General Einstein-Podolsky-Rosen-type entanglement of continuous variables for bosons

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We show that general Einstein-Podolsky-Rosen-type (EPR-type) entanglement of continuous variables with arbitrary eigenvalues for bosons can be yielded. For bosons of nonzero resting mass EPR-type entangled state can be achieved by the use of atomic beam splitters in particles of a position eigenstate and $n-1$ momentum eigenstates. For light field in which resting mass of the photon is zero, approximate EPR-type entanglement can be experimentally generated when we apply optical beam splitters to one position-squeezed coherence state and $n-1$ momentum-squeezed coherence states, this approximate version tends to perfect EPR entanglement in the limit of infinite squeezing.

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I. INTRODUCTION

Quantum entanglement has been identified as a basic resource in achieving tasks of quantum communication and quantum computation [1]. After the first experiments [2] on quantum teleportation using two-mode squeezed states of light [3,4] as well as those [5–7] dealing with entanglement in atomic ensembles [8,9], a significant amount of work has been devoted to develop a quantum information theory of continuous-variable (CV) systems [10]. Photons are considered to have possess abundant capacity to create various types of entanglement including the discrete, the continuous variable, and the combination of both. Various schemes of producing CV multipartite entanglement using light field are proposed recently [11–18], in van Loock and Braunstein's method in Ref. [11], Einstein-Podolsky-Rosen (EPR)-type CV multipartite entanglement is generated from n squeezed modes of the field and combined by appropriately balanced beam splitters (BSs). However, in their schemes they use vacuum modes (momentum-squeezed vacuum modes and position-squeezed vacuum modes) and generate CV multipartite entangled state which tends toward a Greenberger-Horne-Zeilinger (GHZ) state in the limit of infinite squeezing. This state is an eigenstate with total momentum zero and relative positions $x_i-x_j=0$ $(i,j=1,2,\ldots,n)$. In this paper, we show that general EPR-type CV multipartite entanglement can be yielded by the use of optical BSs and momentum-squeezed arbitrary momentum eigenmodes and position-squeezed arbitrary position eigenmode, this entangled state is an eigenstate with total position (center-of-mass position) χ and relative momenta $P_1-P_i=p_i (i=2,3,\ldots,n)$, the eigenvalues χ and p_i can be arbitrary real numbers. The approximate version of this state is experimentally attainable by the use of momentum-squeezed coherence states and position-squeezed coherence state and balanced optical BSs, it tends toward the general perfect EPR-type entangled state in the limit of infinite squeezing.

On the other hand, atomic BSs, the counterparts of optical BSs, have been experimentally realized. Some of the BSs use the momentum transfer, which occurs during a resonant atom-laser interaction [19–23], deviations of some 10 milliradians angle have been achieved in the experiments. Larger angles have been obtained with many-orders Bragg diffraction or with very slow atoms [24–27]. Magnetic interactions lead also to the realization of large angle BSs—for example, by using a Y-shaped current carrying wire [28] or a concave corrugated magnetic reflector [29]. These methods reach to nonzero magnetic momentum atomic levels. In Ref. [30], a cold-atom BS using a far off-resonant atom-laser interaction is reported, which reaches the millimeter range. Such a BS is applicable to all atomic species and offers broader flexibility. Moreover, BS for guided atoms has also been designed by Cassettari *et al.* [31], which enable robust beam splitting and can be integrated into surface-mounted atom optical devices at the mesoscopic scale. In this paper, we show that EPR-type CV multipartite mass-related entanglement for boson whose resting mass is nonzero can be realized by the use of atomic BSs.

II. EPR-TYPE CV ENTANGLEMENT FOR BOSON OF FINITE MASS

For boson of finite mass (the resting mass of particle is nonzero), let $\hat{x}_{\text{c.m.}}=\Sigma_{i=1}^n\mu_i\hat{x}_i$ be n-partite's center-of-mass (c.m.) coordinates, where $\mu_i=m_i/M$ is each particle's reduced mass and $M=\Sigma_{i=1}^n m_i$ is the total mass of n particles. We see that it is permutable with mass-weighted relative momenta, i.e., $[\hat{p}_1/\mu_1-\hat{p}_j/\mu_j,\hat{x}_{\text{c.m.}}]=0$ $(j=2,3,\ldots,n)$, where $\hat{p}_j$ is the momentum of particle j, so they have common eigenstate. We find that in n-mode Fock space the common eigenvector $|\chi,p_2,\ldots,p_n\rangle$ of operators $\hat{x}_{\text{c.m.}}$ and $\hat{p}_1/\mu_1-\hat{p}_j/\mu_j$ $(j=2,3,\ldots,n)$ with eigenvalues χ, $p_2,\ldots,p_n$ (real numbers) reads

