Conditional homodyne detection of light with squeezed quadrature fluctuations

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We discuss the detection of field quadrature fluctuations in conditional homodyne detection experiments and possible sources of error in such an experiment. We also present modifications to these experiments to help eliminate such errors and extend their range of applicability.

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I. INTRODUCTION

Fluctuations of light provide a window on the underlying quantum dynamical evolution of a light-emitting source. Emission of a photon by a light source signals quantum fluctuations in progress, and a measurement that is conditioned on a photodetection allows us to study the time evolution of the fluctuations [1]. For example, measurement of light intensity conditioned on a photodetection, also known as two-time intensity correlation, reveals information regarding bunching and antibunching that is not available in unconditional intensity measurements [2–6].

The conditional measurement of quadrature fluctuations (CMQF) proposed by Carmichael *et al.* [7] reveals in a novel way the nonclassical nature of light from a cavity containing two-level atoms; this has been experimentally observed by Foster *et al.* [8]. Nonclassical effects in conditional intensity and squeezing in light from a degenerate parametric oscillator have been studied [6]. The conventional methods of detecting quadrature squeezing involve unconditional measurements that are degraded by detection inefficiencies and do not explore the time evolution of quadrature fluctuations [1,2,9,10]. The CMQF, on the other hand, is essentially independent of detection efficiency and provides a sensitive probe of the fluctuations' development over time. It has been shown that the conditional measurement can reveal remarkable nonclassical behavior of not only squeezed but also of unsqueezed quadrature fluctuations [6–8].

In Sec. II, we briefly summarize the theoretical concepts underlying a CMQF experiment [6,7]. We then consider intracavity second harmonic generation (ISHG) [11–13] and present a theoretical analysis of the nonclassical features of ISHG quadrature fluctuations.

The CMQF technique achieves a measurement of the quadrature fluctuations of a given source field by cross-correlating a photon count with a balanced homodyne detection. This technique requires the use of auxiliary coherent oscillators (coherent laser sources), and the amplitudes or intensities of the coherent oscillator fields must be set to values that depend on the properties of the source field. It can be shown that the accuracy of the measurement's final results is extremely sensitive to the precision of these adjustments. In some cases, a very small error in these adjustments may give rise to incorrect conclusions about the state of the source field's quadrature fluctuations. We demonstrate this in Sec. III by developing a theoretical model of such an error in the CMQF measurement and exploring its effects on conclusions that might be drawn about the state of the ISHG field.

Weak fields with approximately Gaussian fluctuations are ideal candidates for accurate quadrature fluctuation measurements using the CMQF method, provided that coherent field adjustments can be made sufficiently precise. However, the technique is limited in its applicability to a source field with nonzero third-order moments [14,15], which may obscure the results of the measurement, even with perfect control of coherent light parameters. In Sec. IV, we propose an extension of the CMQF method that can achieve a measurement of the quadrature fluctuations of a completely generic source field while eliminating the effects of third-order fluctuation moments. Additionally, this extended CMQF measurement avoids the need for precise adjustments of coherent laser fields, thus perhaps averting the kinds of error discussed in Sec. III. In Sec. V, we summarize our findings.

II. CONDITIONAL MEASUREMENT OF QUADRATURE FLUCTUATIONS FOR ISHG

The quadrature variables for an optical field with annihilation and creation operators $\hat{a}_s$ and $\hat{a}_s^\dagger$ are defined by

$$\hat{X}_\phi = \tfrac{1}{2}(e^{-i\phi}\hat{a}_s + e^{i\phi}\hat{a}_s^\dagger), \tag{1}$$

$$\hat{Y}_\phi = \frac{1}{2i}(e^{-i\phi}\hat{a}_s - e^{i\phi}\hat{a}_s^\dagger) = \hat{X}_{\phi+\pi/2}, \tag{2}$$

where ϕ is an arbitrary phase [2]. It follows from this quadrature definition that the variances $\langle:(\Delta\hat{X}_\phi)^2:\rangle$ and $\langle:(\Delta\hat{Y}_\phi)^2:\rangle$ are related to the intensity of the field fluctuations $\langle\Delta\hat{a}_s^\dagger\Delta\hat{a}_s\rangle$ by

$$\langle\Delta\hat{a}_s^\dagger\Delta\hat{a}_s\rangle = \langle:(\Delta\hat{X}_\phi)^2:\rangle + \langle:(\Delta\hat{Y}_\phi)^2:\rangle, \tag{3}$$

where colons denote time and normal ordering of the operators enclosed by them. For classical fields, both quadrature variances are always greater than or equal to zero, being equal to zero only in the classical coherent state. For quantum fields, however, the normally ordered variance of a quadrature $\hat{X}_\phi$ can become negative as long as the normally ordered variance of $\hat{Y}_\phi$ increases in such a way that Eq. (3) is still satisfied. In such a case, the quadrature $\hat{X}_\phi$ is said to be squeezed and the field $\hat{a}$ is said to be in a squeezed state [2]. This fact and the fact that the fluctuation intensity is nonnegative lead to the inequality [6,7]

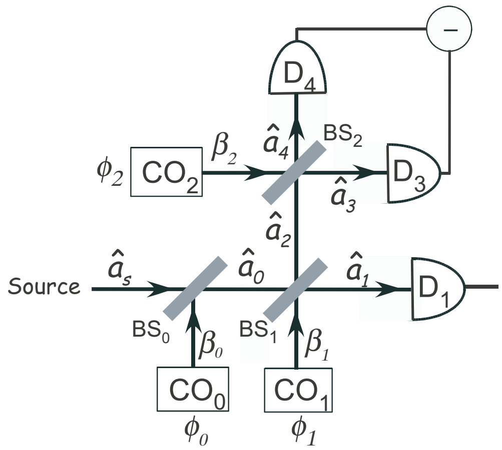

FIG. 1. (Color online) An outline of the setup for conditional measurement of quadrature fluctuations of a source $\hat{a}_s$ with vanishing third-order field correlations.

$$0 \leq \frac{\langle :(\Delta \hat{X}_\phi)^2: \rangle}{\langle \Delta \hat{a}_s^\dagger \Delta \hat{a}_s \rangle} \leq 1. \tag{4}$$

This is satisfied by classical fields but is violated by quantum fields in squeezed states. This inequality has the potential to reveal nonclassical behavior both when the quadrature $\hat{X}_\phi$ is squeezed and when it is unsqueezed (i.e., when it has a nonnegative variance). The lower bound is violated if $\hat{X}_\phi$ is squeezed, whereas the upper bound is violated if $\hat{X}_\phi$ is unsqueezed but nonetheless nonclassical in that its conjugate $\hat{Y}_\phi$ is squeezed.

