Effect of thermal fluctuations on spin degrees of freedom in spinor Bose-Einstein condensates

W. V. Pogosov and K. Machida
Department of Physics, Okayama University, Okayama 700-8530, Japan
(Received 6 February 2006; published 31 August 2006)

We consider the effect of thermal fluctuations on rotating spinor $F=1$ condensates in axially symmetric vortex phases, when all the three hyperfine states are populated. We show that the relative phase among different components of the order parameter can fluctuate strongly due to the weakness of the interaction in the spin channel. These fluctuations can be significant even at low temperatures. Fluctuations of relative phase lead to significant fluctuations of the local transverse magnetization of the condensate. We demonstrate that these fluctuations are much more pronounced for the antiferromagnetic state than for the ferromagnetic one.

DOI: 10.1103/PhysRevA.74.023624 PACS number(s): 03.75.Kk, 03.75.Lm

I. INTRODUCTION

The properties of rotating spinor Bose-Einstein condensates (BECs) attract a lot of attention now. The first examples of these systems with hyperfine spin $F=1$ were found in optically trapped ^{23}Na [1]. The vortex phase diagram of spinor condensates is very rich, since the order parameter has three components in the $F=1$ case and five components in the $F=2$ case. Topological excitations in spinor condensates were studied theoretically in many cases (see, e.g., Refs [2–6]).

At the same time, interest is now growing in temperature effects in atomic condensates. References [7–11] study theoretically the Berezinskii-Kosterlitz-Thouless (BKT) transition associated with the proliferation of thermally excited vortex-antivortex pairs. For instance, in Ref. [8] it was shown that in quasi-two-dimensional condensates the BKT transition can occur at rather low temperatures $T\sim 0.5T_c$, with the number of particles in the system $N\sim 10^4$. Recently, some signatures of a possible BKT phase were also found close to the critical temperature T_c in experimental work [12], where condensates in optical lattice have been studied. Finally, experimental evidence for the BKT transition in trapped condensates was reported in Ref. [13]. References [14,15] deal with the thermal fluctuations of positions of vortices in rotated scalar condensates. Note that, according to the Mermin-Wagner-Hohenberg theorem, Bose-Einstein condensation is not possible in two-dimensional (2D) homogeneous systems. However, application of the trapping potential leads to macroscopic occupation of the ground state of a Bose gas.

The aim of the present paper is to study the effect of thermal fluctuations in rotated quasi-two-dimensional *spinor* condensates. These systems have a specific degree of freedom, associated with the relative angle among different components of the order parameter corresponding to different hyperfine states. In other words, this angle determines the coherence among components of the order parameter. Also it influences the transverse magnetization of the condensate. In this paper, we focus on thermal fluctuations of this angle. Note that experimentally, at the present time, it is possible to study the condensate phase [16–18] (see also Ref [19]). In addition, recently, a new and nondestructive method for measuring the local magnetization of the condensate was proposed and successfully applied in Ref. [20].

We show that the relative angle among hyperfine components of the order parameter in the 2D case can experience strong thermal fluctuation even at low temperatures. The reason is the weakness of the spin energy of the system as compared to interactions in the density channel. Also fluctuations of this angle lead to significant relative fluctuations of the local transverse magnetization of the condensate, which are much larger in the antiferromagnetic case than in the ferromagnetic one.

This paper is organized as follows. In Sec. II, we give a basic formulation of the problem. In Sec. II, we discuss our main results for the fluctuations of angle and spin textures. We conclude in Sec. III.

II. BASIC FORMULATION

We consider a harmonically trapped quasi 2D Bose-Einstein condensate with spin $F=1$. The trapping potential is given by

$$U(r) = \frac{m\omega_\perp^2 r^2}{2}, \tag{1}$$

where $\omega_\perp$ is the trapping frequency, m is the mass of the atom, and r is the radial coordinate. The system is rotated with the angular velocity $\mathbf{\Omega}$, well below the critical rotation speed $\omega_\perp$, and the number of atoms in the cloud is N. In this paper, we restrict ourselves to a range of temperatures much smaller than T_c. Therefore, we can neglect the noncondensate contribution to the free energy of the cloud. The total energy of the system in this approximation coincides with the energy of the condensate. For the number of condensed particles, we use the ideal gas result:

