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Decoherence effects on the quantum spin channels

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An open ended spin chain can serve as a quantum data bus for the coherent transfer of quantum state information. In this paper, we investigate the efficiency of such quantum spin channels which work in a decoherence environment. Our results show that the decoherence will significantly reduce the fidelity of quantum communication through the spin channels. Generally speaking, as the distance increases, the decoherence effects become more serious, which will put some constraints on the spin chains for long distance quantum state transfer.

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I. INTRODUCTION

Quantum computation has the potential to outperform their classical counterparts in solving some intractable problems which would need an exponentially longer time for a classical computer [1,2]. A lot of effort has been devoted to searching for various kinds of real physical systems that may be appropriate for the implementation of quantum computation. One key feature of such physical systems is scalability [3]. There are several prospective candidates for scalable quantum computation, such as optical lattices [4–6], arrays of quantum dots [7–9], and superconducting circuits [10,11]. It is also known that universal quantum computation can be performed by a chain of qubits with nearest neighbor Heisenberg or XY coupling together with some other physical resources [12–15].

In the large-scale quantum computing, how to transmit quantum states from one location to the other location, which is a little similar but not all the same to quantum information distributing [16], is an important problem. The primitive scheme of quantum communication through an unmodulated spin chain is proposed by Bose [17]. It was shown that quantum states can be transferred via an open ended spin chain with ferromagnetic Heisenberg interactions. The fidelity will exceed the highest fidelity for a classical transmission of a quantum state until the chain length N is larger than 80. In Ref. [18], Christandal *et al.* put forward a special class of Hamiltonian that is mirror periodic. Based on a spin chain with such a mirror-periodic Hamiltonian, a perfect quantum state transfer can be achieved. Up to now, there are many other variational schemes for the transfer of quantum states in spin systems [19,20]. In the real physical systems, especially for a solid state system, decoherence and noise is inevitable [1]. For example, in the system of arrays of quantum dots, both the surrounding nuclei spin environment [21] and $1/f$ noise will induce decoherence. Therefore, under the influence of decoherence, how efficient different spin chain channels will work becomes an interesting and important problem. On the other hand, dynamical decoherence properties of many-body systems [22–24] are basically significant by itself. There have been several works about the decoherence and spin chain channels [25–27]. However, the decoherence effects on the efficiency of these quantum spin channels have not yet been *thoroughly* investigated.

In this paper, we calculate the fidelity of quantum communication through the spin chain channels under the influence of decoherence. Two representative kinds of environment model are investigated. One is the one common spin environment [28]. The other is the local independent environment [29]. We show that the efficiency of the spin channels will be significantly lowered by the decoherence environment. As the spin chain length increases, the decoherence effects may become very severe, which suggest some new constraints on the spin chains for long distance quantum state transfers. We mostly concentrate on the Heisenberg spin chain and the mirror-periodic Hamiltonian scheme. However, some of the results are applicable for other schemes of quantum state transfer.

The structure of this paper is as follows. In Sec. II we investigate the efficiency of quantum spin channels in one common spin environment. In Sec. III the situation of a local independent environment is discussed. In Sec. IV are conclusions and some discussions.

II. ONE COMMON SPIN ENVIRONMENT

We start by considering the important decoherence model in spin systems, i.e., one common spin environment. The Hamiltonian of the spin chain with N spins is denoted as H_S. The total z component of the spin is conserved, i.e., $[\Sigma_{i=1}^{N}\sigma_i^z, H_S]=0$. This is true for several important spin chain channels [17,18]. The central system interacts with one common spin environment Ξ [28], which is formed by M independent spins, for large values of M, as depicted in Fig. 1.

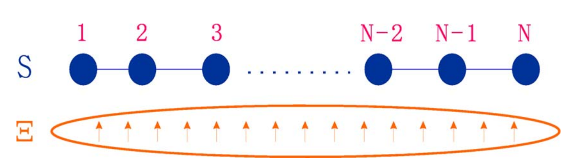

FIG. 1. (Color online) A quantum wire with N spins in the line coupled with one common spin environment.