$$|\chi,p_2,\ldots,p_n\rangle = \pi^{-n/4}\sqrt{\frac{\prod_{i=1}^n\mu_i}{\lambda}}\exp\Bigg[-\frac{1}{2\lambda}\Bigg(\chi^2+\sum_{i=2}^n(\lambda-\mu_i^2)\times\mu_i^2p_i^2-2\sum_{i<j;i,j=2}^n\mu_i^2\mu_j^2p_ip_j\Bigg)+\frac{\sqrt{2}\chi}{\lambda}\sum_{i=1}^n\mu_ia_i^\dagger+\frac{i\sqrt{2}}{\lambda}\sum_{j=2}^n p_j\mu_j^2$$

$$\times\left(\sum_{i=1}^{n}\mu_i a_i^\dagger-\frac{\lambda}{\mu_j}a_j^\dagger\right)+\sum_{i=1}^{n}\left(\frac{1}{2}-\frac{\mu_i^2}{\lambda}\right)a_i^{\dagger 2}-\frac{2}{\lambda}\sum_{i<j,i,j=1}^{n}\mu_i\mu_j a_i^\dagger a_j^\dagger\Bigg]|0\cdots 0\rangle, \qquad (1)$$

where $\lambda=\Sigma_{i=1}^{n}\mu_i^2$, $|0\cdots0\rangle$ is n-mode vacuum state. Using the technique of integration within an ordered product (IWOP) of operators [32] and the normal product form of the n-mode vacuum state $|0\cdots0\rangle\langle0\cdots0| =:\exp(-\Sigma_{i=1}^{n}\hat{a}_i^\dagger\hat{a}_i):$, we can prove that the states $|\chi,p_2,\ldots,p_n\rangle$ span a complete and orthogonal set, i.e., $\int\ldots\int_{-\infty}^{\infty}d\chi dp_2\ldots dp_n|\chi,p_2,\ldots,p_n\rangle\langle\chi,p_2,\ldots,p_n|=1$ and $\langle\chi,p_2,\ldots,p_n|\chi',p_2',\ldots,p_n'\rangle=\delta(\chi-\chi')\delta(p_2-p_2')\cdots\delta(p_n-p_n')$.

By making Fourier integration of $|\chi,p_2,\ldots,p_n\rangle$ over $d\chi$ and the comparison with the expression of the momentum eigenstate, we find the Schmidt decomposition of $|\chi,p_2,\ldots,p_n\rangle$ can be expressed as follows:

$$|\chi,p_2,\ldots,p_n\rangle=\sqrt{\frac{\prod_{i=1}^{n}\mu_i}{2\pi}}\exp\left(\frac{i\chi}{\lambda}\sum_{j=2}^{n}\mu_j^2p_j\right)\times\int_{-\infty}^{+\infty}|\mu_1p\rangle_1\otimes|\mu_2(p-p_2)\rangle_2\otimes\cdots\otimes|\mu_n(p-p_n)\rangle_n e^{-i\chi p}dp, \qquad (2)$$

where $|\mu_1 p\rangle_1$, $|\mu_i(p-p_i)\rangle_i$ $(i=2,3,\ldots,n)$ are eigenstates of momentum operators with eigenvalue $\mu_1p,\mu_i(p-p_i)$. Obviously, this is an EPR-type maximal entangled state of CVs.

