Experimental evidence for an optical spring

A. Di Virgilio,[1] L. Barsotti,[1] S. Braccini,[1] C. Bradaschia,[1] G. Cella,[1] C. Corda,[1] V. Dattilo,[2] I. Ferrante,[1,3] F. Fidecaro,[1,3] I. Fiori,[1] F. Frasconi,[1] A. Gennai,[1] A. Giazotto,[1] P. La Penna,[2] G. Losurdo,[4] E. Majorana,[5] M. Mantovani,[6] A. Pasqualetti,[2] D. Passuello,[1] F. Piergiovanni,[7] A. Porzio,[8] P. Puppo,[5] P. Rapagnani,[5,9] F. Ricci,[5,9] S. Solimeno,[8,10] G. Vajente,[1,11] and F. Vetrano[7]

[1]*INFN Sezione di Pisa, Pisa, Italy*
[2]*EGO, European Gravitational Observatory, Cascina (Pi), Italy*
[3]*Università di Pisa, Pisa, Italy*
[4]*INFN Sezione di Firenze, Sesto Fiorentino, Italy*
[5]*Università di Roma 1, and INFN-Roma 1, Roma, Italy*
[6]*Università di Siena, Siena, Italy*
[7]*Università di Urbino, Urbino, Italy*
[8]*Coherentia, CNR-INFM and CNISM, Napoli, Italy*
[9]*Università di Roma 1, Roma, Italy*
[10]*Università di Napoli and INFN, Napoli, Italy*
[11]*Scuola Normale Superiore, Pisa, Italy*

(Received 23 December 2005; published 26 July 2006)

An optical spring effect has been observed in the motion of a Fabry-Pérot cavity suspended in the Low-Frequency Facility, R&D experiment of the VIRGO Collaboration. The experimental setup consists of a 1-cm-long cavity hanging from a mechanical isolation system, conceived to suppress seismic noise transmission to the optical components of the interferometer. The observed radiation pressure effect corresponds to an optical stiffness k_{opt} ranging between 2.5×10^4 and 6.5×10^4 N/m. This paper reports several measurements of the mirror relative displacement carried out in different working conditions. The measured error signal spectra show broad resonances at frequencies compatible with the optomechanical system. In other runs cavity detuning oscillations have been observed at subhertz frequencies. In these cases the power spectrum of the control loop error signal exhibited a broad resonance with superimposed uniformly spaced peaks. These observations, validated by a simple model, prove that the fluctuations of the optical spring effect can become so large as to exhibit nonlinear features.

DOI: 10.1103/PhysRevA.74.013813 PACS number(s): 42.50.Lc, 04.80.Nn, 07.60.Ly, 42.65.Sf

The radiation pressure will play a crucial role in the next generation of laser interferometric detectors of gravitational waves [1–4]. This is a consequence of the continuous demand for more powerful laser beams to reduce the influence of the photon counting fluctuations. In Fabry-Pérot (FP) cavities detuned from resonance the radiation pressure acting on the two mirrors creates a generally anharmonic optical restoring force, referred to as the optical spring effect. For sufficiently small deviation from resonance, this effect is described by [2] a spring of stiffness k_{opt} coincident with the derivative of the radiation pressure with respect to the phase delay δ (modulus 2π) accumulated by the cavity mode during a round trip:

$$k_{opt}=\frac{8\pi(2R_2+A_2)T_1P_{in}\sqrt{R_1R_2}}{\lambda c}\times\frac{\sin\left(\frac{4\pi\delta}{\lambda}\right)}{\left[1+R_1R_2-2\sqrt{R_1R_2}\cos\left(\frac{4\pi\delta}{\lambda}\right)\right]^2} \quad (1)$$

with R_i, T_i, and A_i the reflectivity, transmissivity, and loss of the ith mirror $(i=1,2)$, P_{in} the cavity input power, λ the light wavelength, and c the speed of light. Since k_{opt} is proportional to $\sin\left(\frac{4\pi\delta}{\lambda}\right)$ it changes sign on passing through a resonance. The cavity reacts to the noise affecting the mirror positions as a mechanical oscillator that is either stable or unstable for positive or negative detuning (δ), respectively. When the fluctuations of δ become very large, the dependence of k_{opt} on δ is no longer negligible and the system behaves as a nonlinear oscillator.

