2 we showed the ratio between the mass gap  $m_1$  of the massive collective modes and the energy gap  $\Delta$  of the fermionic excitations as a function of the coupling  $(p_F a_s)^{-1}$  at  $T=0$ . In the weak coupling limit, the ratio approaches to zero, which reflects the fact that the effect of nonzero  $\alpha$  and hence

nonzero  $H(0, \mathbf{0})$  can be neglected. In this case, we can approximately find five NG modes with linear dispersion law in the energy and momentum region

$$m_1 < q_0 \ll \Delta, \quad m_1 < v_F |\mathbf{q}| \ll \Delta, \quad (52)$$

where  $v_F = p_F/m$  is the Fermi velocity, and our result is consistent with the numerical calculation in [10] where the authors worked in the weak coupling limit and found five NG modes. However, in a wide range of the coupling such as  $-1 < (p_F a_s)^{-1} < 1$  shown in Fig. 2, the mass gap  $m_1$  is of the order of  $\Delta$  and hence the order of  $T_c$ , which means that the effect of nonzero  $\alpha$  cannot be neglected and the mass gap  $m_1$  becomes important for the low-temperature thermodynamics.

We conclude that the abnormal number of NG modes and the mass gap  $m_1$  make sense in a wide range of the coupling and have significant effect on the low-temperature thermodynamics. This situation is quite different from the case of two-flavor color superconductivity in the Nambu-Jona-Lasinio model where the mass gap of the massive modes is very small compared with the energy gap of the quarks [17].

### B. The $I=3$ or $\tilde{T}_8$ sector

For  $I=3$ , the situation is quite conventional. One can check that the functions take the same form as the ones in the two-flavor or U(1) system. Let us first identify that there exists a gapless NG mode corresponding to the broken generator  $\tilde{T}_8$ . To complete this task we need only the explicit form of the functions  $\Pi_3^{ij}$  at  $q_0 = \mathbf{q} = 0$  and check the relation

$$\Pi_3^{11}(0, \mathbf{0})\Pi_3^{22}(0, \mathbf{0}) - \Pi_3^{12}(0, \mathbf{0})\Pi_3^{21}(0, \mathbf{0}) = 0. \quad (53)$$

Using the explicit form of the functions  $\Pi_3^{ij}$  and the gap equation for  $\Delta$ , we obtain the relation

$$\Pi_3^{11}(0, \mathbf{0}) = \Pi_3^{22}(0, \mathbf{0}) = -\Pi_3^{12}(0, \mathbf{0}) = -\Pi_3^{21}(0, \mathbf{0}) \quad (54)$$

at any temperature below  $T_c$ . Hence we have proven that there must be a gapless NG mode corresponding to the broken generator  $\tilde{T}_8$ .

Next we determine the dispersion law of this NG mode. Similarly, we expand the functions  $\Pi_3^{ij}(q_0, \mathbf{q})$  as Taylor series of  $(q_0, \mathbf{q})$  at the point  $(0, \mathbf{0})$  and keep only the leading terms. From the explicit form of the functions  $\Pi_3^{ij}$  evaluated in Appendix B, the Taylor expansions to the lowest order take the following form

$$\begin{aligned} \Pi_3^{11}(q_0, \mathbf{q}) &= A + q_0 B + \mathbf{q}^2 D/2, \\ \Pi_3^{22}(q_0, \mathbf{q}) &= A - q_0 B + \mathbf{q}^2 D/2, \\ \Pi_3^{12}(q_0, \mathbf{q}) &= -A + q_0^2 C/2 + \mathbf{q}^2 E/2, \\ \Pi_3^{21}(q_0, \mathbf{q}) &= -A + q_0^2 C/2 + \mathbf{q}^2 E/2, \end{aligned} \quad (55)$$

where the coefficients  $A, B, C, D, E$  are not explicitly shown. The dispersion law of the NG mode is determined by the equation

$$\begin{aligned} (A + q_0 B + \mathbf{q}^2 D/2)(A - q_0 B + \mathbf{q}^2 D/2) \\ - (-A + q_0^2 C/2 + \mathbf{q}^2 E/2)^2 = 0. \end{aligned} \quad (56)$$

Keeping the lowest order in  $q_0$  and  $\mathbf{q}$ , we obtain a linear dispersion law

$$q_0 = v_s |\mathbf{q}|. \quad (57)$$

The velocity of the NG mode is given by

$$v_s = \sqrt{\frac{A(D+E)}{B^2 - AC}}. \quad (58)$$

Since the functions  $\Pi_3^{ij}$  take the same form as the ones in the two-flavor system with broken U(1) symmetry, the behavior of this NG mode will be the same as the one in the two-flavor system [21,22].