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$$H_{S\Xi} = \frac{1}{2}\sum_{i=1}^{N} \sigma_i^z \otimes \sum_{k=1}^{M} g_k \sigma_k^z. \quad (1)$$

The whole system of the spin chain S and the environment Ξ is described by the Hamiltonian

$$H_{\mathcal{T}} = H_S + H_{S\Xi}. \quad (2)$$

Here the self-Hamiltonian of the environment Ξ is neglected. This simple decoherence model is an important solvable model of decoherence, which is much relevant to quantum information processing [1]. We will demonstrate how efficient the quantum spin chain channels will work in such a decoherence environment.

The quantum state to be transferred is located at the first spin, $|\varphi_{in}\rangle = \alpha|0\rangle + \beta|1\rangle$. The initial state of the central system is

$$|\psi_S(0)\rangle = \alpha|\mathbf{0}\rangle + \beta|\mathbf{1}\rangle \quad (3)$$

with $|\mathbf{0}\rangle = |00,\cdots 0\rangle$, $|\mathbf{1}\rangle = |10,\cdots,0\rangle$. In the following, we will denote $|\mathbf{j}\rangle$ the state in which the jth spin is in the $|1\rangle$ state, while all the other spins are in the $|0\rangle$ state. Without the influence of decoherence, the transfer fidelity at time t is defined as $f_{1N}(t) \coloneqq \langle \mathbf{N}|e^{-iH_s t}|\mathbf{1}\rangle$. Using the calculations in [30], we can write down the final density matrix $\rho_N(t)$ in the absence of the decoherence, just in terms of the initial coefficients of the input state, and the transfer fidelity, as follows:

$$\rho_N(t) = \begin{pmatrix} 1 - |f_{1N}(t)|^2 & \alpha f_{1N}^*(t) \\ \alpha^* f_{1N}(t) & |f_{1N}(t)|^2 \end{pmatrix}. \quad (4)$$

Now we take into account the interaction between the spin chain and the environment. We denote the basis of the environment $\{|m\rangle\langle m|\}$, $\sum_{m=0}^{2^M-1}|m\rangle\langle m| = I_\Xi$, where $|m\rangle = |m_1 m_2,\cdots m_M\rangle$ with $\sigma_k^z|m_k\rangle = (-1)^{m_k}|m_k\rangle$. Since $\sum_{k=1}^{M} g_k \sigma_k^z |m\rangle = \sum_{k=1}^{M}(-1)^{m_k} g_k |m\rangle$, therefore,

$$H_{\mathcal{T}} = \sum_{m=0}^{2^M-1}\left(H_S + \frac{1}{2}B_m \sum_{i=1}^{N}\sigma_i^z\right) \otimes |m\rangle\langle m| \quad (5)$$

with $B_m = \sum_{k=1}^{M}(-1)^{m_k} g_k$. The time evolution operator for the combined system environment is $U(t) = \exp(-itH_{\mathcal{T}})$, i.e.,

$$U(t) = \sum_{m=0}^{2^M-1} U_m(t) \otimes |m\rangle\langle m|, \quad (6)$$

where $U_m(t) = \exp(-itH_S^{(m)})$ with $H_S^{(m)} = H_S + \frac{1}{2}B_m \sum_{i=1}^{N}\sigma_i^z$. We consider the initial state of the spin chain together with the M independent environment spins of the form as Zurek has considered in Ref. [28]:

$$|\psi_{S\Xi}(0)\rangle = |\psi_S(0)\rangle \otimes \sum_{m=0}^{2^M-1} c_m |m\rangle. \quad (7)$$

After arbitrary time t, the evolution of the spin-environment system is $|\psi_{S\Xi}(t)\rangle = U(t)|\psi_{S\Xi}(0)\rangle = \sum_{m=0}^{2^M-1}[U_m(t) \otimes |m\rangle\langle m|]|\psi_{S\Xi}(0)\rangle$. Therefore, the final density matrix of the target spin is

$$\begin{aligned}\rho_N'(t) &= \mathrm{Tr}_{\bar{N}}\mathrm{Tr}_\Xi[|\psi_{S\Xi}(t)\rangle\langle\psi_{S\Xi}(t)|] \\ &= \sum_{m=0}^{2^M-1}|c_m|^2 Tr_{\bar{N}}\{U_m(t)|\psi_S(0)\rangle\langle\psi_S(0)|U_m^\dagger(t)\} \\ &= \sum_{m=0}^{2^M-1}|c_m|^2 \rho_N^{(m)}(t). \end{aligned} \quad (8)$$