One can also establish a classical bound for the two-time autocorrelation function of the quadrature fluctuations $\langle :\Delta \hat{X}_\phi(0) \Delta \hat{X}_\phi(\tau): \rangle$ by using the Schwarz inequality [6,7]

$$\frac{\langle :\Delta \hat{X}_\phi(0) \Delta \hat{X}_\phi(\tau): \rangle}{\langle \Delta \hat{a}_s^\dagger \Delta \hat{a}_s \rangle} \leq \frac{\langle :(\Delta \hat{X}_\phi)^2: \rangle}{\langle \Delta \hat{a}_s^\dagger \Delta \hat{a}_s \rangle}, \tag{5}$$

which holds for all time intervals $\tau \geq 0$. This inequality is not directly related to quadrature squeezing; rather, violation of this inequality is a sign of nonclassical temporal correlations. It is possible for a quadrature $\hat{X}_\phi$ to be unsqueezed (or even for the field $\hat{a}_s$ not to be in a squeezed state) but still violate inequality (5) and, thus, still exhibit this uniquely quantum behavior. Note that, the variance $\langle :(\Delta \hat{X}_\phi)^2: \rangle$ is simply the value of the correlation function $\langle :\Delta \hat{X}_\phi(0) \Delta \hat{X}_\phi(\tau): \rangle$ at $\tau=0$. Thus, the classical inequalities (4) and (5) can be used to probe the nonclassical character of the quadrature fluctuations of any optical field $\hat{a}_s$, provided that its normalized quadrature correlation function $\langle :\Delta \hat{X}_\phi(0) \Delta \hat{X}_\phi(\tau): \rangle / \langle \Delta \hat{a}_s^\dagger \Delta \hat{a}_s \rangle$ can be experimentally measured.

Figure 1 shows the setup for a conditional homodyne detection experiment that can measure this function for a source field $\hat{a}_s$. The coherent oscillators CO$_0$, CO$_1$, and CO$_2$ are assumed to be lasers operating far above threshold, thus emitting coherent-state fields of definite complex amplitudes $\beta_0 = |\beta_0| e^{i\phi_0}$, $\beta_1 = |\beta_1| e^{i\phi_1}$, and $\beta_2 = |\beta_2| e^{i\phi_2}$. Phases of these coherent fields are measured relative to the phase the field $\langle \hat{a}_s \rangle$. The beam splitters BS$_0$, BS$_1$, and BS$_2$ are assumed to be lossless and antisymmetric and to have coefficients of reflection and transmission $R=T=1/2$. A glass plate with a silvered upper surface is an example of such a beam splitter.

The source field is mixed with the field from CO$_0$ at BS$_0$, producing the output field $\hat{a}_0$ given by

$$\hat{a}_0 = \tfrac{1}{\sqrt{2}}(\beta_0 + \hat{a}_s). \tag{6}$$

At BS$_1$, the field $\hat{a}_0$ is mixed with the field from CO$_1$; the resulting output modes are

$$\hat{a}_1 = \tfrac{1}{\sqrt{2}}(\beta_1 + \hat{a}_0), \tag{7}$$

$$\hat{a}_2 = \tfrac{1}{\sqrt{2}}(\beta_1 - \hat{a}_0). \tag{8}$$

The light in mode $\hat{a}_1$ is sent to the photoelectric detector D$_1$, and that in mode $\hat{a}_2$ is mixed with the light from CO$_2$ at the beam splitter BS$_2$. The output fields at BS$_2$ are then given by

$$\hat{a}_3 = \tfrac{1}{\sqrt{2}}(\hat{a}_2 + \beta_2), \tag{9}$$

$$\hat{a}_4 = \tfrac{1}{\sqrt{2}}(\hat{a}_2 - \beta_2), \tag{10}$$

and the light in these modes is sent to the photoelectric detectors D$_3$ and D$_4$.

This configuration allows for a balanced homodyne measurement of mode $\hat{a}_2$ conditioned on a photodetection of mode $\hat{a}_1$. A photodetection by D$_1$ is made to initialize a sampling of the homodyne current in a standard "start" or "stop" scheme [15], and when averaged over a large number of samples, the measurement will result in a function of τ (time elapsed since a photodetection of $\hat{a}_1$) given by [6,7]

$$F(\tau) = \frac{\langle :\hat{n}_1(0)[\hat{n}_3(\tau) - \hat{n}_4(\tau)]: \rangle}{\langle \hat{n}_1 \rangle}, \tag{11}$$

where $\hat{n}_i = \hat{a}_i^\dagger \hat{a}_i$ is the number operator for the field mode $\hat{a}_i$. Using Eqs. (6)–(10) for the field modes and the definition of the quadrature variable $\hat{X}_\phi$ from Eq. (1), and recalling that $\Delta \hat{O} = \hat{O} - \langle \hat{O} \rangle$ for any operator $\hat{O}$, we find

