High-fidelity transmission of quantum polarization states through birefringent optical fibers

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High fidelity transmission of quantum polarization states is essential for realizing quantum networks. The degenerate Ψ^- state is an eigenstate of a transmission fiber. In this paper we propose a method for preparing the degenerate Ψ^- state using spontaneous four wave mixing (SFWM) in a fiber Sagnac loop excited by two orthogonally polarized pump waves. This configuration includes a 45° Faraday rotator, which creates a relative phase of π between the two counterpropagating entangled photon pairs with indistinguishability both in frequencies and generation paths. We confirm that only the amplitude is degraded in the output wave form of nonlinear sum frequency generation in the Bell state analyzer after propagation through the fiber due to the polarization mode dispersion.

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Currently, only seven photons in a signal pulse, which correspond to the received power of −50.5 dBm, are sufficient to realize error free transmission in the most sophisticated 10 Gb/s return-to-zero differential phase shift keying (RZ-DPSK) modulation scheme with the forward error correction code [1]. It is highly desirable to be able to use quantum correlations between these photons to overcome the limitations of classical communication scheme. High-fidelity transmission of the quantum polarization state is essential for realizing quantum networks. Since a fiber has a temporally unstable Jones matrix due to variation in its birefringence, the initial polarization state of a photon, (such as one photon of Alice's EPR pair that is transmitted to Bob through the fiber in the quantum teleportation [2]), cannot be preserved. The scattering of Bell polarization states in propagation through a fiber was discussed preliminarily to show that the degenerate asymmetric Bell state Ψ^- is an eigenstate of the lossless fiber and its eigenvalue is independent of the fiber birefringence [3]. Bell states have been generated in a fiber by employing a fiber loop with a polarization beam splitter [4], and by using the Sagnac configuration [5]. In both methods, however, the polarization entangled states are not degenerate which means that they have to be separated by a filter into signal and idler photons so that their phase and polarization states can be controlled, and then be recombined to Ψ^- state.

In this report, we propose a method for generating degenerate Ψ^- state in a fiber by the addition of only a 45° Faraday rotator and two frequency-separated orthogonal pump waves, to the Sagnac configuration described in Ref. [5]. We also discuss Ψ^- transmission though a fiber having polarization mode dispersion (PMD).

Figure 1 illustrates the proposed degenerate Ψ^- transmission system, which consists of a generator of degenerate Ψ^- state, a transmission fiber and the Bell state analyzer for detecting Ψ^- state [6]. Figure 2 depicts the degenerate Ψ^- generator which consists of two pump sources having frequencies ω_1 and ω_2, and a loop fiber in the Sagnac configuration and a 45° Faraday rotator. We suppose two orthogonally polarized pump waves having frequencies ω_1 and ω_2. The frequency separation of these two pump frequencies is sufficiently large to make the contribution of nonlinear processes, such as $2\omega_1-\omega_3$, negligible near the zero dispersion frequency ω_o [7]. The dominant nonlinear processes are $\chi^{(3)}E_1^xE_2^yE_3^{x*}$ for the idler E_4^y (or $\chi^{(3)}E_1^xE_2^yE_3^{y*}$ for the idler E_4^x) and $\chi^{(3)}E_1^xE_2^yE_4^{y*}$ for the signal E_3^x (or $\chi^{(3)}E_1^xE_2^yE_4^{x*}$ for the signal E_3^y), which generate orthogonally polarized photon pair, having frequencies $\omega_3=\omega_4$. These photon pairs have indistinguishable frequencies and generation paths in the clockwise (CW) and counterclockwise (CCW) directions in the Sagnac configuration. Here we assume the bandwidth of an optical filter is sufficiently narrow to limit the frequency indistinguishability in degenerate $\omega_3=\omega_4$.

The output entangled polarization state with the relative phase ϕ_r is described as follows:

$$\Psi = \alpha|V\rangle_3|H\rangle_4 + \beta e^{j\phi_r}|H\rangle_3|V\rangle_4, \tag{1}$$

where the relative phase ϕ_r is calculated as follows [4]:

$$\phi_r = (k_{1H}+k_{2V}-k_{3H}-k_{4V})x + (k_{1H}+k_{2V}-k_{3V}-k_{4H})y, \tag{2}$$

where k_{3H} denotes the wave number of the signal having horizontal polarization $|H\rangle_3$, for example. Also x denotes the