Now we show that this state can be achieved for boson by the use of atomic BSs. Applying the atomic BS operations $\hat{B}_{n-1,n}(\theta_{n-1})\times\cdots\times\hat{B}_{2,3}(\theta_2)\hat{B}_{1,2}(\theta_1)$ to a position eigenstate $|x\rangle_1$ in particle 1 and $n-1$ momentum eigenstates $|y_j\rangle_j$ $(j=2,3,\ldots,n)$, in particles 2 through n yields the entangled state

$$\begin{aligned}|\chi,p_2,\ldots,p_n\rangle &=\hat{B}_{n-1,n}(\theta_{n-1})\times\cdots\times\hat{B}_{2,3}(\theta_2)\hat{B}_{1,2}(\theta_1)|x\rangle_1|y_2\rangle_2\cdots|y_n\rangle_n\\ &=\sqrt{\frac{\prod_{i=1}^{n}\mu_i}{2\pi}}\exp\left(\frac{i\chi}{\lambda}\sum_{j=2}^{n}\mu_j^2p_j\right)\\ &\quad\times\int_{-\infty}^{+\infty}|\mu_1p\rangle_1\otimes|\mu_2(p-p_2)\rangle_2\\ &\quad\otimes\cdots\otimes|\mu_n(p-p_n)\rangle_n e^{-i\chi p}dp,\end{aligned} \qquad (3)$$

where $\theta_j=\sin^{-1}\frac{\mu_j}{\sqrt{\Sigma_{i=j}^{n}\mu_i^2}}$ $(j=1,2,\ldots,n-1)$, the operator $\hat{B}_{j,k}(\theta_j)$ which describes the operations of atomic BS on particles j and k reads $\hat{B}_{j,k}(\theta_j)=\exp[\frac{\theta_j}{2}(\hat{a}_j^\dagger\hat{a}_ke^{i\phi}-\hat{a}_j\hat{a}_k^\dagger e^{-i\phi})]$. The relations between parameters $\chi,p_2,\ldots,p_n$ and $x,y_2,\ldots,y_n$ obey identities

$$x\sin\theta_1+y_2 i\cos\theta_1=\frac{\mu_1}{\lambda}\left(\chi+i\sum_{j=2}^{n}p_j\mu_j^2\right),$$

$$\sin\theta_k\left[x\prod_{j=1}^{k-1}\cos\theta_j-i\sum_{j=2}^{k}\left(y_j\sin\theta_{j-1}\prod_{m=j,m\le k-1}^{k-1}\cos\theta_m\right)\right]+iy_{k+1}\cos\theta_k=\frac{\mu_k}{\lambda}\left[\chi+i\left(\sum_{j=2}^{n}p_j\mu_j^2-p_k\lambda\right)\right],$$

$$k=2,3,\ldots,n-1,$$

$$x\prod_{j=1}^{n-1}\cos\theta_j-i\sum_{j=2}^{n-1}\left(y_j\sin\theta_{j-1}\prod_{m=j}^{n-1}\cos\theta_m\right)-iy_n\sin\theta_{n-1}=\frac{\mu_n}{\lambda}\left[\chi+i\left(\sum_{j=2}^{n}p_j\mu_j^2-p_n\lambda\right)\right]. \qquad (4)$$

Thus we obtain the EPR-type CV entanglement for boson of nonzero mass by the use of atomic BSs, in the following, we show that EPR-type CV entanglement for light can also be obtained by using squeezed coherence states and balanced optical BSs.

III. EPR-TYPE CV ENTANGLEMENT FOR LIGHT FIELD

For light field, the resting mass of photon is zero. A single frequency mode of the electric field (for a single polarization) reads

$$\hat{E}_k(\vec{r},t)=E_0(\hat{a}_ke^{i(\vec{k}\cdot\vec{r}-\omega_kt)}+\hat{a}_k^\dagger e^{-i(\vec{k}\cdot\vec{r}-\omega_kt)}), \qquad (5)$$

where E_0 is a constant which contains all the dimensional prefactors. We can rewrite the mode as

$$\hat{E}_k(\vec{r},t)=\sqrt{2}E_0[\hat{x}_k\cos(\omega_kt-\vec{k}\cdot\vec{r})+\hat{p}_k\sin(\omega_kt-\vec{k}\cdot\vec{r})]. \qquad (6)$$