$$\begin{aligned} F(\tau) = {}& |\beta_2| [\sqrt{2} |\beta_1| \cos(\phi_1 - \phi_2) - |\beta_0| \cos(\phi_0 - \phi_2) \\ & - |\langle \hat{a}_s \rangle| \cos \phi_2] - \frac{|\beta_2|}{4\langle \hat{n}_1 \rangle} [2 |\langle \hat{a}_s \rangle| \langle :\Delta \hat{X}_0(0) \Delta \hat{X}_{\phi_2}(\tau): \rangle \\ & + 2|\beta_0| \langle :\Delta \hat{X}_{\phi_0}(0) \Delta \hat{X}_{\phi_2}(\tau): \rangle + \sqrt{8} |\beta_1| \\ & \times \langle :\Delta \hat{X}_{\phi_1}(0) \Delta \hat{X}_{\phi_2}(\tau): \rangle + \langle :\Delta \hat{a}_s^\dagger(0) \Delta \hat{X}_{\phi_2}(\tau) \Delta \hat{a}_s(0): \rangle], \end{aligned} \tag{12}$$

where the mean photon number $\langle \hat{n}_1 \rangle$ is given by

$$\begin{aligned} \langle \hat{n}_1 \rangle = {}& \tfrac{1}{4} [|\langle \hat{a}_s \rangle|^2 + \langle \Delta \hat{a}_s^\dagger \Delta \hat{a}_s \rangle + |\beta_0|^2 + 2|\beta_1|^2 \\ & + 2 |\langle \hat{a}_s \rangle| |\beta_0| \cos \phi_0 + \sqrt{8} |\langle \hat{a}_s \rangle| |\beta_1| \cos \phi_1 \\ & + \sqrt{8} |\beta_0| |\beta_1| \cos(\phi_1 - \phi_0)]. \end{aligned} \tag{13}$$

For the single measurement $F(\tau)$ to be useful for detecting quadrature squeezing, we must first place a restriction on the types of source fields that can be used. We require that third-order moments in the source field fluctuations $\Delta\hat{a}_s$ and $\Delta\hat{a}_s^\dagger$ be equal to zero, so that the last term in Eq. (12) will vanish. This requirement is satisfied exactly if the field fluctuations are Gaussian or otherwise symmetric about the mean and approximately if the source field is weak [7]. Second, we require that the experimenter make three precision adjustments to the coherent oscillator fields, which we label adjustments (i), (ii), and (iii) for future reference.

Adjustment (i): the field amplitude from CO_0 is chosen to have the same magnitude as the source field's mean amplitude $\langle\hat{a}_s\rangle$ but opposite phase

$$|\beta_0| = |\langle\hat{a}_s\rangle|, \quad \phi_0 = \pi \Rightarrow \beta_0 = \langle\hat{a}_s\rangle e^{i\pi}. \tag{14}$$

As can be seen from Eq. (6), this will cause the field $\hat{a}_0$ to be of zero mean amplitude.

Adjustment (ii): the mean photon number from CO_1 is made equal to the mean photon number in mode $\hat{a}_0$:

$$|\beta_1|^2 \equiv \langle\hat{n}_0\rangle = \tfrac{1}{2}[|(\langle\hat{a}_s\rangle + \beta_0)|^2 + \langle\Delta\hat{a}_s^\dagger\Delta\hat{a}_s\rangle] = \tfrac{1}{2}\langle\Delta\hat{a}_s^\dagger\Delta\hat{a}_s\rangle, \tag{15}$$

which is now due only to the source field fluctuations, as follows from Eqs. (6) and (14).

Adjustment (iii): the fields from CO_1 and CO_2 are made to have the same phase:

$$\phi_1 = \phi_2 = \phi. \tag{16}$$

This common phase is to be varied in order to change the phase of the quadrature amplitudes that will be measured. The magnitude of the field amplitude from CO_2 ($|\beta_2|$) may be chosen arbitrarily.

Substituting Eqs. (14)–(16) into Eq. (12) (minus the final term), we find that $F(\tau)$ simplifies to

$$F(\tau,\phi) = |\beta_2|\sqrt{\langle\Delta\hat{a}_s^\dagger\Delta\hat{a}_s\rangle}\left[1 - \frac{\langle:\Delta\hat{X}_\phi(0)\Delta\hat{X}_\phi(\tau):\rangle}{\langle\Delta\hat{a}_s^\dagger\Delta\hat{a}_s\rangle}\right]. \tag{17}$$

Here, we have explicitly taken into account the fact that F also depends on the phase parameter ϕ, which now corresponds to the phase of the quadrature amplitude fluctuations that are measured. Apart from the arbitrary field amplitude $|\beta_2|$, F now depends only on τ, ϕ, and the properties of the source field. F can be normalized by its limit as $\tau\to\infty$. In this limit, the two factors in the quadrature correlation function become uncorrelated, and the average $\langle:\Delta\hat{X}_\phi(0)\Delta\hat{X}_\phi(\tau):\rangle$ becomes the product of the averages $\langle\Delta\hat{X}_\phi(0)\rangle\langle\Delta\hat{X}_\phi(\tau)\rangle$, each of which vanishes. We can therefore write

$$F(\infty,\phi) \equiv \lim_{\tau\to\infty} F(\tau,\phi) = |\beta_2|\sqrt{\langle\Delta\hat{a}_s^\dagger\Delta\hat{a}_s\rangle} \tag{18}$$

and use this quantity to introduce the normalized function

$$f(\tau,\phi) \equiv \frac{F(\tau,\phi)}{F(\infty,\phi)} = 1 - \frac{\langle:\Delta\hat{X}_\phi(0)\Delta\hat{X}_\phi(\tau):\rangle}{\langle\Delta\hat{a}_s^\dagger\Delta\hat{a}_s\rangle}. \tag{19}$$

The function $f(\tau,\phi)$ is determined entirely from laboratory measurement. It is of the form in Eq. (19) as long as the experiment is performed without error and third-order moments in $\Delta\hat{a}_s$ and $\Delta\hat{a}_s^\dagger$ are zero. It can therefore be used along with the classical inequalities (4) and (5) to detect squeezing and other nonclassical features of the source field's quadrature fluctuations. The inequalities can easily be rewritten in terms of $f(\tau,\phi)$

$$0 \le f(0,\phi) \le 1, \tag{20}$$

$$f(\tau,\phi) \ge f(0,\phi). \tag{21}$$

Violation of inequality (20) for any value of ϕ indicates that the source field is in a squeezed state. More specifically, a violation of the upper bound indicates that the quadrature $\hat{X}_\phi$ is squeezed, whereas a violation of the lower bound is nonclassical in that it implies the squeezing of $\hat{Y}_\phi$, the conjugate of $\hat{X}_\phi$. A violation of inequality (21) indicates nonclassical temporal correlations, both when $\hat{X}_\phi$ is both squeezed and when it is unsqueezed.