This paper reports the observation of the optical spring effect in a cavity stabilized by means of a Pound-Drever-Hall (PDH) system and hanging from a VIRGO superattenuator (SA) [5–9] mounted on the so-called Low-Frequency Facility (LFF) apparatus at the VIRGO interferometer [10–12]. Under suitable conditions, resonances have appeared in the PDH spectrum, signaling the onset of optical spring oscillations (see Fig. 1). In some cases the oscillations have been large enough to offer evidence of nonlinear behavior of the optomechanical oscillator. The resonance emerges from the noise pedestal thanks to the SA system, which drastically reduces the seismic noise, developed to push down to 10 Hz the VIRGO antenna detection bandwidth. To our knowledge this is the first direct observation of linear and nonlinear optical spring effects in a suspended cavity with massive mirrors.

So far, the optical spring has been reported twice. First it was observed with a 1.2 g mirror mounted on a cantilever flexure [13]. Later, it was inferred from the transfer function

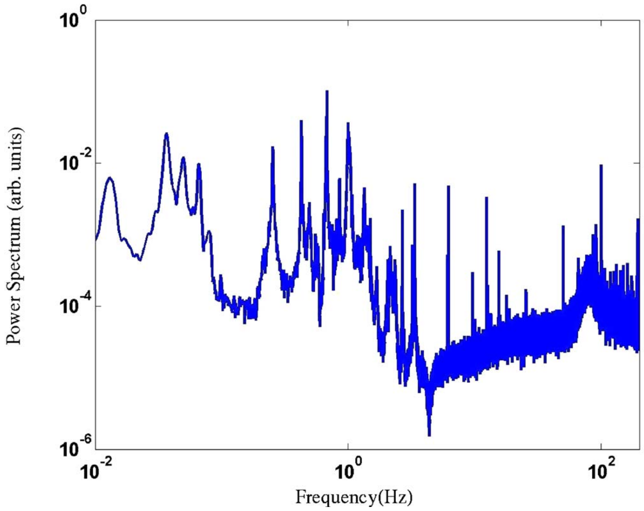

FIG. 1. (Color online) Typical error signal power spectrum. The optical spring effect appears as a broad resonance at ∼80 Hz.

of the force applied to the end mirror of a suspended cavity [14]. In the latter case the resonance peak was not observed directly due to the high gain of the control loop in the optical spring band (below 100 Hz). Direct observation of the optical spring requires a small control loop gain at low frequency. This condition has been achieved in the reported case by suspending the cavity in the SA, which reduces the seismic-noise-induced motion down to a few micrometers below 2 Hz, so allowing a feedback unity-gain frequency of 150 Hz.

To clearly identify the observed resonance as an optical spring effect, several runs were made for different cavity static detunings. As predicted by the change of sign of k_{opt} in passing from positive to negative detunings, stable and unstable behaviors have been observed, with the instability manifested by the impossibility of locking the cavity. As further evidence, on increasing the control bandwidth the system behaves nonlinearly, exhibiting a time modulation of k_{opt}.

The experimental apparatus consists of a 1-cm-long FP cavity hanging on a SA chain [5]. The last stage of the suspension is sketched in Fig. 2, a more detailed description being provided in [10,11]. The optical parameters of the FP cavity are given in Table I.