We note that $\left[\frac{1}{2}B_m \sum_{i=1}^{N}\sigma_i^z, H_S\right] = 0$. It can be observed that if the environment is in the state $|m\rangle$, the output just undergoes a Z rotation by some angle B_m, i.e.,

$$\rho_N^{(m)}(t) = \begin{pmatrix} 1 - |f_{1N}(t)|^2 & \alpha f_{1N}^*(t) e^{-iB_m t} \\ \alpha^* f_{1N}(t) e^{iB_m t} & |f_{1N}(t)|^2 \end{pmatrix}. \quad (9)$$

Consequently, the only change to the final density matrix caused by the decoherence is to reduce the off-diagonal elements by a factor of $\gamma(t) = \sum_{m=1}^{2^M-1}|c_m|^2 e^{iB_m t}$ and $\gamma^*(t)$. The efficiency of the quantum spin channel is characterized by the fidelity averaged over all pure state in the Bloch sphere, that is $F'(t) = 1/4\pi \int \mathrm{Tr}[\rho_N'(t)|\varphi_{in}\rangle\langle\varphi_{in}|]d\Omega$.

We denote the character function of the environment as $\eta(B) = \sum_{m=0}^{2^M-1}|c_m|^2 \delta(B - B_m)$. After some straightforward calculation, we can write the average fidelity of the quantum spin channel in the spin environment as

$$F'(t) = \frac{1}{2} + \frac{|f_{1N}(t)|^2}{6} + \frac{|f_{1N}(t)|}{3}\int \cos(Bt + \phi)\eta(B)dB, \quad (10)$$

where $\phi = \arg\{f_{1N}(t)\}$. For a general spin environment of large M values, the character function $\eta(B)$ is approximately Gaussian [28], that is $\eta(B) = \exp(-B^2/\vartheta)/\sqrt{\pi\vartheta}$. Then $\int \cos(Bt)\eta(B)dB = e^{-\vartheta t^2/4}$, and the average fidelity becomes

$$F'(t) = \frac{1}{2} + \frac{|f_{1N}(t)|^2}{6} + \frac{|f_{1N}(t)|\cos\phi}{3}e^{-\vartheta t^2/4}. \quad (11)$$

A. Heisenberg spin chain

We first consider the Bose primitive scheme. There are N spins in the line with ferromagnetic Heisenberg interactions, labeled $1,2,\ldots,N$. The Hamiltonian of the spin chain [17] is $H_S = -J\sum_{i=1}^{N-1}\vec{\sigma}_i \cdot \vec{\sigma}_{i+1} - B\sum_{i=1}^{N}\sigma_i^z$. For the situation without considering the influence of decoherence environment, by choosing the magnetic fields B as some special value B_c, one can make the transfer fidelity $f_{1N}(t) = \langle \mathbf{N}|e^{-itH_S}|\mathbf{1}\rangle \in \mathcal{R}$, i.e., $\phi = \arg\{f_{1N}(t)\} = 0$, and then maximize the original average fidelity. Therefore, the average fidelity of the quantum spin channel in the spin environment is

$$F'(t) = \frac{1}{2} + \frac{f_{1N}^2(t)}{6} + \frac{f_{1N}(t)}{3}e^{-\vartheta t^2/4}. \quad (12)$$

We depict the above average fidelity for $N=3,5,8,10$ in Fig. 2. Compared with the original fidelity [17] $F(t) = \frac{1}{2} + f_{1N}^2(t)/6 + f_{1N}(t)/3$, it can be seen that the decoherence environment will obviously reduce the efficiency of

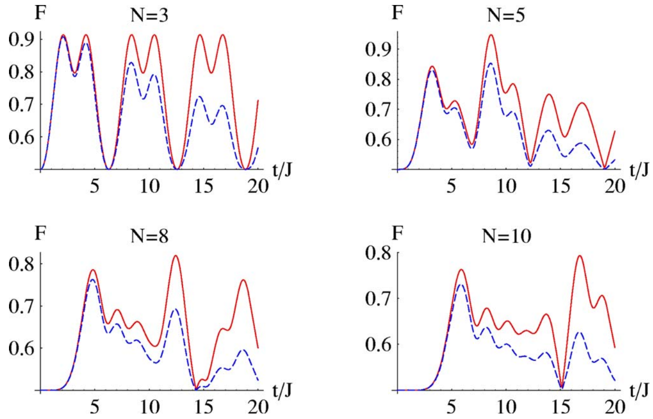

FIG. 2. (Color online) The average fidelity of quantum spin channels in the one common spin environment (dashed) and without decoherence (solid) as functions of time t/J for different distances $N=3,5,8,10$. The Gaussian parameter $\vartheta/J^2=0.02$.