To demonstrate the use of $f(\tau,\phi)$ for detecting nonclassical effects and to develop a theoretical analysis of the quadrature fluctuations in this system, we consider the light produced by ISHG, using a theoretical model presented in Refs. [11–13]. Second harmonic generation occurs when two photons of frequency ω combine to form a single photon of frequency 2ω inside a nonlinear crystal. In ISHG, the crystal is placed in an optical cavity that is resonant at both the fundamental frequency ω and the second harmonic frequency 2ω, and the fundamental mode is excited by an injected classical signal. Using the positive-P representation [16], the annihilation and creation operators of the cavity's fundamental mode, which we write as $\hat{a}_s$ and $\hat{a}_s^\dagger$, can be mapped to the complex field amplitudes α_s and α_{s*} given by [11–13]

$$\hat{a}_s \to \alpha_s = \sqrt{n_o}[\sqrt{\bar{n}} + i(u_1 + u_2)], \tag{22}$$

$$\hat{a}_s^\dagger \to \alpha_{s*} = \sqrt{n_o}[\sqrt{\bar{n}} + i(u_1 - u_2)]. \tag{23}$$

Here, n_o is the threshold photon number, $\bar{n}$ is the mean photon number in the fundamental mode in units of n_o, and u_1 and u_2 are two real Gaussian random variables with zero mean and correlation functions given by

$$\langle u_i(t)u_j(t')\rangle = \delta_{ij}\frac{\gamma\bar{n}}{4n_o\lambda_i}e^{-\lambda_i|t-t'|}, \quad i,j = 1,2, \tag{24}$$

where γ is the cavity linewidth at the fundamental frequency and the decay constants λ_1 and λ_2 are given by

$$\lambda_1 = \gamma(1 + 3\bar{n}), \quad \lambda_2 = \gamma(1 + \bar{n}). \tag{25}$$

Normally ordered averages of $\hat{a}_s$ and $\hat{a}_s^\dagger$ can be calculated by taking the corresponding averages of α_s and α_{s*} with respect to the positive-P function.

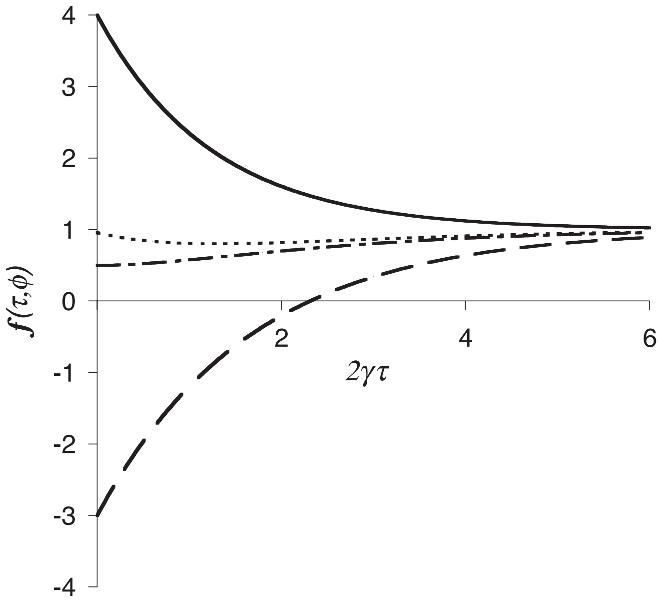

FIG. 2. Normalized conditional homodyne measurement $f(\tau,\phi)$ as a function of $2\gamma\tau$ for the ISHG with system parameters $n_o=10^6$ and $\bar{n}=0.2$ and phases $\phi=0$ (solid line), $\phi=11\pi/8$ (dotted line), $\phi=\pi/4$ (dashed-dotted line), and $\phi=\pi/2$ (dashed line).

Because the ISHG field has Gaussian fluctuations, third-order moments in $\Delta\hat{a}_s$ and $\Delta\hat{a}_s^\dagger$ are zero. We can therefore take this field to be the source field $\hat{a}_s$ in the experiment outlined above and find the resulting function $f(\tau,\phi)$ by direct use of Eq. (19). Using Eqs. (22) through (24) to describe the ISHG field, we find

$$f(\tau,\phi) = 1 - \frac{\langle u_2^2\rangle e^{-\lambda_2\tau}\sin^2\phi - \langle u_1^2\rangle e^{-\lambda_1\tau}\cos^2\phi}{\langle u_2^2\rangle - \langle u_1^2\rangle}, \qquad (26)$$

where $\langle u_1^2\rangle \equiv \langle [u_1(0)]^2\rangle$ and $\langle u_2^2\rangle \equiv \langle [u_2(0)]^2\rangle$ are simply the variances of u_1 and u_2 as given by Eq. (24).

Figure 2 shows plots of $f(\tau,\phi)$ vs $2\gamma\tau$ for $\phi=0$, $11\pi/48$, $\pi/4$, and $\pi/2$ for typical values of the ISHG parameters ($n_o=10^6$ and $\bar{n}=0.2$). The quantity $2\gamma\tau$ is the number of cavity lifetimes elapsed since the initialization of the correlator. Using the inequality (20) for classical bounds on the initial values of these functions, we can immediately see that the ISHG field is in a squeezed state; the fact that $f(0,0)$ exceeds 1 (by a factor of 4) indicates that the quadrature $\hat{X}_0$ is squeezed. Additionally, the value of $f(0,\pi/2)$ is -3, thus violating the lower bound of inequality (20); it is noteworthy that a measurement of the quadrature $\hat{X}_{\pi/2}$ reveals quadrature squeezing even though $\hat{X}_{\pi/2}$ itself is unsqueezed.