where the position and momentum operators $\hat{x}_k=(1/\sqrt{2})(\hat{a}_k+\hat{a}_k^\dagger)$, $\hat{p}_k=(1/\sqrt{2}i)(\hat{a}_k-\hat{a}_k^\dagger)$ are a conjugate pair of quadratures which represent the in-phase and out-of-phase components of the electric-field amplitude of the single mode k, respectively. We note that, for n-mode system, the operators $\hat{x}\equiv\Sigma_{i=1}^{n}\hat{x}_i$ and $\hat{p}_1-\hat{p}_j$ $(j=2,3,\ldots,n)$ are permutable with each other, the common eigenvector $|\chi p_2p_3\ldots p_n\rangle$ of them with eigenvalues $\chi,p_2,\ldots,p_n$ (real numbers) reads

$$\begin{aligned}|\chi p_2p_3\cdots p_n\rangle=\frac{1}{\sqrt{n}\pi^{n/4}}\exp\Bigg[&-\frac{1}{2n}(n-1)\sum_{j=2}^{n}p_j^2-\frac{1}{2n}\chi^2\\ &+\frac{1}{n}\sum_{j<k,j,k=2}^{n}p_jp_k+\frac{\sqrt{2}\chi}{n}\sum_{i=1}^{n}\hat{a}_i^\dagger+\frac{i\sqrt{2}}{n}\\ &\times\sum_{j=2}^{n}p_j\left(\sum_{i=1}^{n}\hat{a}_i^\dagger-n\hat{a}_j^\dagger\right)\\ &-\frac{2}{n}\sum_{j<k=1}^{n}\hat{a}_j^\dagger\hat{a}_k^\dagger-\left(\frac{1}{n}-\frac{1}{2}\right)\sum_{j=1}^{n}\hat{a}_j^{\dagger 2}\Bigg]|00\cdots0\rangle.\end{aligned} \qquad (7)$$

It also makes up a complete and orthogonal set. The Schmidt decomposition of $|\chi p_2 p_3 \ldots p_n\rangle$ can be expressed as follows:

$$|\chi p_2 p_3 \cdots p_n\rangle = \frac{1}{\sqrt{2\pi}} \exp\left(\frac{i}{n}\chi\sum_{j=2}^{n} p_j\right)\int_{-\infty}^{+\infty} |p\rangle_1 \otimes |p-p_2\rangle_2 \otimes \cdots \otimes |p-p_n\rangle_n e^{-i\chi p} dp, \tag{8}$$

where $|p\rangle_1$, $|p-p_i\rangle_i$ $(i=2,3,\ldots,n)$ are eigenstates of momentum operators with eigenvalues $p, p-p_i$.

From (8) we see that the state is an EPR-type CV maximal entangled state for light field.

We now investigate if this state can be realistically achieved experimentally. Applying the optical BS operations $\hat{B}_{n-1,n}(\theta_{n-1})\times\cdots\times\hat{B}_{2,3}(\theta_2)\hat{B}_{1,2}(\theta_1)$ to a position eigenstate in mode 1 and $n-1$ momentum eigenstates in modes 2 through n, i.e., applying them to $|x\rangle_1|y_2\rangle_2\cdots|y_n\rangle_n$, the outgoing state reads

$$\begin{aligned}|\chi p_2 \ldots p_n\rangle &= \hat{B}_{n-1,n}(\theta_{n-1}) \times \cdots \times \hat{B}_{2,3}(\theta_2)\hat{B}_{1,2}(\theta_1) \times |x\rangle_1|y_2\rangle_2\cdots|y_n\rangle_n \\ &= \frac{1}{\sqrt{2\pi}} \exp\left(\frac{i}{n}\chi\sum_{j=2}^{n} p_j\right)\int_{-\infty}^{+\infty} |p\rangle_1 \otimes |p-p_2\rangle_2 \otimes \cdots \otimes |p-p_n\rangle_n e^{-i\chi p} dp,\end{aligned} \tag{9}$$

where $\theta_j=\sin^{-1}\frac{1}{\sqrt{n-j+1}}$, BS operator $\hat{B}_{j,k}(\theta_j)=\exp\left[\frac{\theta_j}{2}(\hat{a}_j^\dagger\hat{a}_k e^{i\phi}-\hat{a}_j\hat{a}_k^\dagger e^{-i\phi})\right]$, the relations between parameters $\chi, p_2,\ldots,p_n$ and $x, y_2,\ldots,y_n$ obey the identities

$$x\sin\theta_1 + y_2 i\cos\theta_1 = \frac{1}{n}\left(\chi + i\sum_{j=2}^{n} p_j\right),$$