The measured cavity finesse ranged between 4000 and 7000 (nominal value 6300), while the radiation pressure force at resonance was $\simeq 5\ \mu$N. The spherical 3.5-cm-diameter mirror is embedded in a large steel cylinder playing the role of the VIRGO test mass (VM in Fig. 2). The flat mirror (AX, auxiliary mirror) is hung, by means of an independent three-stage suspension, on the last mechanical filter of the chain, called Filter 7. The longitudinal motion is controlled by acting on the VM mirror using four coil-magnet actuators. The coils are screwed on a suspended recoil mass (RM) and the magnets are glued on the back of the mirror holder. This setup is identical to the one implemented in the VIRGO interferometer [5]. The cavity, the input beam, the longitudinal control loop scheme, and the acquired signals are indicated in Fig. 2. The laser beam, frequency stabilized

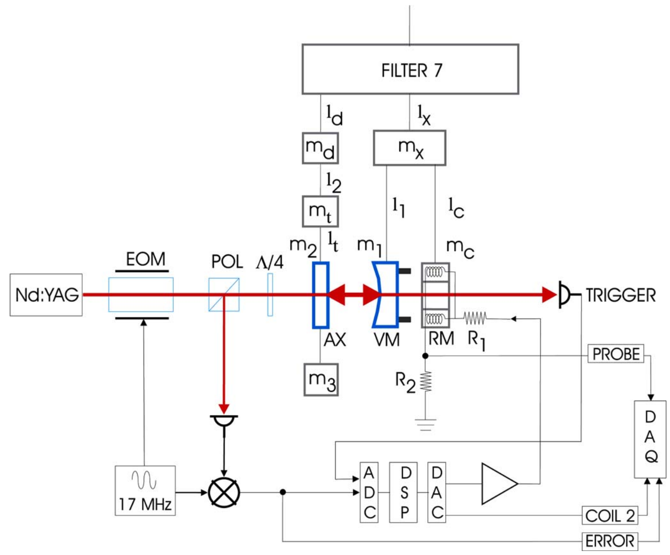

FIG. 2. (Color online) Schematic of the esperimental setup showing the optical layout, the control loop, and the acquisition system (DAQ) as seen from Filter 7. Nd:YAG indicates a neodymium-doped yttrium aluminum garnet laser, EUM an electro-optic modulator, and PUL a polarizer.

by means of a reference cavity and phase modulated at 17 MHz, is injected into the cavity controlled by a Pound-Drever-Hall system [15]. The linear zone of the readout signal corresponds to a mirror travel of $\simeq 10^{-10}$ m [16]. The zero of the control signal does not necessarily correspond to a zero-detuning situation [17]. The control loop is based on a DSP digital signal processor (DSP) electronic board, developed for the VIRGO suspension chain. The error signal, coming from the mixer (see Fig. 2), is amplified and then sent to a 16-bit analog-to-digital converter (ADC). The signal, filtered by the DSP, is sent to four channels of a 20-bit digital-to-analog (DAC) board and then to the four coil drivers to control the cavity. At the same time the error signal is acquired by a DAQ (see Fig. 2) in a LABVIEW environment. Two other signals are usually acquired: the first one, labeled "coil 2," monitors the voltage applied in parallel to the four coils; the second one, labeled "probe," measures the current flowing through one of the coils.

In order to understand the optical spring dynamics, a simple one-dimensional mechanical model, based on the parameters of Table II, has been developed assuming massless wires and pointlike masses. The model considers two branches, each including one cavity mirror, of suspended masses hanging from Filter 7 (see Fig. 2) of the SA and includes the feedback loop. The first one (AX) includes four masses suspended in series, while the second one (VM) is composed of a simple pendulum plus two equal pendula

TABLE I. Optical characteristics of the suspended Fabry–Pérot cavity.

R_1	0.9991	A_1	0.00001
T_1	0.0009	P_{in}	200 mW
R_2	0.9999	A_2	0.00001

TABLE II. Mechanical characteristics of the LFF system with masses and wire lengths defined in Fig. 2.