The function $f(\tau,\pi/4)$ violates neither of the bounds of inequality (20), and the behavior of $\hat{X}_{\pi/4}$ that it reveals is characteristic of the typical classical quadrature amplitude. The initial value of $f(\tau,11\pi/48)$ is also within classical bounds, but the quantum nature of $\hat{X}_{11\pi/48}$ can nonetheless be seen in its time development. We see that $f(\tau,11\pi/48)$ violates the Schwarz inequality (21) as it first decreases from its initial value before increasing toward the steady-state value of 1. This observation highlights a key advantage of the ability to make a conditional measurement; whereas nonconditional measurements would conclude only that $\hat{X}_{11\pi/48}$ is unsqueezed, $f(\tau,11\pi/48)$ reveals a temporal quantum phenomenon of great interest. This same violation of the Schwarz inequality can be seen in the function $f(\tau,0)$.

Each function plotted in Fig. 2 is a representative of one of the four classes of quadrature amplitude states that can be distinguished by the classical conditions (20) and (21), showing that each of the two inequalities can be satisfied or violated independently of the other and that such phenomena can be readily observed with the CMQF technique. In addition to providing this significantly expanded view of quadrature amplitude fluctuations in comparison to nonconditional measurements, $f(\tau,\phi)$ is found to be largely unaffected by detection inefficiencies due to its normalization in Eq. (19).

III. ANALYSIS OF POTENTIAL SOURCES OF ERROR

If any of the adjustments given by Eqs. (14)–(16) in Sec. II is not executed precisely, the measured function F will not be of the form shown in Eq. (17). Instead, it will contain extra terms that prevent the experimenter from obtaining an accurate measurement of the normalized quadrature correlation function. We develop an appraisal of the room for error in this experiment by exploring the ways in which imprecise adjustments to the coherent oscillator fields affect the conclusions drawn from the measurement.

We consider here the effects of an inexact execution of only adjustment (i) in the field amplitude of CO_0. This choice leads to the greatest number of complications in the measurement. These complications essentially encompass those created by inexact executions of adjustments (ii) and (iii). Though the following analysis is carried out for the specific experiment outlined in Sec. II, it can be shown that the qualitative results (such as the order of magnitude of error terms introduced) are relevant to any similar experiment that measures the normalized quadrature correlation function in a conditional homodyne detection experiment and requires precise adjustments to the amplitudes or intensities of coherent oscillator fields [6,7].

To model a potential error in the execution of adjustment (i), we suppose that $|\beta_0|$, the magnitude of the CO_0 field amplitude, is adjusted to

$$\beta_0 = \langle\hat{a}_s\rangle_{\text{error}}e^{i\pi} = (1-\eta)\langle\hat{a}_s\rangle e^{i\pi}. \qquad (27)$$

This error may arise from a faulty measurement of $\langle\hat{a}_s\rangle$ or from a misadjustment of β_0 itself. In general, η can be complex, accounting for misadjustment of both amplitude and phase of β_0. For simplicity of the discussion, we assume the error parameter η to be real. Complex η does not lead to qualitatively new results.

We assume that no additional sources of error are present; following the faulty adjustment of $|\beta_0|$, the experiment is carried out precisely as prescribed in Sec. II. In particular, adjustment (ii) is executed by setting $|\beta_1|^2=\langle\hat{n}_0\rangle$, and adjustment (iii) by matching the phases of β_1 and β_2, as in Eq.

(16). We assume that no errors concerning phases have been introduced, so that the relative phases of π for β_0 and ϕ for β_1 and β_2 are unambiguous. However, the mean photon number $\langle \hat{n}_0 \rangle$ depends on the new β_0, and thus, $|\beta_1|^2$ will no longer be given by Eq. (15) but by

$$|\beta_1|^2 = \langle \hat{n}_0 \rangle = \frac{1}{2}(\eta^2 |\langle \hat{a}_s \rangle|^2 + \langle \Delta \hat{a}_s^\dagger \Delta \hat{a}_s \rangle), \tag{28}$$

which follows from Eqs. (6) and (27). Note that, while a proper tuning of CO_0 would leave $\langle \hat{n}_0 \rangle$ depending only on the source field fluctuation intensity $\langle \Delta \hat{a}_s^\dagger \Delta \hat{a}_s \rangle$, the improper tuning adds an extra term proportional to the square of the error parameter.

We again assume a source field for which third-order moments in $\Delta \hat{a}_s$ and $\Delta \hat{a}_s^\dagger$ are zero and, thus, omit the last term in Eq. (12). Following the prescription of Sec. II, normalized function $f'(\tau, \phi, \eta)$ is found to be

$$f'(\tau,\phi,\eta) \equiv \frac{F'(\tau,\phi,\eta)}{F'(\infty,\phi,\eta)} = 1 - \frac{1}{2\langle \hat{n}_1 \rangle}\left(1 - \frac{\eta |\langle \hat{a}_s \rangle| \cos\phi}{\sqrt{2\langle \hat{n}_0 \rangle}}\right)^{-1} \times \Big[\langle : \Delta \hat{X}_\phi(0) \Delta \hat{X}_\phi(\tau) : \rangle + \frac{\eta |\langle \hat{a}_s \rangle| \langle : \Delta \hat{X}_0(0) \Delta \hat{X}_\phi(\tau) : \rangle}{\sqrt{2\langle \hat{n}_0 \rangle}}\Big], \tag{29}$$

where the function resulting from the conditional homodyne measurement under the effects of the adjustments, which we label $F'(\tau, \phi, \eta)$, is obtained by substituting Eqs. (27), (28), and (16) into Eq. (12). Here, $\langle \hat{n}_0 \rangle$ is given by Eq. (28) and $\langle \hat{n}_1 \rangle$ is given by

$$\langle \hat{n}_1 \rangle = \langle \hat{n}_0 \rangle + \eta |\langle \hat{a}_s \rangle| \sqrt{\frac{\langle \hat{n}_0 \rangle}{2}} \cos\phi. \tag{30}$$