quantum communication through the spin chain channels, especially for the target spin of long distance. Because if the time of transfer is longer, the decoherence effects are more severe and more quantum state information will be lost. In fact, without the influence of decoherence, the critical spin chain length is $N_c=80$[17], i.e., if the spin chain length $N \leq N_c$, the fidelity of quantum communication through the spin channel will exceed $\frac{2}{3}$, which is the highest fidelity for classical transmission of the state [31]. However, even though the environment parameter ϑ is small, the critical spin chain length is significantly reduced. We list the critical spin chain length for several environment parameters in Table I. Therefore, if the spin chain length is large, to achieve satisfactory efficiency of quantum state transfer, new constraints, e.g., larger coupling strength J, is necessary.

We now investigate entanglement distribution through the above open ended spin channel in the one common spin environment. Two particles $\mathcal{A}$ and $\mathcal{B}$ are initially in the entangled state $|\psi^{\dagger}_{\mathcal{AB}}\rangle=(|01\rangle+|01\rangle)/\sqrt{2}$. We set $\mathcal{B}$ as the first site of the spin chain channel, then after some time t entanglement will be established between $\mathcal{A}$ and the target spin, i.e., the Nth spin. What we are interested in is the amount of distributed entanglement between the $\mathcal{A}$ and the Nth spin. As pointed out in Ref. [17], the spin chain, without the influence of environment, acts as an amplitude damping quantum channel, i.e.,

$$\rho_{\mathcal{A}N}(t)=\sum_{i=0,1}(I\otimes M_i)|\psi^{\dagger}_{\mathcal{AB}}\rangle\langle\psi^{\dagger}_{\mathcal{AB}}|(I\otimes M_i^{\dagger}) \tag{13}$$

with $M_0=|0\rangle\langle 0|+f_{1N}(t)|1\rangle\langle 1|$ and $M_1=[1-|f_{1N}(t)|^2]^{1/2}|0\rangle\langle 1|$. In the same way as discussed above, the final entangled state becomes $\rho'_{\mathcal{A}N}(t)=\{(1-\lambda^2)|00\rangle\langle 00|+\lambda^2|01\rangle\langle 01|+|10\rangle\langle 10|$

TABLE I. Critical spin chain length N_c for different environment parameters ϑ/J^2.

ϑ/J^2	0	0.0002	0.001	0.002	0.005	0.01	0.015	0.02
N_c	80	32	29	26	22	18	16	15

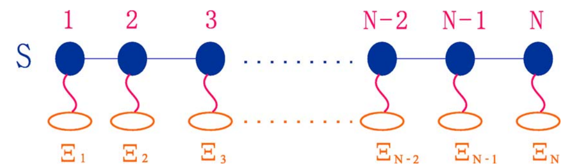

FIG. 3. (Color online) A quantum wire with N spins in the line coupled with local independent environment.

$+\zeta|01\rangle\langle 10|+\zeta^{*}|10\rangle\langle 01|\}/2$, where $\lambda=|f_{1N}(t)|$ and $\zeta=\int\lambda\exp(-iB't)\eta(B')dB'=\lambda e^{-\vartheta t^2/4}$. Therefore, the distributed entanglement measured by concurrence [32] is,

$$\xi'=\xi_0 e^{-\vartheta t^2/4}, \tag{14}$$

where $\xi_0=\lambda$ is the distributed entanglement without decoherence.