$$\sin\theta_k\left[x\prod_{j=1}^{k-1}\cos\theta_j - i\sum_{j=2}^{k}\left(y_j\sin\theta_{j-1}\prod_{m=j,m\le k-1}^{k-1}\cos\theta_m\right)\right] + iy_{k+1}\cos\theta_k = \frac{1}{n}\left[\chi + i\left(\sum_{j=2}^{n} p_j - np_k\right)\right],$$

$$k=2,3,\ldots,n-1,$$

$$x\prod_{j=1}^{n-1}\cos\theta_j - i\sum_{j=2}^{n-1}\left(y_j\sin\theta_{j-1}\prod_{m=j}^{n-1}\cos\theta_m\right) - iy_n\sin\theta_{n-1} = \frac{1}{n}\left[\chi + i\left(\sum_{j=2}^{n} p_j - np_n\right)\right]. \tag{10}$$

Thus the entangled state $|\chi p_2\cdots p_n\rangle$ is generated from n eigenmodes of positions and momentum and balanced optical BSs. Unfortunately, it is an unphysical and unnormalizable state for light field, since the momentum wave function for the state $|\chi p_2\cdots p_n\rangle$ is

$$\begin{aligned}\psi(p_1',p_2',\ldots,p_n') &= \langle p_n'|\cdots\langle p_2'|\langle p_1'|\chi p_2\cdots p_n\rangle \\ &= \frac{1}{\sqrt{2\pi}}\exp\left(-\frac{i}{n}\chi\sum_{j=1}^{n} p_j'\right)\delta(p_1'-p_2'-p_2) \times\delta(p_1'-p_3'-p_3)\cdots\delta(p_1'-p_n'-p_n),\end{aligned} \tag{11}$$

(where $\langle p_j'|$ is momentum eigenstate of mode j) which is δ function and is not square integrable. However, the approximate version of this state is achievable experimentally. Noting that the position-squeezed coherence state reads [33]

$$\begin{aligned}|\alpha,r\rangle &= D(\alpha)S(r)|0\rangle \\ &= \mathrm{sech}^{1/2}\, r\exp\left(-\frac{|\alpha|^2}{2} - \frac{\alpha^{*2}}{2}\tanh\lambda + (\alpha^*\tanh r+\alpha)a^\dagger - \frac{a^{\dagger 2}}{2}\tanh r\right)|0\rangle,\end{aligned} \tag{12}$$

where $D(\alpha)$ is the displacement operator and $S(r)=\exp\left[\frac{r}{2}(a^2-a^{\dagger 2})\right]$ is the position squeeze operator with squeeze parameter r. Let $\alpha=x/\sqrt{2}$, then

$$\begin{aligned}|x,r\rangle &= D(x/\sqrt{2})S(r)|0\rangle \\ &= \mathrm{sech}^{1/2}\, r\exp\left(-(\tanh r+1)\frac{x^2}{4} + \frac{x}{\sqrt{2}}(\tanh r+1)a^\dagger - \frac{a^{\dagger 2}}{2}\tanh r\right)|0\rangle.\end{aligned} \tag{13}$$

For infinite squeezing $r\to\infty$, $\tanh r\to 1$, then

$$\exp\left(-(\tanh r+1)\frac{x^2}{4} + \frac{x}{\sqrt{2}}(\tanh r+1)a^\dagger - \frac{a^{\dagger 2}}{2}\tanh r\right)|0\rangle \to \exp\left(-\frac{x^2}{2} + \sqrt{2}xa^\dagger - \frac{a^{\dagger 2}}{2}\right)|0\rangle$$

which is just the position eigenstate with the eigenvalue x. Similarly, the momentum-squeezed coherence state is $D(\beta)S'(r)|0\rangle$, here $D(\beta)$ is the displacement operator and $S'(r)=\exp\left[-\frac{r}{2}(a^2-a^{\dagger 2})\right]$ is the momentum squeeze operator. Let $\beta=ip/\sqrt{2}$, we have

$$\begin{aligned}\|p,r\rangle &= D(ip/\sqrt{2})S'(r)|0\rangle \\ &= \mathrm{sech}^{1/2}\, r\exp\left(-(\tanh r+1)\frac{p^2}{4} + \frac{ip}{\sqrt{2}}(\tanh r+1)a^\dagger + \frac{a^{\dagger 2}}{2}\tanh r\right)|0\rangle.\end{aligned} \tag{14}$$