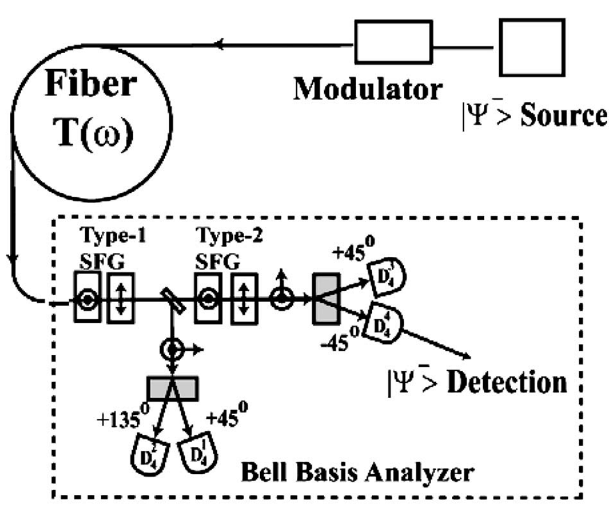

FIG. 1. Eigenstate Ψ^- optical fiber communication system.

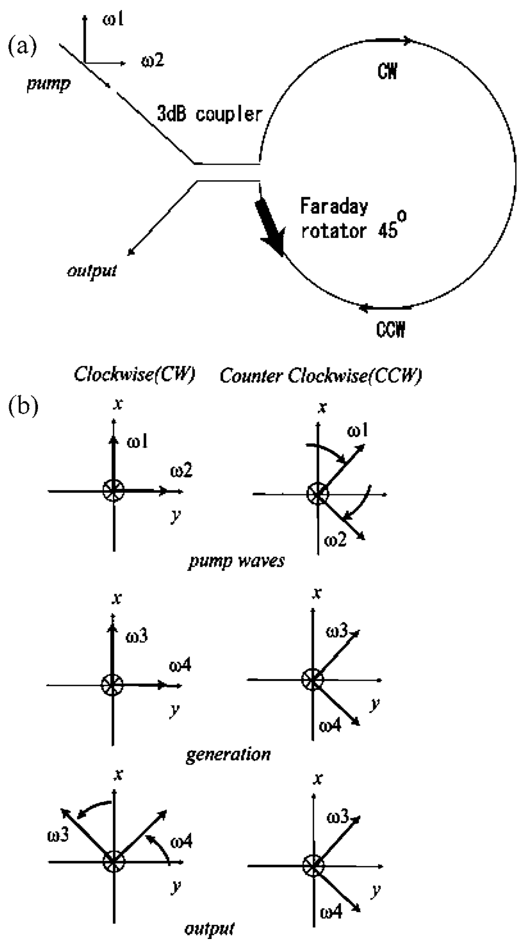

FIG. 2. Bell polarization state Ψ^- generation in the Sagnac configuration. (a) Ψ^- generator configuration: the orthogonal polarized pump waves generate entangled copolarized waves using SFWM. Two items have been added to the nonlinear Sagnac configuration of Ref. [5]: two orthogonal pumps with frequencies ω_1 and ω_2, and a 45° Faraday rotator to the relative phase ϕ_r. (b) Relative phase of π between the CW and the counter CCW waves is realized by the configuration with a 45° Faraday rotation, with indistinguishability. The top row shows the polarizations for the CW and CCW pump waves. The second row shows the polarizations of the CW and CCW converted waves. The bottom row shows the polarizations of the CW and CCW converted waves, at the output of the Ψ^- generator.

distance from the 3 dB coupler to the point where the signal and idler photon pair are generated in the CW direction, while y denotes the corresponding distance for the photon pair generated in the CCW direction. Since the coefficients of x and y are zero when the phase matching condition is satisfied, the relative phase ϕ_r is negligible when there is a small dispersion slope [4].

By inserting a 45° Faraday rotator, the relative phase ϕ_r can be converted equivalently to π: Immediately after entering the loop, the pump waves coupled into the CW propagation are considered to be rotated through 45° in the right-hand coordinate system shown in Fig. 2. The signal and idler photons in the CW propagation are generated having copolarized states with the pump waves [7], so that the signal and idler should be indistinguishable [8]. On the other hand, photon pairs generated CCW propagation having states copolarized with the CCW pump waves which are rotated by 45° in the right-hand coordinate system, immediately before exiting the loop. The relative phase ϕ_r as observed at the output port of the loop fiber as shown in Fig. 2 is recognized to be π, so that the entangled polarization state at the output is the degenerate Ψ^- state, which is indistinguishable with respect to the CW and CCW directions and also to the signal and idler frequencies.