$$N(T) = N\left[1 - \left(\frac{T}{T_c}\right)^2\right]. \tag{2}$$

At the same time,

$$\frac{\hbar\omega_\perp}{k_B T_c} = \sqrt{\frac{\zeta(2)}{N}}, \tag{3}$$

where $\zeta(2)$ is the Riemann zeta function, $\sqrt{\zeta(2)}\approx 1.28$. Equations (2) and (3) remain accurate even for the case of interacting particles [21]. We also introduce a dimensionless temperature $t=T/T_c$. Since we are considering low temperatures $T\sim 0.1T_c$, the temperature dependence of the condensed particle number can be neglected, $N(T)\simeq N$.

The order parameter in the $F=1$ condensate has three components Ψ_j ($j=-1,0,1$). The free energy of the system can be written as [22,23]

$$F = \hbar\,\omega_\perp N \int dS[\Psi_j^*\hat{h}\Psi_j + 2\pi g_n \Psi_j^*\Psi_k^*\Psi_j\Psi_k + 2\pi g_s \Psi_j^*\Psi_l^*(F_a)_{jk}(F_a)_{lm}\Psi_k\Psi_m - i\boldsymbol{\Omega}\cdot\Psi_j^*(\boldsymbol{\nabla}\times\mathbf{r})\Psi_j], \tag{4}$$

where the integration is performed over the system area, repeated indices are summed, F_a ($a=x,y,z$) is the angular momentum operator, which can be expressed in a matrix form through the usual Pauli matrices, and $\hat{h}$ is the one-body Hamiltonian, given by

$$\hat{h} = -\frac{\nabla^2}{2} + \frac{r^2}{2}. \tag{5}$$

The constants g_n and g_s characterize interactions in the density and spin channels and are given by

$$g_n = \frac{(a_0 + 2a_2)n_z}{3}, \tag{6}$$

$$g_s = \frac{(a_2 - a_0)n_z}{3}, \tag{7}$$

where a_0 and a_2 are the scattering lengths for atoms with total spin 0 and 2, and n_z is the concentration of atoms in the longitudinal direction. In real spinor condensates, $|g_s| \ll |g_n|$, since $a_0 \approx a_2$. Typically, $|g_s/g_n| \sim 0.001-0.01$, and this ratio can be tuned. In this paper, we study the case of a relatively dilute condensate and take $g_n=10$. We will consider different values of N but at fixed value of the interaction parameter g_n. This is possible, since, in the case of a single-layer cloud, we can always tune the trapping frequency in the longitudinal direction, keeping g_n constant. To ensure the regime of quasi-two-dimensionality, we can also tune $\omega_\perp$. In this case, we have to change the rotation speed to keep the dimensionless rotation speed the same, and the temperature to fix the dimensionless t. In real atomic condensates, a_0 is approximately several nanometers. The most realistic value of N for this g_n is close to 10^3, and to illustrate the effect of N we will consider the range $10^2 \lesssim N \lesssim 10^4$.

The total magnetization of the condensate is fixed:

$$M = \int dS |\Psi_j|^2 j. \tag{8}$$

The magnetization M is normalized in terms of N and the maximum of $|M|$ is equal to 1. One also has to take into account the normalization condition for the order parameter:

$$\int dS\, \Psi_j \Psi_j^* = 1. \tag{9}$$

The spatial profiles of all the components of the order parameter in the equilibrium can be found from the condition of the minimum of energy (4). It is also convenient to introduce the longitudinal l_z and transverse l_{tr} local magnetizations of the condensate:

$$l_z = |\Psi_1|^2 - |\Psi_{-1}|^2, \tag{10}$$

$$l_{tr}^2 = l_x^2 + l_y^2 = 2|\Psi_0|^2|\Psi_1|^2 + 2|\Psi_0|^2|\Psi_{-1}|^2 + 4(\Psi_0^2\Psi_1^*\Psi_{-1}^* + \mathrm{c.c.}). \tag{11}$$