Comparing the expression for $f'(\tau, \phi, \eta)$ with Eq. (19), we can see that the error in the definition of $|\beta_0|$ substantially changes the functional form of $f'(\tau, \phi, \eta)$ from that of $f(\tau, \phi)$, adding both ϕ- and τ-dependent terms. Each of these terms contains at least one factor of η, so that Eq. (29) reduces to Eq. (19) when $\eta = 0$. However, if $\langle \hat{a}_s \rangle_{\text{error}}$ is different from $\langle \hat{a}_s \rangle$, η will be nonzero, and using the function $f'(\tau, \phi, \eta)$ in Eq.(29) and the classical inequalities (20) and (21) to characterize quadrature squeezing may lead to invalid conclusions.

To demonstrate these effects, we again consider the fundamental mode produced by ISHG. The explicit form of the function $f'(\tau, \phi, \eta)$ for the ISHG field is too lengthy to display here, but it can be found by substituting the following four equations—which were derived by taking the positive-P averages of α_s and α_{s*} in Eqs. (22) and (23)—into Eqs. (28)–(30) while referencing Eqs. (24) and (25)

$$\langle \hat{a}_s \rangle = \sqrt{n_o \bar{n}}, \tag{31}$$

$$\langle \Delta \hat{a}_s^\dagger \Delta \hat{a}_s \rangle = n_o(\langle u_2^2 \rangle - \langle u_1^2 \rangle), \tag{32}$$

$$\langle : \Delta \hat{X}_\phi(0) \Delta \hat{X}_\phi(\tau) : \rangle = \langle u_2^2 \rangle e^{-\lambda_2 \tau} \sin^2\phi - \langle u_1^2 \rangle e^{-\lambda_1 \tau} \cos^2\phi, \tag{33}$$

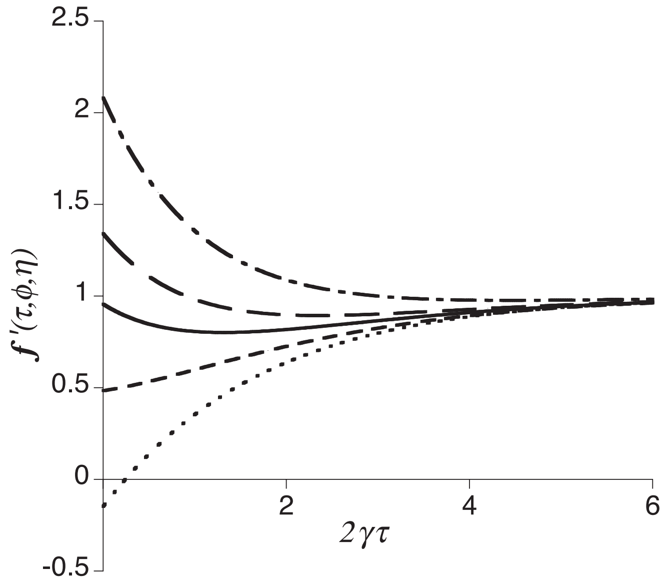

FIG. 3. Conditional homodyne measurement $f'(\tau, 11\pi/48, \eta)$ as a function of $2\gamma\tau$ for the ISHG with system parameters $n_o = 10^6$ and $\bar{n} = 0.2$ and error parameters $\eta = -2 \times 10^{-4}$ (dotted lines), $\eta = -5 \times 10^{-5}$ (dashed lines), $\eta = 4 \times 10^{-5}$ (long-dashed lines), $\eta = 2 \times 10^{-4}$ (dotted-dashed lines). The error-free measurement $f(\tau, 11\pi/48) = f'(\tau, 11\pi/48, 0)$ (solid) is also shown for comparison.

$$\langle : \Delta \hat{X}_0(0) \Delta \hat{X}_\phi(\tau) : \rangle = - n_o \cos\phi \langle u_1^2 \rangle e^{-\lambda_1 \tau}. \tag{34}$$

Figure 3 shows plots of the function $f'(\tau, 11\pi/48, \eta)$ resulting from a measurement of the ISHG system, with the same system parameters used in Sec. II. The figure displays both the error-free measurement $f'(\tau, 11\pi/48, 0) = f(\tau, 11\pi/48)$ and multiple measurements $f'(\tau, 11\pi/48, \eta)$ with a few small positive and negative values of the error parameter η. As noted above, the function $f(\tau, 11\pi/48)$ violates inequality (21) but not inequality (20), indicating that the quadrature $\hat{X}_{11\pi/48}$ exhibits nonclassical temporal correlations while neither $\hat{X}_{11\pi/48}$ nor its conjugate is squeezed. Yet, all four of the functions $f'(\tau, 11\pi/48, \eta)$ in Fig. 3 would lead to qualitatively different conclusions about $\hat{X}_{11\pi/48}$, indicating that it is squeezed ($\eta = 4 \times 10^{-5}$, 2×10^{-4}), that its conjugate is squeezed ($\eta = -2 \times 10^{-4}$), or that it is entirely classical in nature ($\eta = -5 \times 10^{-5}$).