Mass	kg	Wir	mm
m_d	71.72	l_d	1100
m_t	0.08	l_t	250
m_2	0.296	l_2	500
m_3	2.5	l_3	300
m_x	80	l_x	1130
m_1	27.61	l_1	700
m_c	64.14	l_c	500

hanging from it. The feedback transfer function accounts for the filtering by the DSP card and the coil impedance (coupling constant $\alpha=3$ mN/A); the DSP filter has a pole at 0.5 Hz, one zero at 30 Hz, and two poles at 13 kHz. The noise is assumed to enter through either the suspension or the mirrors.

The first evidence of an optical spring effect emerged from the observation that it was possible to lock the cavity only for positive detunings (with the cavity longer than the closest resonance), corresponding to positive k_{opt}. The control loop did not integrate at low frequency, so that the dc response of the system gave a constant value compensating static forces. A static detuning, different from run to run, ranging between 10^{-11} and 10^{-12} m, was measured. Using these values and the optical parameters of Table I in Eq. (1) gave a stiffness constant k_{opt} ranging between 70 000 and 10 000 N/m.

The measured error signal spectra were characterized by broad peaks due to the optical spring. Figure 1 shows a typical power spectrum with a resonance centered around 80 Hz. From the reported spectrum a seismic noise motion confined below 3 Hz is inferred, with evident antiresonance at 4.1 Hz. While a theoretical calculation is able to predict the resonance frequency and the relative linewidth [2,3], the measured one depends on the control loop parameters. In particular it has been observed that, as expected, the resonance changes its position in accordance with the change of the static detuning. Figure 3 shows the spectra for different static detunings. Scattered points represent the resonance profiles as computed by the above simple model. Assuming a white noise force acting on the mirror AX, the static force and k_{opt} have been fitted by matching the resonance position. The resulting stiffness values are in agreement with the cavity performances. Similar results can be obtained by assuming that the excitation originates from the electronics, while a contribution from the suspension top is incompatible with the measured spectra. The model reproduces as well the antiresonance at 4 Hz. A more detailed discussion on the origin of the noise in the optical spring band goes beyond the scope of the present paper.

The data presented above have been taken with a control loop bandwidth of 150 Hz. In these cases the system behaved as a linear one [18], in the sense that different parts of the frequency spectrum were uncorrelated, and the data followed Gaussian distributions. For larger bandwidths, a significantly large residual motion at low frequency (about 0.37 Hz) has been observed (see bottom curve in Fig. 4) and the data statistics are no longer Gaussian. Consequently, peaks regularly spaced at 0.37 Hz appeared in the noise spectra. The analysis showed that several higher-frequency resonances were correlated with the low-frequency motion. One of the strongest resonances occurred at 12.34 Hz. This peak, due to the high quality factor of the AX suspension, is well reproduced by the mechanical model of the LFF. In Fig. 4 the upper trace shows the 12.34 Hz resonance, bandpass filtered around the resonance with 1 Hz bandwidth, while the lower one shows the oscillation of the cavity length. The consequent modulation of k_{opt} is at the origin of the series of peaks equally spaced at 0.37 Hz shown in Fig. 5.

To evaluate the effects of the time dependence of k_{opt} a simplified model was developed by considering two simple

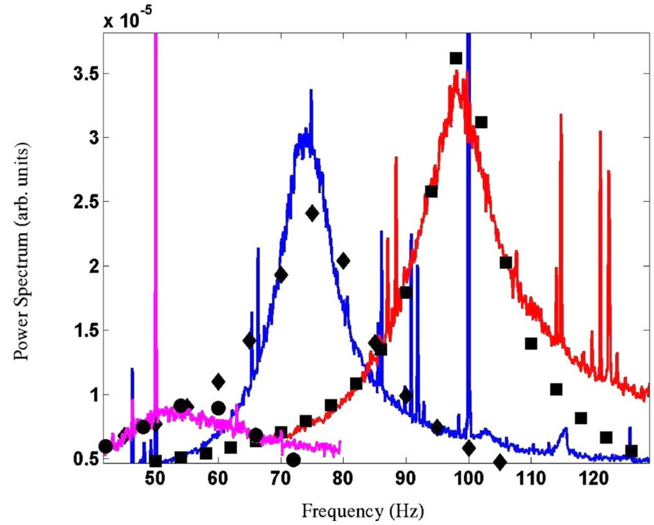

FIG. 3. (Color online) Power spectra of the error signal around optical spring peaks measured during three different runs. Scattered points have been computed by entering the respective static detunings in the mechanical model.