B. Mirror-periodic Hamiltonian

Now we consider the perfect state transfer channels, i.e., the mirror-periodic Hamiltonian scheme in one common spin environment. The N spins in the line with XY coupling is described by the Hamiltonian [18] $H_S=\Sigma_{i=1}^{N-1}J_i/2(\sigma_i^x\sigma_{i+1}^x+\sigma_i^y\sigma_{i+1}^y)$, where $J_i=\omega\sqrt{i(N-i)}/2$. The most important property of the mirror-periodic Hamiltonian is that $e^{-itH}\psi(s_1,s_2,\ldots,s_{N-1},s_N)=(\pm)\psi(s_N,s_{N-1},\ldots,s_2,s_1)$ for some time t. For the above Hamiltonian H_S, the transfer fidelity is $f_{1N}(t)=[-i\sin(\omega t/2)]^{N-1}$, i.e., perfect quantum state transfer will be achieved at a constant time $t=\pi/\omega$ for arbitrary spin chain distance. According to the above Eqs. (8) and (9), we can easily get the average fidelity

$$F'(t)=\tfrac{2}{3}+\tfrac{1}{3}e^{-\vartheta t^2/4}. \tag{15}$$

Since the optimal transfer time $t=\pi/\omega$ is constant, the average fidelity is independent on the spin chain length N. This is different from the situation of the Heisenberg spin chain channel.

III. LOCAL INDEPENDENT ENVIRONMENT

In this section, we will consider another representative decoherence model, the local independent environment. Each individual spin of the central system S interacts independently with the local environment Ξ_i, as depicted in Fig. 3. The decoherence process of the multispin system can be described by a general quantum master equation of Lindblad form [33],

$$\frac{\partial}{\partial t}\rho=-i[H_S,\rho]+\sum_{i=1}^{N}(\mathbf{I}\otimes\cdots\otimes\mathbf{I}\otimes\mathcal{L}_i\otimes\mathbf{I}\otimes\cdots\otimes\mathbf{I})\rho. \tag{16}$$

The superoperator $\mathcal{L}_i$ describes the independent interaction of the ith spin with the local environment.

It is known that macroscopic systems are more fragile under the influence of the decoherence environment. In the situation of local independent decoherence environment, this can be demonstrated in an explicit way as follows.

The phenomenological analysis solution of the quantum master equation in Eq. (15) can be written as $\rho(t) = \rho(0) + \int_0^t \{-i[H_S, \rho(t')] + \sum_{i=1}^N \mathcal{L}_i \rho(t') dt'\}$. And the ideal state without decoherence is $\rho_0(t) = \rho(0) + \int_0^t \{-i[H_S, \rho_0(t')]\}$. If the time $t = \delta t$ is short enough, the difference between the real and ideal state of the central system is $\rho(t) - \rho_0(t) = \sum_{i=1}^N \mathcal{L}_i \rho(0) \delta t$. Therefore, it is obvious that the state deviation will become larger as N increases for most kinds of decoherence model.

A. Mirror-periodic Hamiltonian

We first consider the mirror-periodic Hamiltonian scheme in the local independent dephasing and damping channels. The dephasing process corresponds to the situation where only phase information is lost [1], without energy exchange. The superoperator for the dephasing channel [29] is

$$\mathcal{L}_i \rho = -\frac{\gamma_i}{2}(\rho - \sigma_i^z \rho \sigma_i^z). \tag{17}$$

For simplicity, we assume that the system-environment coupling strength $\gamma_i = \gamma$ are the same for all spins.

In the case of quantum state transfer, the initial state of the system is $|\psi_S(0)\rangle = \alpha|\mathbf{0}\rangle + \beta|\mathbf{1}\rangle$. The state transfer dynamics under the influence of the dephasing channel is completely determined by the evolution in the *zero* and *single* excitation subspace $\mathcal{H}_{0\oplus 1}$. Therefore, we only need to solve the above master equation in this $(N+1)$-dimensional subspace. When restricted to the subspace $\mathcal{H}_{0\oplus 1}$,

$$H_S = \sum_{i=1}^{N-1} J_i(|\mathbf{i}\rangle\langle \mathbf{i+1}| + |\mathbf{i+1}\rangle\langle \mathbf{i}|),$$

$$\sigma_i^z = \mathbf{I}_{\mathrm{N+1}} - 2|\mathbf{i}\rangle\langle \mathbf{i}|. \tag{18}$$