This analysis demonstrates that small errors in the adjustment of the CO_0 field amplitude can make the behavior of $f'(\tau, \phi, \eta)$ vastly different from that of $f(\tau, \phi)$. We can gain an understanding of the sources of these large variations by considering the dependence of $f'(\tau, \phi, \eta)$ on the error parameter η. We note that the measurement is twice normalized by quantities that depend on the mean photon number $\langle \hat{n}_0 \rangle$—once by $\langle \hat{n}_1 \rangle$ in Eq. (11) and again by $F'(\infty, \phi, \eta)$ in Eq. (29). A portion of the dependence of $\langle \hat{n}_1 \rangle$ on $\langle \hat{n}_0 \rangle$ arises straightforwardly from Eq. (13), but the other portion and the dependence of $F'(\infty, \phi, \eta)$ on $\langle \hat{n}_0 \rangle$ arise from the adjustment of the CO_1 field amplitude in Eq. (28). Then, a comparison of Eqs. (15) and (28) shows that the sensitivity

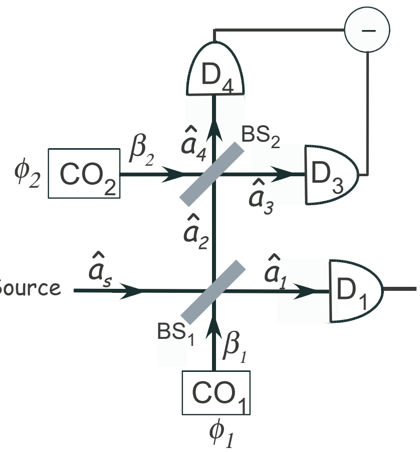

FIG. 4. (Color online) An outline of the setup for modified conditional measurement of quadrature fluctuations of a source $\hat{a}_s$.

of $\langle\hat{n}_0\rangle$ to changes in η depends on the relative magnitudes of the fluctuation intensity $\langle\Delta\hat{a}_s^\dagger\Delta\hat{a}_s\rangle$ and the square of the mean amplitude $\langle\hat{a}_s\rangle$, which, in an error-free execution of the experiment, is intended to be completely removed from $\langle\hat{n}_0\rangle$ by interference with a properly tuned CO_0 field. We find that, if $|\langle\hat{a}_s\rangle|^2 \gg \langle\Delta\hat{a}_s^\dagger\Delta\hat{a}_s\rangle$, then even small values of η can produce significant variations in $\langle\hat{n}_0\rangle$ and thus potentially large variations in $f'(\tau,\phi,\eta)$. For ISHG, it is true that $|\langle\hat{a}_s\rangle|^2 \gg \langle\Delta\hat{a}_s^\dagger\Delta\hat{a}_s\rangle$, as Eqs. (31) and (32) show that $|\langle\hat{a}_s\rangle|^2/\langle\Delta\hat{a}_s^\dagger\Delta\hat{a}_s\rangle \approx 2\times10^7$. Thus, we see that the large amplifications of initially tiny errors demonstrated above are results of the experiment's reliance on the exact cancellation, by interference with a coherent oscillator field of precisely tuned amplitude, of a field amplitude that is commonly much larger than the fluctuations the experiment is designed to measure. We also see that this reliance is compounded by a required adjustment of the intensity of a second coherent oscillator field.

IV. CONDITIONAL MEASUREMENT OF QUADRATURE FLUCTUATIONS FOR NON-GAUSSIAN FIELDS

The results of Sec. III suggest that it might be advantageous to devise a method for observing the time evolution of field quadrature fluctuations that does not require precision adjustments to the amplitudes or intensities of coherent oscillator fields. Another desirable feature in such a method would be the ability to deal with source fields for which third-order moments in the source field fluctuations $\Delta\hat{a}_s$ and $\Delta\hat{a}_s^\dagger$ are not intrinsically equal to zero. In this section, we propose a modification of the measurement scheme of Sec. II that achieves these goals.

The setup for the modified experiment is shown in Fig. 4, which is identical to Fig. 1 except for the absence of the beam splitter BS_0, the coherent oscillator CO_0, and the field mode $\hat{a}_0$. We have left the indices on the remaining components of the setup unchanged to maintain a parallel between this experiment and the Sec. II experiment. The coherent oscillators CO_1 and CO_2 still emit fields with definite complex amplitudes $\beta_1=|\beta_1|e^{i\phi_1}$ and $\beta_2=|\beta_2|e^{i\phi_2}$, and the beam splitters BS_1, and BS_2 are still lossless and antisymmetric and have coefficients of reflection and transmission $R=T=1/2$. The field modes numbered 1 and 2, are modified, and are given by

$$\hat{a}_1 = \tfrac{1}{\sqrt{2}}(\beta_1 + \hat{a}_s), \tag{35}$$

$$\hat{a}_2 = \tfrac{1}{\sqrt{2}}(\beta_1 - \hat{a}_s). \tag{36}$$

As shown in Fig. 4, the light in modes $\hat{a}_1$, $\hat{a}_3$, and $\hat{a}_4$ is sent to the photodetectors D_1, D_3, and D_4.

As in Sec. II, the configuration Fig. 4 allows for a balanced homodyne measurement of mode $\hat{a}_2$ conditioned on a photodetection of mode $\hat{a}_1$. This results in a function that we now label $G(\tau,\phi_1,\phi_2)$

$$G(\tau,\phi_1,\phi_2) = \frac{\langle:\hat{n}_1(0)[\hat{n}_3(\tau)-\hat{n}_4(\tau)]:\rangle}{\langle\hat{n}_1\rangle}. \tag{37}$$

Using Eqs. (35), (36), (9), and (10), we find

$$\begin{aligned} G(\tau,\phi_1,\phi_2) = {} & \sqrt{2}|\beta_2|[|\beta_1|\cos(\phi_1-\phi_2) - |\langle\hat{a}_s\rangle|\cos\phi_2] \\ & - \frac{|\beta_2|}{\sqrt{2}\langle\hat{n}_1(\phi_1)\rangle}[2|\beta_1|\langle:\Delta\hat{X}_{\phi_1}(0)\Delta\hat{X}_{\phi_2}(\tau):\rangle \\ & + 2|\langle\hat{a}_s\rangle|\langle:\Delta\hat{X}_0(0)\Delta\hat{X}_{\phi_2}(\tau):\rangle \\ & + \langle:\Delta\hat{a}_s^\dagger(0)\Delta\hat{X}_{\phi_2}(\tau)\Delta\hat{a}_s(0):\rangle], \end{aligned} \tag{38}$$

where the mean photon number $\langle\hat{n}_1\rangle$, which is a function of ϕ_1, is given by

$$\langle\hat{n}_1(\phi_1)\rangle = \tfrac{1}{2}[|\beta_1|^2 + |\langle\hat{a}_s\rangle|^2 + \langle\Delta\hat{a}_s^\dagger\Delta\hat{a}_s\rangle + 2|\beta_1||\langle\hat{a}_s\rangle|\cos\phi_1]. \tag{39}$$