The spin energy in this case can be represented as

$$F_{spin} = 2\pi g_s \hbar\,\omega_\perp N \int dS (l_z^2 + l_{tr}^2). \tag{12}$$

In this paper, we restrict ourselves only to the case of axially symmetric phases, when the moduli of all the components of the order parameter are independent of the azimuthal angle and depend only on the radial coordinate r. Note that equilibrium vortex phases in this situation were studied in Refs. [2,3] for the spin $F=1$ condensate and in Ref. [6] for the $F=2$ system. For axially symmetric phases, each component of the order parameter can be represented as

$$\Psi_j(r,\varphi) = f_j(r)\exp(-iL_j\varphi - i\delta_j), \tag{13}$$

where φ is the polar angle, L_j is the winding number, and δ_j is the relative phase. We will denote such phases as (L_{-1}, L_0, L_1). As shown in Ref. [3], the axial symmetry of the solution implies that the winding numbers satisfy the relation $L_1 + L_{-1} = 2L_0$. In this case, according to Eqs. (11) and (12), the spin energy depends on the relative angle $\chi = 2\delta_0 - \delta_1 - \delta_{-1}$.

It is important to note that only the spin contribution to the total energy (4) depends on the phases δ_j via the spin-mixing term. For the stationary state, which is a local minimum of the Gross-Pitaevskii functional (4), the value of χ is determined by the sign of the interaction constant in spin channel g_s. For positive g_s (antiferromagnetic case), a minimum of F_{spin} is attained at $\chi=\pi$, whereas for negative g_s (ferromagnetic case) $\chi=0$.

III. RESULTS AND DISCUSSION

According to the results of Ref. [3], for the antiferromagnetic state ($g_s>0$), the phases $(-1,0,1)$ and $(1,1,1)$ are energetically favorable in the region of small and moderate values of magnetization M. The phase $(-1,0,1)$ is realized at low rotation frequencies Ω and $(1,1,1)$ at higher Ω. In Ref. [2], it was shown that the $(0,1,2)$ state is favorable in the ferromagnetic case ($g_s<0$) in the region of moderate values of Ω and M. In these phases, all three hypefine states are populated. Fluctuations of χ have a meaning only in this case, since the χ-dependent part of the energy is equal to zero identically, if one of the components of the order parameter is zero. In this paper, we will concentrate on these three vortex states, since they are appropriate candidates for the illustration of the effect of thermal fluctuations. Note that in homogeneous spin-1 condensate atoms populate only two or one hyperfine state(s); they can populate three states only if the system is trapped and experiences rotation, which generates vortices.

An important feature of real atomic spinor Bose-Einstein condensates is the weakness of the spin interactions comparing to the interaction in the density channel ($|g_s| \ll |g_n|$). At the same time, the coherence among the different components of the order parameter (the angle χ) is fully determined by the spin interaction. The angle χ also influences the transverse magnetization of the condensate, as seen from Eq. (11). Note that the longitudinal component of magnetization is independent of χ.

The smallness of g_s compared to g_n leads to the fact that thermal fluctuations of the relative angle χ become significant at much lower temperatures than fluctuations of the density of particles. Therefore, at relatively low temperatures, one can assume that the moduli of all the components of the order parameter remain fixed (which can also be checked numerically), whereas χ is fluctuating. For the case of small fluctuations of χ, one can use the harmonic approximation and represent the deviation of the energy of the system from equilibrium, $\delta F = F(\chi_0+\delta\chi) - F(\chi_0)$, as a quadratic function in terms of the deviation of the angle χ from the equilibrium $\delta\chi = \chi - \chi_0$:

$$\delta F = 2\pi g_s \hbar\, \omega_\perp N I[\cos(\chi_0+\delta\chi) - \cos(\chi_0)] \approx \pi |g_s| \hbar\, \omega_\perp N I (\delta\chi)^2, \tag{14}$$

where $I = \int dS(f_1 f_{-1} f_0^2)$. Under these assumptions, the average square of the deviation of χ from the equilibrium is given by

$$\langle(\delta\chi)^2\rangle_T = \frac{\int d(\delta\chi)(\delta\chi)^2 \exp(-\delta F/k_B T)}{\int d(\delta\chi)\exp(-\delta F/k_B T)}. \tag{15}$$