At time $T = \pi/\omega$, the reduced density of the Nth spin is

$$\rho^{(N)}(\pi/\omega) = \begin{pmatrix} 1 - \rho_{NN}(\pi/\omega) & \rho_{0N}(\pi/\omega) \\ \rho_{0N}^*(\pi/\omega) & \rho_{NN}(\pi/\omega) \end{pmatrix}, \tag{19}$$

where $\rho_{NN}(\pi/\omega) = \langle \mathbf{N}|\rho(\pi/\omega)|\mathbf{N}\rangle$ and $\rho_{0N}(\pi/\omega) = \langle \mathbf{0}|\rho(\pi/\omega)|\mathbf{N}\rangle$. Therefore, the probability of an excitation transfer from the first spin to the Nth spin at time $t = \pi/\omega$ is $P = \rho_{NN}(\pi/\omega)$. And the fidelity between the real and ideal transferred quantum state, i.e., $|\psi_{ideal}(\pi/\omega)\rangle = \alpha|0\rangle + (-i)^{N-1}\beta|1\rangle$, is

$$F(\pi/\omega, \alpha, \beta) = [1 - \rho_{NN}(\pi/\omega)]|\alpha|^2 + \rho_{NN}(\pi/\omega)|\beta|^2 + 2\,\mathrm{Re}\{(-i)^{N-1}\rho_{0N}(\pi/\omega)\alpha^*\beta\}. \tag{20}$$

The efficiency of quantum communication through the above quantum spin channel is characterized by the average fidelity over all pure states in the Bloch sphere $F(\pi/\omega) = \frac{1}{4\pi}\int F(\pi/\omega, \alpha, \beta) d\Omega$. In the mirror-periodic Hamiltonian scheme, we solve the above differential equations numerically, and depict the probability of an excitation transfer and the average fidelity in Figs. 4 and 5. Though the transfer time is constant $t = \pi/\omega$ for any spin chain distance N, the probability of an excitation transfer and the average fidelity will still decay as the spin chain length increases. Due to the decoherence effects, the mirror-periodic Hamiltonian scheme cannot achieve perfect quantum state transfer again. However, using larger nearest-neighbor interaction ω, i.e., relative smaller system-environment coupling strength γ/ω, we will increase the efficiency of quantum communication in the decoherence environment. The pure damping channel corresponds to the decay process of the central system coupled to a thermal bath at zero temperature [1,29]. The superoperator for the damping channel is

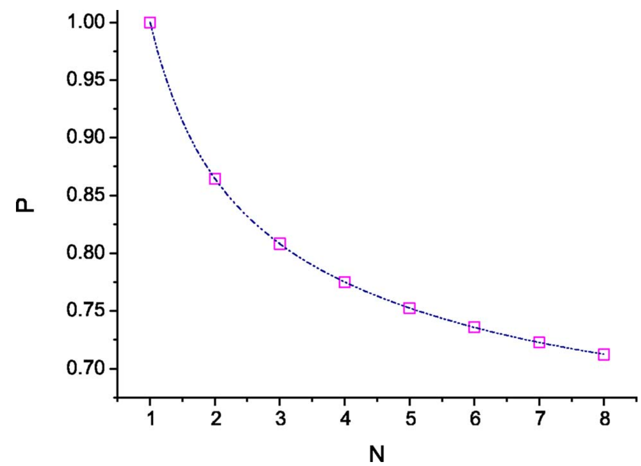

FIG. 4. (Color online) The probability of an excitation transfer as a function of the spin chain length N for the dephasing channel in the local independent model. The system-environment coupling strength $\gamma/\omega = 0.1$.

$$\mathcal{L}_i \rho = -\frac{\gamma_i}{2}(\sigma_i^- \sigma_i^+ \rho + \rho \sigma_i^- \sigma_i^+ - 2\sigma_i^+ \rho \sigma_i^-), \tag{21}$$

where $\sigma_i^{\pm} = (\sigma_i^x \pm i\sigma_i^y)/2$, with the system-environment coupling strength $\gamma_i = \gamma$ for all spins.