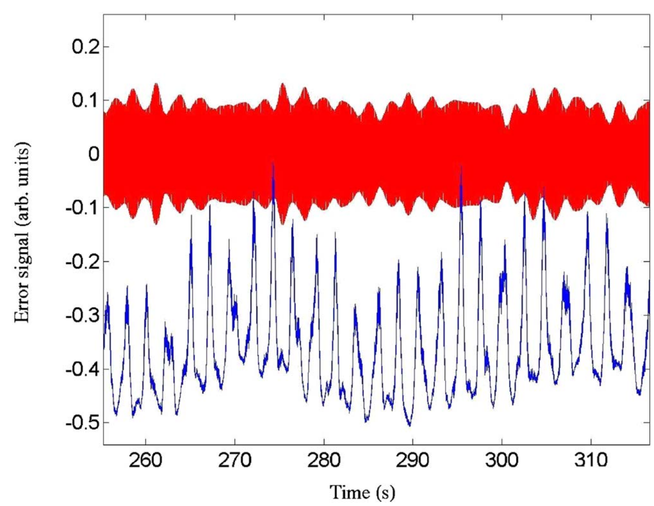

FIG. 4. (Color online) Histogram of the closed-loop cavity detuning, indicating a large swinging at 0.37 Hz (bottom) and the 12.34 Hz mechanical resonance (top).

damped pendula, with test masses m_1 and m_2, coordinates x_1 and x_2, and wire lengths l_1 and l_2. The system obeys the following equations of motion quadratic in $x_1 - x_2$:

$$f_1 = m_1 x_1'' + \gamma x_1' + \frac{g x_1}{l_1} + \left(k_{opt} + \frac{1}{2} \frac{d}{d\delta} k_{opt} \bigg|_{\delta_0} (x_1 - x_2) \right)(x_1 - x_2), \qquad (2)$$

$$f_2 = m_2 x_2'' + \gamma x_2' + g \frac{x_2 + d_{stat}}{l_2} + \left(k_{opt} + \frac{1}{2} \frac{d}{d\delta} k_{opt} \bigg|_{\delta_0} (x_2 - x_1) \right)(x_2 - x_1) + f_{loop}(x_2 - x_1), \qquad (3)$$

with γ (assumed equal to 0.005 N s/m) the damping factor, g the acceleration of gravity, d_{stat} the static deviation of the pendulum from its rest position, and f_{loop} the feedback force as a function of $x_2 - x_1$. The system has been integrated by using the nonlinear differential equations package of MATHEMATICA, introducing two resonances at 0.3 and 12 Hz, respectively, for the AX suspension, and imposing stiffness parameters consistent with the LFF cavity and typical initial conditions of the experiment. The computed second-order term of radiation pressure stiffness $\frac{d}{d\delta}(k_{opt})$ was of the order of 10^{16} N/m^2 in agreement with the experimental findings.

Evidence of an optical rigidity between 25 000 and 65 000 N/m, compatible with theoretical expectations was found by analyzing the LFF data collected with a cavity of finesse $\simeq$6000. Since the resonance was close to the unity-gain point of the feedback loop, the detuning spectrum displayed a clearly visible peak with width a function of the control-loop parameters. Moreover, during some runs with large control-loop bandwidth, an anharmonic behavior of the optical spring was observed, described by a second-order stiffness term $\simeq 10^{16}$ N/m^2, a value compatible with that obtained from a nonlinear model of the suspended cavity [19–22].