The integrals in Eq. (15) can be calculated analytically. After taking into account Eq. (3), we get

$$\langle(\delta\chi)^2\rangle_T = \frac{t}{1.28\sqrt{N}4|g_s|I}. \tag{16}$$

We also introduce a quantity $\Delta\chi = \sqrt{\langle(\delta\chi)^2\rangle_T}$, which can be considered as the average deviation of the angle χ from the equilibrium. We see that $\Delta\chi$ depends on the dimensionless temperature $t = T/T_c$, the number of particles N, and the integral I. For a given vortex phase, I is also a function of the total magnetization M. It is important to emphasize that the scaling relation (16) has meaning only if g_n is independent of N, as discussed above. In order to calculate I, we use a variational method, which was previously applied by us in Ref. [6] to evaluate the energies of various axially symmetric vortex phases in the spin $F=2$ condensate. In this approach, each component of the order parameter is modeled by a trial function and values of the variational parameters are found from the condition of the minimum of total energy.

In Fig. 1 we plot the calculated dependence of $\Delta\chi$ (measured in degrees) as a function of the number of particles in the system for different vortex phases at $t=0.1$ and $g_n=10$. This value of g_n is close to typical experimental ones ($a_0 \approx 5$ nm, $n_z \approx 2$ nm^{-1}); see also the calculations of Refs. [3,6]. We assume that, for the $(-1,0,1)$ and $(1,1,1)$ states, $g_s = 0.01 g_n$ and $M=0.1$, whereas for $(0,1,2)$, $g_s = -0.01 g_n$ and $M=0.5$. Note that $\Delta\chi$ for a particular phase is independent of Ω, since I has the same property. We see that even for quite low temperatures, $\Delta\chi$ can be rather large and the coherence among different components of the order parameter is practically destroyed. For smaller values of $|g_s|$, fluctuations of χ are, of course, even stronger. To illustrate the effect of temperature, in the inset to Fig. 1, we show the dependence of $\Delta\chi$ on T for the $(0,1,2)$ phase (curve 1), the $(1,1,1)$ phase (curve 2), and the $(-1,0,1)$ phase (curve 3) at fixed number of atoms $N=1000$, $g_s = -0.05 g_n$ for the first curve and $g_s = 0.05 g_n$ for the two others. Note that $\Delta\chi$ is almost independent of the total magnetization M of the condensate.

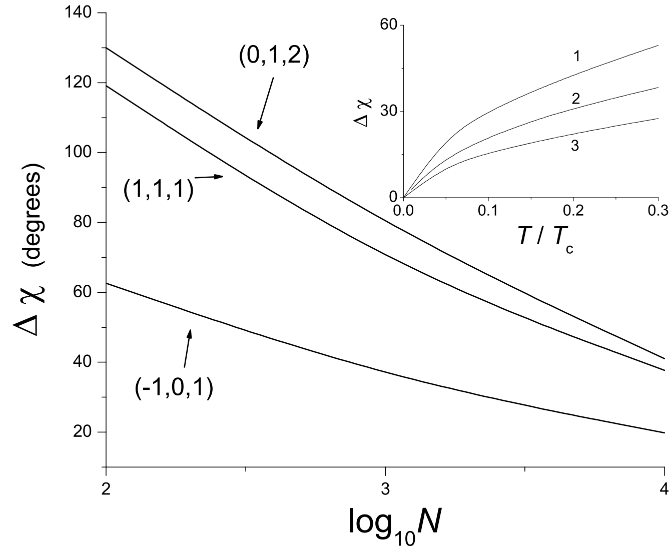

FIG. 1. Dependence of $\Delta\chi$ (in degrees) on the number of particles in the system for different vortex phases at fixed value of interaction constant $g_n=10$ (see in the text) and $t=0.1$. In the $(-1,0,1)$ phase, $g_s=0.01g_n$, $M=0.1$; in the $(1,1,1)$ state, $g_s=0.01g_n$, $M=0.1$; in the $(0,1,2)$ state, $g_s=-0.01g_n$, $M=0.5$. Inset shows $\Delta\chi$ as a function of temperature for the $(0,1,2)$ phase (curve 1), the $(1,1,1)$ phase (curve 2), and the $(-1,0,1)$ phase (curve 3) at the same values of M, g_n, and t. The number of atoms is $N=1000$; interaction constants are $g_s=-0.05g_n$ for the first curve and $g_s=0.05g_n$ for the two others.