### V. CAN WE RECOVER FIVE NG MODES?

We have shown that in a three-flavor Fermi gas with SU(3) gauge symmetry that there are only three gapless NG modes in the superfluid state. A natural question we may ask is the possibility to recover the five NG modes. In this section we will argue that there is no way to obtain five NG modes in such a system.

First, one may criticize that the abnormal number of NG modes may be due to the specific choice of the symmetry breaking direction  $\Delta_1 = \Delta_2 = 0, \Delta_3 \equiv \Delta$ . If we take the following symmetry breaking direction

$$\Delta_1 = \Delta_2 = \Delta_3 = \Delta/\sqrt{3}, \quad (59)$$

we have automatically  $n_1 = n_2 = n_3$  and hence  $\langle Q_8 \rangle = 0$ , and we may expect five NG modes with this choice. However, as we have emphasized, the physical quantities such as the ground state and the number and dispersion laws of the NG modes do not depend on the specific choice of the symmetry breaking direction. In the symmetric case with  $\Delta_1 = \Delta_2 = \Delta_3$ , the broken and unbroken generators will be changed, and correspondingly  $\langle Q_1 \rangle, \langle Q_4 \rangle$ , and  $\langle Q_6 \rangle$  are nonzero. If one works with this choice, he will certainly obtain the same number and dispersion laws of the NG modes as we have obtained.

Second, we may relax the constraint of equal chemical potentials for the three flavors. For instance, with the choice  $\Delta_1 = \Delta_2 = 0$  and  $\Delta_3 \equiv \Delta$ , we can set  $\mu_1 = \mu_2 = \mu$  for flavors 1 and 2 and  $\mu_3 = \mu + \mu_0$  for flavor 3. By requiring  $n_1 = n_2 = n_3$  we can guarantee that all  $\langle Q_a \rangle$  are zero and the condition  $\langle [Q_i, Q_j] \rangle = 0$  is satisfied for any  $i$  and  $j$ . However, in this case, the chemical potentials must be adjusted, a proper value of  $\mu_0 \neq 0$  is needed, and the SU(3) symmetry of the model Lagrangian is explicitly broken with the broken generators  $T_4, T_5, T_6, T_7$ . An explicit calculation like the one in [19] shows that the four NG modes corresponding to the  $I = 1, 2$  sector obtain a mass gap  $\mu_0$ . In the BCS region where the quantity  $\alpha$  in the equal chemical potential system is small, the corresponding  $\mu_0$  is also small. Such a phenomenon can be regarded as the spontaneous breaking of the approximate symmetry, and the corresponding collective

modes with a small mass gap  $\mu_0$  can be considered as pseudo-NG modes like the case of color superconductivity in the Nambu-Jona-Lasinio model [19]. However, since the nonzero  $\mu_0$  explicitly breaks the SU(3) symmetry, a specific choice of the symmetry breaking direction may be dangerous [20]. In fact, if we calculate the susceptibilities

$$\left. \frac{\partial^2 \Omega}{\partial \Delta_1^2} \right|_{\Delta_1=\Delta_2=0} = \left. \frac{\partial^2 \Omega}{\partial \Delta_2^2} \right|_{\Delta_1=\Delta_2=0}, \quad (60)$$

we find they are negative for  $\mu_0 \neq 0$ . This indicates that if the numbers of the three flavor are fixed, the specific choice of the pairing pattern is forbidden. In this case, we should solve the chemical potentials  $\mu_1, \mu_2, \mu_3$  and the condensates  $\Delta_1, \Delta_2, \Delta_3$  from a large set of equations.

In conclusion, we can never find five NG modes in the SU(3) system. Only in the BCS limit we can obtain five approximate NG modes with linear dispersions via neglecting the effect of nonzero  $\alpha$ .

## VI. SUMMARY

We have investigated the superfluidity and the associated NG modes in an atomic Fermi gas with three degenerate hyperfine states. In our model, the pairing occurs in the  $s$  wave and flavor antitriplet channel, and the chemical potentials are constrained to be equal due to chemical equilibrium. In the superfluid state, there are both gapped and gapless fermionic excitations, i.e., paired and unpaired fermions. Only in the BEC region where the chemical potential becomes negative, the unpaired fermions disappear at zero temperature. Once the pairs are condensed, the SU(3) symmetry is spontaneously broken down to a SU(2) subgroup with five broken generators. We showed that there are only three NG modes, the one corresponding to the diagonal generator is conventional and has linear dispersion law, and the other two have quadratic dispersion law. The additional two expected NG modes obtain a mass gap. While the mass gap is very small compared with the energy gap  $\Delta$  of the fermions in the BCS limit, it is of the order of  $\Delta$  in a wide range of the coupling  $(p_F a_s)^{-1}$  and can be reached in experiments of atomic Fermi gas. As a consequence, the abnormal number of the NG modes, the quadratic dispersion law, and the mass gap have significant effect on the low-temperature thermodynamics of the three-flavor Fermi gas.