To isolate information about the source field quadrature fluctuations in this scheme, the measurement $G(\tau,\phi_1,\phi_2)$ is taken twice for every quadrature amplitude $\hat{X}_\phi$ to be observed. First, the experimenter must measure the function $G(\tau,\phi,\phi)$, simply matching the phases of the two coherent oscillators. Second, the experimenter must measure $G(\tau,\phi+\pi,\phi)$, leaving the phase of the CO_2 field unchanged but shifting the phase of the CO_1 field by one half period. Additionally, the steady-state mean photon numbers $\langle\hat{n}_1(\phi)\rangle$ and $\langle\hat{n}_1(\phi+\pi)\rangle$ must be measured along with each pair of conditional measurements. This can be done at the photodetector D_1 without disturbing the setup. The field amplitudes $|\beta_1|$ and $|\beta_2|$ can both be arbitrarily chosen and held fixed throughout the entire series of measurements, as long as their values are known. Noting from Eq. (1) that $\hat{X}_{\phi+\pi}$ is simply equal to $-\hat{X}_\phi$, we can see from Eqs. (38) and (39) that a measurement of the source field quadrature fluctuations can be constructed from the four measurements $G(\tau,\phi,\phi)$, $G(\tau,\phi+\pi,\phi)$, $\langle\hat{n}_1(\phi)\rangle$, and $\langle\hat{n}_1(\phi+\pi)\rangle$ and the values of $|\beta_1|$ and $|\beta_2|$ via the following equation:

$$H(\tau,\phi) \equiv \langle:\Delta\hat{X}_\phi(0)\Delta\hat{X}_\phi(\tau):\rangle = \frac{1}{\sqrt{8}|\beta_1||\beta_2|}\{\langle\hat{n}_1(\phi)\rangle \times[G(\infty,\phi,\phi) - G(\tau,\phi,\phi)] - \langle\hat{n}_1(\phi+\pi)\rangle \times[G(\infty,\phi+\pi,\phi) - G(\tau,\phi+\pi,\phi)]\}, \quad (40)$$

where we have labeled the quadrature correlation function $H(\tau,\phi)$ for compactness. The experimentally measurable quantity $G(\infty,\phi_1,\phi_2)$ in Eq. (40) is simply $G(\tau,\phi_1,\phi_2)$ with the time-correlated fluctuation terms set equal to zero

$$G(\infty,\phi_1,\phi_2) = \sqrt{2}|\beta_2|[|\beta_1|\cos(\phi_1-\phi_2) - |\langle\hat{a}_s\rangle|\cos\phi_2]. \quad (41)$$

Written in terms of $H(\tau,\phi)$, the classical inequalities (4) and (5) are

$$H(0,\phi) \geq 0, \quad (42)$$

$$H(\tau,\phi) \leq H(0,\phi). \quad (43)$$

The inequalities for function $H(\tau,\phi)$ are similar to the inequalities for the function $f(\tau,\phi)$ given in Eqs. (20) and (21), except that $H(\tau,\phi)$ does not have an upper bound. However, advantage of the inequalities for $H(\tau,\phi)$ is that they can be applied to any field source. Figure 5 shows plots of $H(\tau,\phi)$ versus $2\gamma\tau$ with $\phi=0$, $11\pi/48$, $\pi/4$, and $\pi/2$ for the same ISHG parameters used in Sec. II. Through the classical inequalities (42) and (43), these plots highlight the nonclassical behaviors of the ISHG.

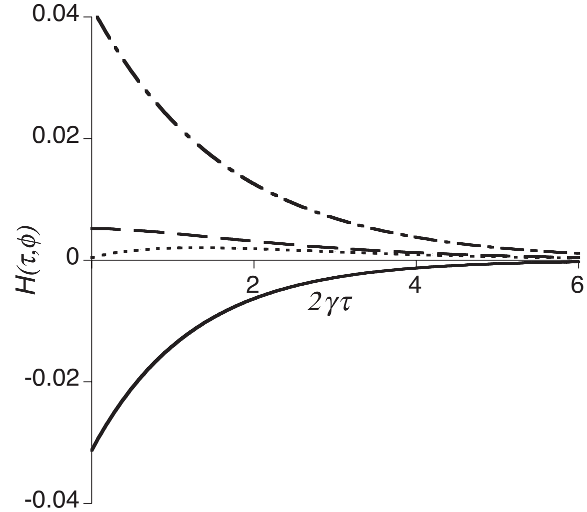

FIG. 5. Conditional homodyne measurement $H(\tau,\phi)$ as a function of $2\gamma\tau$ for the ISHG with system parameters $n_o=10^6$ and $\bar{n}=0.2$ and phases $\phi=0$ (solid lines), $\phi=11\pi/8$ (dotted lines), $\phi=\pi/4$ (dashed lines), and $\phi=\pi/2$ (dotted-dashed lines).

V. SUMMARY

The analyses of Sec. II show clear deviations of the ISHG system from the expectations of a classical optical field, indicating the presence of uniquely quantum phenomena, such as quadrature squeezing and nonclassical temporal correlations. We have shown in Sec. III, however, that these phenomena can sometimes be very easily obscured by only minuscule experimental errors in the adjustment of coherent oscillator intensities in the standard scheme for detecting optical quadrature fluctuations. The measurement scheme proposed in Sec. IV could help to eliminate such sources of error while revealing the same nonclassical behaviors, since it is not sensitive to the intensities of auxiliary coherent fields. Though there are still potential sources of experimental error in this scheme, such as in the adjustments to the phases of coherent field sources, it presents an improvement in detecting quadrature squeezing and other nonclassical effects in optical systems that are non-Gaussian in character.

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