As we already pointed out, fluctuations of χ lead to fluctuations of l_{tr}. In the harmonic approximation, one can express the average deviation of $|l_{tr}|$ from the equilibrium $\langle\delta|l_{tr}|\rangle_T$ through the deviation of χ:

$$\frac{\langle\delta|l_{tr}|\rangle_T}{|l_{tr}|} = (-1)^u \frac{1}{2}\langle(\delta\chi)^2\rangle_T \frac{f_1 f_{-1}}{[f_1 + (-1)^{u+1} f_{-1}]^2}, \tag{17}$$

where $u=0$ for the antiferromagnetic case and $u=1$ for the ferromagnetic one. If in the antiferromagnetic state the total magnetization is not large, $M \lesssim 0.5$, one can expect that $(f_1 - f_{-1})^2 \ll f_1 f_{-1}$, and, therefore, even small fluctuations of χ lead to strong relative fluctuations of $|l_{tr}|$. At the same time, for the ferromagnetic case, $u=1$ in this equation, and relative fluctuations of $|l_{tr}|$ are much smaller.

We have calculated $\langle\delta|l_{tr}|\rangle_T$ for different vortex phases and our calculations revealed that $\langle\delta|l_{tr}|\rangle_T/|l_{tr}|$ is almost independent of the radial coordinate r for vortex phases $(-1,0,1)$ and $(1,1,1)$. This is due to the fact that $|L_{-1}| = |L_1|$ for these states; therefore, $f_1(r)$ is nearly proportional

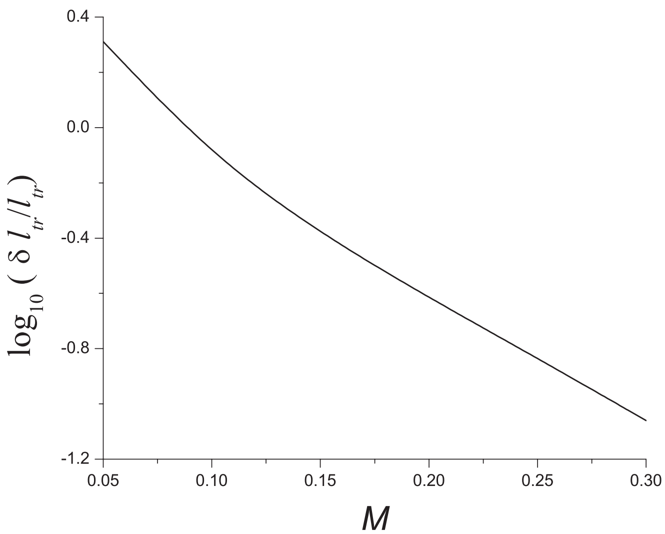

FIG. 2. Dependence of $\langle\delta|l_{tr}|\rangle_T/|l_{tr}|$ on the total magnetization for the $(-1,0,1)$ phase at $N=1000$; $g_n=10$, $t=0.1$, $g_s=0.01g_n$.