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## APPENDIX A: THE FUNCTIONS $\Pi_I^j(q)$ FOR $I=1,2$

In this Appendix we evaluate the functions  $\Pi_I^j(q)$  for  $I=1,2$ . From the relations  $\Pi_I^{22}(q)=\Pi_I^{11}(-q)$  and  $\Pi_I^{12}(q)=\Pi_I^{21}(q)=0$ , we need to evaluate  $\Pi_I^{11}$  only. After the Matsubara frequency summation and the analytical continuation  $i\nu_n \rightarrow q_0 + i\varepsilon$ , we obtain

$$\begin{aligned} \Pi_I^{11}(q) = & \frac{1}{g} + \frac{1}{2} \int \frac{d^3 \mathbf{p}}{(2\pi)^3} \left[ \frac{1-f(E_p)-f(\xi_{p-q})}{q_0-E_p-\xi_{p-q}} u_p^2 \right. \\ & + \frac{f(E_p)-f(\xi_{p-q})}{q_0+E_p-\xi_{p-q}} v_p^2 + \frac{1-f(E_p)-f(\xi_{p+q})}{q_0-E_p-\xi_{p+q}} u_p^2 \\ & \left. + \frac{f(E_p)-f(\xi_{p+q})}{q_0+E_p-\xi_{p+q}} v_p^2 \right] \end{aligned} \quad (A1)$$

with the coherent coefficients  $u_p^2$  and  $v_p^2$  defined as  $u_p^2=(1+\xi_p/E_p)/2$  and  $v_p^2=(1-\xi_p/E_p)/2$ . In the superfluid phase with  $\Delta \neq 0$ , using the gap equation for  $\Delta$ , we can express it as

$$\Pi_I^{11}(q) = q_0 H(q) + J(q) \quad (A2)$$

with the functions  $H(q)$  and  $J(q)$  defined as

$$\begin{aligned} H(q) = & \int \frac{d^3 \mathbf{p}}{(2\pi)^3} \frac{1}{2E_p} \left[ \frac{1-f(E_p)-f(\xi_{p-q})}{q_0-E_p-\xi_{p-q}} - \frac{f(E_p)-f(\xi_{p-q})}{q_0+E_p-\xi_{p-q}} \right. \\ & \left. + \frac{1-f(E_p)-f(\xi_{p+q})}{q_0-E_p-\xi_{p+q}} - \frac{f(E_p)-f(\xi_{p+q})}{q_0+E_p-\xi_{p+q}} \right], \\ J(q) = & \int \frac{d^3 \mathbf{p}}{(2\pi)^3} \left[ \frac{\xi_p-\xi_{p-q}}{2E_p} \left( \frac{1-f(E_p)-f(\xi_{p-q})}{q_0-E_p-\xi_{p-q}} \right. \right. \\ & \left. \left. - \frac{f(E_p)-f(\xi_{p-q})}{q_0+E_p-\xi_{p-q}} \right) + \frac{\xi_p-\xi_{p+q}}{2E_p} \left( \frac{1-f(E_p)-f(\xi_{p+q})}{q_0-E_p-\xi_{p+q}} \right. \right. \\ & \left. \left. - \frac{f(E_p)-f(\xi_{p+q})}{q_0+E_p-\xi_{p+q}} \right) \right]. \end{aligned} \quad (A3)$$

We now list some properties of the functions  $H(q)$  and  $J(q)$  which are useful to determine the dispersion laws. It is easy to observe that the functions  $H$  and  $J$  are both even functions of  $\mathbf{q}$ ,

$$H(q_0, \mathbf{q}) = H(q_0, -\mathbf{q}), \quad J(q_0, \mathbf{q}) = J(q_0, -\mathbf{q}). \quad (A4)$$

The function  $H(0, \mathbf{0})$  can be expressed as

$$H(0, \mathbf{0}) = - \int \frac{d^3 \mathbf{p}}{(2\pi)^3} \frac{1}{E_p} \left[ \frac{1-f(E_p)-f(\xi_p)}{E_p+\xi_p} + \frac{f(E_p)-f(\xi_p)}{E_p-\xi_p} \right]. \quad (A5)$$

By using the identities

$$\frac{1}{E_p E_p + \xi_p} = \frac{2v_p^2}{\Delta^2}, \quad \frac{1}{E_p E_p - \xi_p} = \frac{2u_p^2}{\Delta^2}, \quad (A6)$$

we derive the relation between  $H(0, \mathbf{0})$  and  $\langle Q_8 \rangle$ ,

$$H(0, \mathbf{0}) = - \frac{n_1 + n_2 - 2n_3}{\Delta^2} = - \frac{\sqrt{3} \langle Q_8 \rangle}{\Delta^2 V}. \quad (A7)$$