For infinite squeezing $r\to\infty$, $\tanh r\to 1$, then

$$\exp\left(-(\tanh r+1)\frac{p^2}{4} + \frac{ip}{\sqrt{2}}(\tanh r+1)a^\dagger + \frac{a^{\dagger 2}}{2}\tanh r\right)|0\rangle \to \exp\left(-\frac{p^2}{2} + i\sqrt{2}pa^\dagger + \frac{a^{\dagger 2}}{2}\right)|0\rangle$$

which is just the momentum eigenstate with the eigenvalue p. Thus, applying the above "n beam splitter" to one position-squeezed coherence state with squeezing parameter r_1 and $n-1$ momentum-squeezed coherence states with squeezing parameter r_2 $[e^{r_1}\approx(n-1)e^{r_2}]$ yields the approximate version of the state $|\chi p_2\cdots p_n\rangle$, this approximate state is experimentally feasible with current technology. Perfect EPR-type entangled state is achieved for infinite squeezing in the position-squeezed mode $r_1\to\infty$ and $n-1$ momentum-squeezed modes $r_2\to\infty$.

As an example, we show an application of the above entangled state in quantum teleportation. We now try to figure out a protocal so that an unknown quantum state can be teleported to the receiver. We take the approximate two-mode EPR entangled state $|\chi p_2\rangle_{12}=\hat{B}_{1,2}(\frac{\pi}{4})|x,r\rangle_1\otimes\|p,r\rangle_2$ ($\chi=\sqrt{2}x$, $p_2=\sqrt{2}p$) as quantum channel. Let Alice and Bob share modes 1 and 2 of $|\chi p_2\rangle_{12}$, respectively. Supposing Alice wants to teleport state $|\psi\rangle_3$ to Bob, then the total initial state is $|\chi p_2\rangle_{12}\otimes|\psi\rangle_3$. Alice makes a measurement with the projection basis being $|\chi' p_2'\rangle_{13}\,{}_{13}\langle\chi' p_2'|$, the projection yields ${}_{13}\langle\chi' p_2'|\chi p_2\rangle_{12}\otimes|\psi\rangle_3$. Thus the state of mode 2 becomes $\zeta\hat{U}|\psi\rangle_2$ in the limit of infinite squeezing, where ζ is a complex number, unitary operator $\hat{U}=e^{-i\hat{P}_2(\chi+\chi')}e^{i\hat{X}_2(p_2-p_2')}$. Alice then informs Bob of the measurement outcomes via the classical channel, after receiving the classical information, Bob performs the unitary transformation $\hat{U}^{-1}$. In this way Bob has his mode in the state as the state to be teleported with the fidelity $F=1$. For finite squeezing the state $|\psi\rangle_3$ cannot be teleported faithfully, the fidelity of the teleportation is similar to the results in Ref. [11].

IV. CONCLUDING REMARK

We have considered EPR-type entangled states of CV for boson, these states whose eigenvalues are arbitrary are different from the multipartite EPR entangled states introduced in Ref. [11], the latter whose eigenvalues are zero is GHZ state and is obtained by applying BSs to the zero-momentum and zero-position eigenstates, its physical approximate version is yielded from the squeezed vacuum modes. We show that general EPR-type entanglement of CVs with arbitrary eigenvalues for boson can be yielded by the use of BSs. For nonzero resting mass boson, EPR-type entangled state can be achieved by using atomic BSs in particles of one position eigenstate and $n-1$ momentum eigenstates. For light field, perfect EPR-type entangled state which is the common eigenstate of total position $\hat{x}\equiv\sum_{i=1}^{n}\hat{x}_i$ and relative momenta $\hat{p}_1-\hat{p}_j$, $(j=2,3,\ldots,n)$ can be yielded by applying optical BSs to eigenstates of position and momenta operators, however, it is unphysical. Fortunately the approximate version of this state can be experimentally generated when we apply optical BSs to one position-squeezed coherence state and $n-1$ momentum-squeezed coherence states, it tends to perfect EPR entanglement for infinite squeezing.

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