to $f_{-1}(r)$, and, according to Eq. (15), $\langle\delta|l_{tr}|\rangle_T/|l_{tr}|$ should only slightly depend on r. In Fig. 2 we present $\langle\delta|l_{tr}|\rangle_T/|l_{tr}|$ as a function of total magnetization of the condensate for $(-1,0,1)$ state at $t=0.1$, $g_s=0.01g_n$ (antiferromagnetic case), and $N=1000$. We see that relative fluctuations of the transverse magnetization can be significant even at low temperature. The value of $\langle\delta|l_{tr}|\rangle_T/|l_{tr}|$ decreases with increase of M. This result is natural, since the condensate becomes more polarized with growing M. The absolute value of $\langle\delta|l_{tr}|\rangle_T$ also remains sizable. Although the value of the fractional quantity $\langle\delta|l_{tr}|\rangle_T/|l_{tr}|$ grows with decrease of M, the value of $|l_{tr}|$ itself becomes smaller. Therefore, we found that the most appropriate value of M to observe fluctuations of the transverse magnetization is around $M=0.2$, where both $\langle\delta|l_{tr}|\rangle_T/|l_{tr}|$ and $|l_{tr}|$ are high: $\langle\delta|l_{tr}|\rangle_T/|l_{tr}|\gtrsim 0.1$, whereas l_{tr} is comparable to the longitudinal magnetization l_z in the fully polarized state at $M=1$, where it should be easily detectable experimentally. The value of $\langle\delta|l_{tr}|\rangle_T/|l_{tr}|$ depends also on the vortex phase; we found that in the $(1,1,1)$ state it is even much larger than in the $(-1,0,1)$ state.

We also have calculated $\langle\delta|l_{tr}|\rangle_T$ for the ferromagnetic $(0,1,2)$ phase. As might be expected, in this case, the relative fluctuations of $|l_{tr}|$ are much weaker. Physically, this is because $|l_{tr}|$ is proportional to the ferromagnetic order parameter [6], which is responsible for the ferromagnetic ordering. Therefore, one can expect that in the ferromagnetic phase this order parameter is more robust with respect to thermal fluctuations than in the antiferromagnetic one. In addition, the average deviation of $|l_{tr}|$ from equilibrium is negative and its modulus grows with increase of M, in contrast to the antiferromagnetic system.

Thermal fluctuations should also be important in the case of the $F=2$ condensate, where there are two interaction constants in the spin channel and two characteristic angles. Therefore, one can expect more complicated behavior, as compared to the $F=1$ condensate. For instance, in the homogeneous $F=2$ system, a cyclic state can have the lowest energy; in this case atoms populate three hyperfine states, and the spin energy depends on the coherence among them. The fluctuation problem for this system was analyzed in Ref. [24]. A new method to create such entangled states in spin-1 condensate was recently applied experimentally in Ref. [19], where microwave energy was injected into the system. As a result, particles redistribute from states of spin -1 to spin-0 and -1 states, and all three magnetic sublevels become populated. The spin-mixing dynamics in the $F=1$ condensate was studied theoretically in Ref. [25].

Note that in Eq. (14) we have assumed that the fluctuating χ is spatially independent, which is not true in general case. However, the spatial gradients of χ give some additional contribution to the kinetic energy of the system, which is much larger than the spin energy. Therefore, gradients of χ result in a rather large increase of total energy, and we can neglect them for the trapped system, at least for our range of parameters. In other words, the healing length for χ far exceeds the Thomas-Fermi radius of the system, and, therefore, although χ is fluctuating inside the cloud, it remains nearly constant [24], except for the surface layer, where the density of particles is low.

Thermal fluctuations of χ should also be noticeable in three-dimensional condensates at low and moderate temperatures. In general, the dependences of the number of condensed particles on the reduced temperature and critical temperature on the total number of atoms for the 3D case are similar to those in the 2D system, which are described by Eqs. (2) and (3). The main difference is the powers of t and N in the right-hand sides of Eqs. (2) and (3), which are 3 and $-1/3$ ($\hbar\omega_\perp/kT_c\sim N^{-1/3}$), respectively. However, in this case one has to take accurately into account the possibility of long-wavelength fluctuations of χ in longitudinal direction and the formation of kinks [24].

IV. CONCLUSIONS

In this paper, we have studied the effect of thermal fluctuations on the coherence among different components of the order parameter in a quasi-2D rotating $F=1$ Bose-Einstein condensate, when all three hyperfine states are populated. Different axially symmetric vortex phases were considered. We have shown that the deviation of the relative phase $\chi=2\delta_0-\delta_1-\delta_{-1}$ from equilibrium can be very significant even at low temperatures, much smaller than T_c. Fluctuations of the relative angle induce sizable fluctuations of the spin texture, namely, local transverse magnetization of the condensate. We have shown that these fluctuations are much more pronounced in the antiferromagnetic case than in the ferromagnetic one. The direct and nondestructive method recently proposed in Ref. [20] for the imaging of spinor BEC spatial magnetization (or some of its modification) can be applied for the experimental study of the thermal fluctuations of spin textures, since it enables multiple-shot imaging and one can directly observe the dynamics of a single sample.