An additional merit of this scheme is the easiness of eliminating the residual pump waves by just employing an optical band pass filter for the degenerate frequencies of ω_3 and ω_4.

For broadband applications, the polarization mode dispersion of the fiber degrades the fidelity of the Bell state Ψ^-. The positive frequency electric field operators for the photon pair generated by the SFWM process are described as follows [9]:

$$\hat{E}_3^+(t) = E_0 \int_{-\infty}^{+\infty} d\varepsilon e^{-\varepsilon^2/2\sigma^2} e^{-i(\omega_0+\varepsilon)t} \hat{a}_3, \tag{3}$$

$$\hat{E}_4^+(t) = E_0 \int_{-\infty}^{+\infty} d\varepsilon e^{-\varepsilon^2/2\sigma^2} e^{-i(\omega_0-\varepsilon)t} \hat{a}_4, \tag{4}$$

where the photon pair is assumed to have temporal wave forms determined by the spectrum limiting optical filter $g(\omega)=e^{-(\omega-\omega_0)^2/2\sigma^2}$, and to satisfy the energy conservation condition $2\omega_0=\omega_1+\omega_2=\omega_3+\omega_4$. The signal frequency is denoted by $\omega_3=\omega_0+\varepsilon$, while the idler frequency is denoted by $\omega_4=\omega_0-\varepsilon$. Finally, $\hat{a}_3$ and $\hat{a}_4$ are the annihilation operators of the signal ω_3 and idler ω_4 waves, respectively.

The transmission of the photon pair through an optical fiber is described by the tensor product

$$\frac{1}{\sqrt{2}}\left[\begin{pmatrix}\tilde{E}_{11}^{3+}(t)\\ \tilde{E}_{21}^{3+}(t)\end{pmatrix}\otimes\begin{pmatrix}\tilde{E}_{12}^{4+}(t)\\ \tilde{E}_{22}^{4+}(t)\end{pmatrix}-\begin{pmatrix}\tilde{E}_{12}^{3+}(t)\\ \tilde{E}_{22}^{3+}(t)\end{pmatrix}\otimes\begin{pmatrix}\tilde{E}_{11}^{4+}(t)\\ \tilde{E}_{21}^{4+}(t)\end{pmatrix}\right] = \frac{1}{\sqrt{2}}\begin{pmatrix}\tilde{E}_{11}^{3+}\tilde{E}_{12}^{4+}-\tilde{E}_{12}^{3+}\tilde{E}_{11}^{4+}\\ \tilde{E}_{11}^{3+}\tilde{E}_{22}^{4+}-\tilde{E}_{12}^{3+}\tilde{E}_{21}^{4+}\\ \tilde{E}_{21}^{3+}\tilde{E}_{12}^{4+}-\tilde{E}_{22}^{3+}\tilde{E}_{11}^{4+}\\ \tilde{E}_{21}^{3+}\tilde{E}_{22}^{4+}-\tilde{E}_{22}^{3+}\tilde{E}_{21}^{4+}\end{pmatrix}, \tag{5}$$

where the amplitude of the positive frequency electric field operator $\tilde{E}_{11}^{3+}(t)$ after the fiber propagation is calculated by

$$\tilde{E}_{11}^{3+}(t) = E_0 \int_{-\infty}^{+\infty} T_{11}(\varepsilon) e^{-\varepsilon^2/2\sigma^2} e^{-i(\omega_0+\varepsilon)t} d\varepsilon \tag{6}$$

for example, and $T_{11}(\varepsilon)$ is the matrix component of the Jones matrix $T(\omega)$ of the fiber is denoted by

$$T(\omega) = \exp\{i\bar{\phi}(\omega)\hat{\sigma}_3\}\exp\{i\bar{\Theta}(\omega)\hat{\sigma}_2\}\exp\{i\bar{\psi}(\omega)\hat{\sigma}_3\}\exp\{iX(\omega)\}. \tag{7}$$

The Pauli's operators are denoted by $\sigma_i(i=1,2,3)$ while $X(\omega)$ is the phase shift due to polarization independent dispersion. The Euler's generalized phase shifts can be expanded in Taylor series as follows [10,11]:

$$\bar{\Theta}(\omega) = \bar{\Theta}(\omega_0) + \bar{\alpha}_1(\omega - \omega_0) + \frac{1}{2!}\bar{\alpha}_2(\omega - \omega_0)^2 + \cdots$$

$$\bar{\phi}(\omega) = \bar{\phi}(\omega_0) + \bar{\beta}_1(\omega - \omega_0) + \frac{1}{2!}\bar{\beta}_2(\omega - \omega_0)^2 + \cdots$$

$$\bar{\psi}(\omega) = \bar{\psi}(\omega_0) + \bar{\gamma}_1(\omega - \omega_0) + \frac{1}{2!}\bar{\gamma}_2(\omega - \omega_0)^2, \tag{8}$$

where these Taylor expansion coefficients are termed the basic polarization mode dispersion parameters.