In the similar way as the dephasing channel, when restricted to the subspace $\mathcal{H}_{0\oplus 1}$,

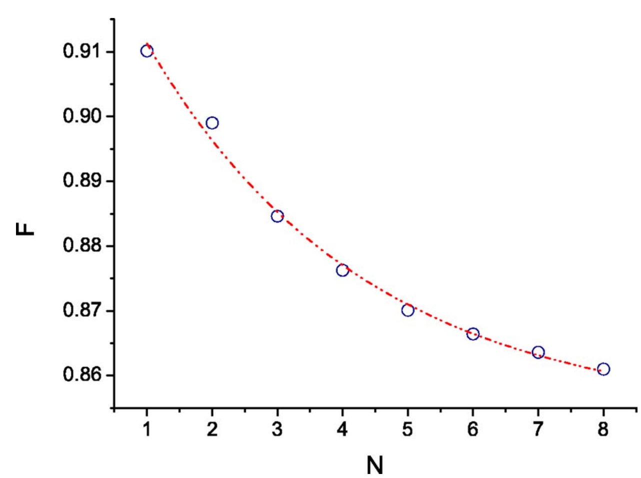

FIG. 5. (Color online) The fidelity as a function of the spin chain length N for the dephasing channel in the local independent model. The system-environment coupling strength $\gamma/\omega = 0.1$.

TABLE II. The maximum probability of an excitation transfer for different spin chain length N in the local independent model. The system-environment coupling strength $\gamma/J=0.1$.

N	2	3	4	5	6	7	8
Dephasing P	0.965	0.708	0.664	0.535	0.431	0.379	0.338
Damping P	0.928	0.681	0.678	0.556	0.420	0.371	0.333

$$\sigma_i^- = |\mathbf{i}\rangle\langle\mathbf{0}|, \sigma_i^+ = |\mathbf{0}\rangle\langle\mathbf{i}|, \sigma_i^-\sigma_i^+ = |\mathbf{i}\rangle\langle\mathbf{i}|. \tag{22}$$

The reduced density of the Nth spin and the average fidelity in Eqs. (18) and (19) are applicable to the damping channel also. By solving the corresponding differential master equations numerically, we can obtain the probability of an excitation transfer and the average fidelity for different spin chain lengths N. Unlike the situation of the dephasing channel, we find that at time $t=\pi/\omega$, the probability of an excitation transfer is independent on the spin chain length, that is $\rho_{NN}(\pi/\omega)=e^{-\gamma\pi}$. Moreover, the average fidelity $F(\pi/\omega)$ is independent on the spin chain length also. This result is somewhat surprising. Whether this property is only held by the mirror-periodic Hamiltonian in a pure damping environment is an interesting open problem, which will be investigated in detail in our following work.

B. Heisenberg spin chain

For the Heisenberg spin chain channel in the local independent dephasing and damping environment, we can express the system Hamiltonian H_S in the subspace $\mathcal{H}_{0\oplus 1}$ and solve the quantum master equations in the similar way as discussed above. We list the maximum probability of an excitation transfer P for different spin chain length N in Table II. The behavior of the average fidelity F is similar to the probability of an excitation transfer P.

It can be seen that the maximum probability of an excitation transfer, i.e., the efficiency of the Heisenberg spin chain channel, will decay as the spin chain length N increases not only for the dephasing channel but also for the damping channel. This is slightly different from the situation of mirror-periodic Hamiltonian, where the probability of an excitation transfer and the average fidelity are independent on the spin chain length.

IV. CONCLUSIONS AND DISCUSSIONS

In conclusion, we have investigated the efficiency of quantum communication through the spin chain channels under the influence of a decoherence environment. We focus on the Heisenberg spin chain and the mirror-periodic Hamiltonian scheme. Two representative decoherence models are considered, one is the common spin environment, the other is local independent environment. It can be seen that the efficiency of the quantum wires will be significantly lowered by the external decoherence environment. Generally speaking, the decoherence effects become more serious for larger spin chain lengths. However, for a different spin chain Hamiltonian and decoherence models, the results will be somewhat different.

In Ref. [26], it has been shown that in some specific environment, quantum state transfer is possible with the same fidelity and only reasonable slowing. However, for more general decoherence models as considered in this paper, we show that, to achieve highly efficient long distance quantum state transfer in a decoherence environment, the time of transfer [34] becomes a crucial factor and more constraints on the spin system Hamiltonian are requisite. We should resort to a new encoding strategy for the protection of quantum state information during the transfer along the spin chain channels. Besides, we provide some possible evidence for the unusual dynamical decoherence properties of the mirror-periodic Hamiltonian, which may lead to some valuable utilities of this special class of Hamiltonian.

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