The derivative of the function  $H$  with respect to  $q_0$  at  $q_0=0, \mathbf{q}=\mathbf{0}$  can be written as

$$\left. \frac{\partial H(q_0, \mathbf{q})}{\partial q_0} \right|_{(0,0)} = - \int \frac{d^3 \mathbf{p}}{(2\pi)^3} \frac{1}{E_p} \left[ \frac{1 - f(E_p) - f(\xi_p)}{(E_p + \xi_p)^2} - \frac{f(E_p) - f(\xi_p)}{(E_p - \xi_p)^2} \right]. \quad (\text{A8})$$

The function  $J(q)$  can be expressed as

$$J(q) = - \frac{q^2}{2m} H(q) + \frac{1}{m} \int \frac{d^3 \mathbf{p}}{(2\pi)^3} \frac{\mathbf{p} \cdot \mathbf{q}}{2E_p} \left[ \frac{1 - f(E_p) - f(\xi_{p-q})}{q_0 - E_p - \xi_{p-q}} - \frac{f(E_p) - f(\xi_{p-q})}{q_0 + E_p - \xi_{p-q}} - \frac{1 - f(E_p) - f(\xi_{p+q})}{q_0 - E_p - \xi_{p+q}} + \frac{f(E_p) - f(\xi_{p+q})}{q_0 + E_p - \xi_{p+q}} \right], \quad (\text{A9})$$

from which we obtain  $J(q_0, \mathbf{0}) = 0$  and

$$\left. \frac{\partial^n J(q_0, \mathbf{q})}{\partial q_0^n} \right|_{(0,0)} = 0 \quad (\text{A10})$$

for any integer  $n$ . For the derivative with respect to  $\mathbf{q}$ , only the second derivative is nonzero,

$$\left. \frac{\partial^2 J(q_0, \mathbf{q})}{\partial \mathbf{q}^2} \right|_{(0,0)} = - \frac{1}{m} H(0, \mathbf{0}), \quad \left. \frac{\partial^n J(q_0, \mathbf{q})}{\partial \mathbf{q}^n} \right|_{(0,0)} = 0, \quad n \neq 2. \quad (\text{A11})$$

## APPENDIX B: THE FUNCTIONS $\Pi_i^j(q)$ FOR $i=3$

In this Appendix we evaluate the functions  $\Pi_i^j(q)$  for  $I=3$ . From the relations  $\Pi_3^{22}(q) = \Pi_3^{11}(-q)$  and  $\Pi_3^{21}(q) = \Pi_3^{12}(q)$ , we need to evaluate  $\Pi_3^{11}$  and  $\Pi_3^{12}$  only. Completing the Matsubara frequency summation and performing a shifting  $\mathbf{p} \rightarrow \mathbf{p} - \mathbf{q}/2$ , we obtain

$$\Pi_3^{11}(q) = \frac{1}{g} + \int \frac{d^3 \mathbf{p}}{(2\pi)^3} \left[ \left( \frac{u_{p-q/2}^2 u_{p+q/2}^2}{q_0 - E_{p-q/2} - E_{p+q/2}} - \frac{v_{p-q/2}^2 v_{p+q/2}^2}{q_0 + E_{p-q/2} + E_{p+q/2}} \right) [1 - f(E_{p-q/2}) - f(E_{p+q/2})] + \left( \frac{v_{p-q/2}^2 u_{p+q/2}^2}{q_0 + E_{p-q/2} - E_{p+q/2}} - \frac{u_{p-q/2}^2 v_{p+q/2}^2}{q_0 - E_{p-q/2} + E_{p+q/2}} \right) \times [f(E_{p-q/2}) - f(E_{p+q/2})] \right],$$

$$\Pi_3^{12}(q) = \Delta^2 \int \frac{d^3 \mathbf{p}}{(2\pi)^3} \left[ \left( \frac{1}{q_0 - E_{p-q/2} - E_{p+q/2}} - \frac{1}{q_0 + E_{p-q/2} + E_{p+q/2}} \right) \frac{1 - f(E_{p-q/2}) - f(E_{p+q/2})}{2E_{p-q/2} E_{p+q/2}} + \left( \frac{1}{q_0 + E_{p-q/2} - E_{p+q/2}} - \frac{1}{q_0 - E_{p-q/2} + E_{p+q/2}} \right) \frac{f(E_{p-q/2}) - f(E_{p+q/2})}{2E_{p-q/2} E_{p+q/2}} \right]. \quad (\text{B1})$$

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