The Bell analyzer after Kim [6] gives the $|\Psi^-\rangle$ detector output for the input entangled state of Eq. (5), considering the $-45°$ projection of the type II sum frequency generation (SFG) output shown in Fig. 1

$$D_4(t) = \frac{(\chi^{(2)})^2}{2}\left|\left(\{\tilde{E}_{11}^{3+}(t)\tilde{E}_{22}^{4+}(t) - \tilde{E}_{12}^{4+}(t)\tilde{E}_{21}^{3+}(t)\} - \{\tilde{E}_{12}^{3+}(t)\tilde{E}_{21}^{4+}(t) - \tilde{E}_{11}^{4+}(t)\tilde{E}_{22}^{3+}(t)\}\right|^2, \tag{9}$$

where the contents of the first bracket corresponds to the SFG output of the second row of the tensor product of Eq. (5) generated by the first type II SFG in horizontal SFG polarization of Kim's configuration, and the second bracket corresponds to the SFG output of the third row of the tensor product of Eq. (5) generated by the second type II SFG in vertical SFG polarization, which are projected onto the $-45°$ direction relative to the horizontal. The other rows in the tensor product cannot generate single photons by SFG due to the type II SFG crystal symmetry.

A numerical example for the evaluation of $D_4(t)$ is shown in Fig. 3. The $D_4(t)$ is degraded in amplitude but the wave form shows no degradation, in the case of isolated pulse. For σ=3 *Thz* the detection pulse $D_4(t)$ is degraded by 12 dB in intensity for $\bar{\alpha}_2=\bar{\beta}_2=\bar{\gamma}_2=1$ ps^2 and $\bar{\alpha}_1=\bar{\beta}_1=\bar{\gamma}_1=1$ ps, but the broadening of pulse width is negligible. However, the constituent signals such as $\tilde{E}_{11}^{3+}(t)$ exhibit a delay equal to the half width of the original pulse. These deformations in the constituent components result from the scattering of Ψ^- to the other Bell states. It should be noted that this feature is suitable for time synchronization.

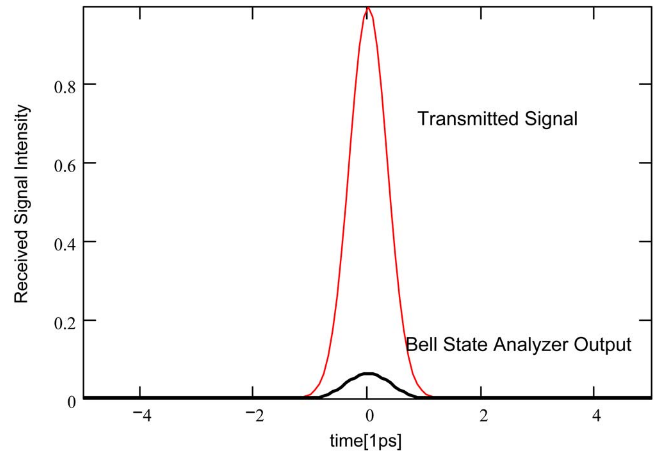

FIG. 3. (Color online) The output wave form of the bell state analyzer. For the Gauss wave form limiting filter with σ=3 Thz at the transmitter, the intensity of the detection pulse $D_4(t)$ of the bell state analyzer is degraded by 12 dB in intensity due to $\bar{\alpha}_2=\bar{\beta}_2=\bar{\gamma}_2=1$ ps^2 and $\bar{\alpha}_1=\bar{\beta}_1=\bar{\gamma}_1=1$ ps, but broadening of the pulse width is negligible.

In conclusion, we propose a system for generating the degenerate Ψ^- state in an optical fiber. The propagation of the degenerate Ψ^- state as the eigenstate of the optical fiber is analyzed. It should be noted that for practical narrow bandwidth applications the scattering of Ψ^- is negligible after propagation through an optical fiber.

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