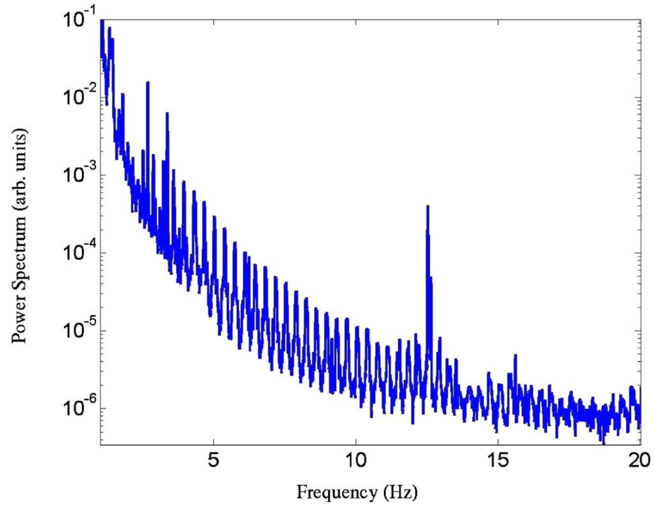

FIG. 5. (Color online) High peaks due to the high nonlinearity of the optical spring evidenced by the swinging at 0.37 Hz.

Thanks are due to R. Castaldi, M. Giorgi, N. Christensen, L. H. Holloway, Wei Tou Ni and his students, L. Cabiddu, D. S. Rabeling, F. Nenci, R. Macchia, F. Paoletti, M. Perciballi, S. Di Franco, R. Cosci, and C. Magazzù.

[1] A. Buonanno and Y. Chen, Class. Quantum Grav. **18**, L95 (2001).
[2] V. B. Braginsky, M. L. Gorodetsky, and F. Ya. Khalili, Phys. Lett. A **232**, 340 (1997).
[3] V. B. Braginsky and F. Ya. Khalili, Phys. Lett. A **257**, 341 (1999).
[4] D. Vitali *et al.*, Phys. Rev. A **65**, 063803 (2002).
[5] Class. Quantum Grav. **21** (2004), special issue.
[6] K. Yamamoto *et al.*, Class. Quantum Grav. **19**, 1689 (2002).
[7] Y. Hadjar *et al.*, Europhys. Lett. **47**, 545 (1999).
[8] V. Leonhardt *et al.*, Class. Quantum Grav. **21**, S1127 (2004).
[9] Bram Slagmolen, Ph.D. thesis, ANU, Canberra, 2005 (unpublished).
[10] A. Di Virgilio *et al.*, in *Proceedings of the Amaldi Conference*, Okinawa, Japan, 2005, special issue of J. Phys. A (to be published).
[11] A. Di Virgilio *et al.*, Phys. Lett. A **322**, 1 (2004).
[12] A. Di Virgilio *et al.*, Phys. Lett. A **316**, 1 (2003).
[13] B. S. Sheard *et al.*, Phys. Rev. A **69**, 051801 (2004).
[14] T. Corbit *et al.*, Report No. LIGO-P050045-00-R (unpublished).
[15] R. W. P. Drever *et al.*, Appl. Phys. B: Photophys. Laser Chem. **31**, 97 (1983).
[16] http://www.virgo.infn.it/
[17] G. Cella *et al.*, following paper, Phys. Rev. A **74**, 013814 (2006).
[18] A. Swami, J. M. Mendel, and C. L. Nikias, *Higher-Order Spectral Analysis Toolbox* (The Mathworks, 1998).
[19] J. Chreighton, e-print gr-qc/9712004; F. Echeverria, Phys. Rev. D **40**, 3194 (1989); C. Miller, Astrophys. J. **581**, 438 (2002).
[20] A. Dorsel *et al.*, Phys. Rev. Lett. **51**, 1550 (1983).
[21] R. S. Tucker *et al.*, IEEE J. Sel. Top. Quantum Electron. **8**, 88 (2002).
[22] A. Samojlenko, Ph.D. thesis, MSU, 2003 (unpublished).