ACKNOWLEDGMENTS

The authors acknowledge useful discussions with T. K. Ghosh and T. Mizushima. W.V.P. is supported by the Japan Society for the Promotion of Science.

[1] J. Stenger, D. M. Stamper-Kurn, H. J. Miesner, A. P. Chikkatur, and W. Ketterle, Nature (London) **396**, 345 (1999).
[2] T. Mizushima, K. Machida, and T. Kita, Phys. Rev. Lett. **89**, 030401 (2002).
[3] T. Isoshima and K. Machida, Phys. Rev. A **66**, 023602 (2002).
[4] J. W. Reijnders, F. J. M. van Lankvelt, K. Schoutens, and N. Read, Phys. Rev. Lett. **89**, 120401 (2002).
[5] K. Kasamatsu, M. Tsubota, and M. Ueda, Int. J. Mod. Phys. B **19**, 1835 (2005).
[6] W. V. Pogosov, R. Kawate, T. Mizushima, and K. Machida, Phys. Rev. A **72**, 063605 (2005).
[7] A. Trombettoni, A. Smerzi, and P. Sodano, New J. Phys. **7**, 57 (2005).
[8] T. P. Simula, M. D. Lee, and D. A. W. Hutchinson, Philos. Mag. Lett. **85**, 395 (2005).
[9] M. Holzmann, G. Baym, J. P. Blaizot, and F. Laloe, e-print cond-mat/0508131.
[10] T. P. Simula and P. B. Blakie, Phys. Rev. Lett. **96**, 020404 (2006).
[11] D. Schumayer and D. A. W. Hutchinson, e-print cond-mat/0601500.
[12] S. Stock, Z. Hadzibabic, B. Battelier, M. Cheneau, and J. Dalibard, Phys. Rev. Lett. **95**, 190403 (2005).
[13] Z. Hadzibabic, P. Kruger, M. Cheneau, B. Battelier, and J. Dalibard, Nature (London) **441**, 1118 (2006).
[14] Y. Castin, Z. Hadzibabic, S. Stock, J. Dalibard, and S. Stringari, Phys. Rev. Lett. **96**, 040405 (2006).
[15] W. V. Pogosov and K. Machida, Phys. Rev. A **74**, 023622 (2006).
[16] Y. J. Wang, D. Z. Anderson, V. M. Bright, E. A. Cornell, Q. Diot, T. Kishimoto, M. Prentiss, R. A. Saravanan, S. R. Segal, and S. Wu, Phys. Rev. Lett. **94**, 090405 (2005).
[17] M. H. Wheeler, K. M. Mertes, J. D. Erwin, and D. S. Hall, Phys. Rev. Lett. **93**, 170402 (2004).
[18] M. Saba, T. A. Pasquini, C. Sanner, Y. Shin, W. Ketterle, and D. E. Pritchard, Science **307**, 1945 (2005).
[19] M. S. Chang, Q. Qin, W. Zhang, L. You, and M. S. Chapman, Nature Physics **1**, 111 (2005).
[20] J. M. Higbie, L. E. Sadler, S. Inouye, A. P. Chikkatur, S. R. Leslie, K. L. Moore, V. Savalli, and D. M. Stamper-Kurn, Phys. Rev. Lett. **95**, 050401 (2005).
[21] C. Gies, B. P. van Zyl, S. A. Morgan, and D. A. W. Hutchinson, Phys. Rev. A **69**, 023616 (2004).
[22] T. Ohmi and K. Machida, J. Phys. Soc. Jpn. **67**, 1822 (1998).
[23] T. L. Ho, Phys. Rev. Lett. **81**, 742 (1998).
[24] W. V. Pogosov and K. Machida, Phys. Rev. A **74**, 023611 (2006).
[25] H. Pu, C. K. Law, S. Raghavan, J. H. Eberly, and N. P. Bigelow, Phys. Rev. A **60**, 1463 (1999).