Comment on "Interference-induced gain in the Autler-Townes doublet of a V-type atom in a cavity" and "Cavity implementation of quantum interference in a Λ-type atom"

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In a couple of papers [Phys. Rev. A, **63**, 033818 (2001); **63**, 023810 (2001)], P. Zhou, S. Swain, and L. You studied the cavity-induced modifications to the Autler-Townes spectrum of three-level atoms due to the phenomenon of quantum interference. They demonstrated that probe gain can occur in either of the Autler-Townes doublet due to the cavity-induced interference among spontaneous decay channels. In this Comment, we show that this conclusion is incorrect and it contradicts the thermodynamic equilibrium conditions of the atomic system in steady state.

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When two closely lying energy levels of an atom are coupled by the same vacuum modes to other levels, the quantum interference takes place in the spontaneous emission of atoms. For three-level atoms, this can lead to many remarkable features such as population trapping, dark lines in emission spectrum, fluorescence quenching, etc. However, all these effects assume an existence of nonorthogonal dipole moments of the atomic transitions for the interference to occur. Since this condition is rarely met in many atomic systems, various alternative methods were later proposed to bypass the condition of nonorthogonal dipole moments.

In Refs. [1,2], Zhou, Swain, and You considered the three-level atoms in the V-type and Λ-type schemes to be interacting with a preselected polarization mode of a cavity. In this case, it can be shown that the effect of interference is not sensitive to the orientations of atomic dipoles. Zhou and coworkers further predicted interference-assisted population trapping, population inversion and probe gain in the Autler-Townes spectrum. However, our analysis of these works refutes their claim by showing that their numerical results are not consistent with analytical results of the atomic density matrix in steady state. To be specific, we give, in this comment, the details of our analysis for the case of three-level atom in the V-type configuration studied in [1].

I. MODEL SYSTEM AND BASIC EQUATIONS

We first review the model system and basic density matrix equations considered in Ref. [1]. A V-type three-level atom with ground state $|0\rangle$ and excited doublet $|1\rangle$, $|2\rangle$ interacts with a single mode cavity field as shown in Fig. 1. Let ω_{10}, ω_{20} denote the energy separations between the excited and ground levels of the atom. In a frame rotating with the average atomic transition-frequency $\omega_o=(\omega_{10}+\omega_{20})/2$, the total Hamiltonian of the atom-cavity interaction is given by

$$H=\frac{1}{2}\omega_{21}(A_{22}-A_{11})+\delta a^{\dagger}a+[i(g_1A_{01}+g_2A_{02})a^{\dagger}+\text{H.c.}]. \tag{1}$$

Here, a and $a^{\dagger}$ are the photon annihilation and creation operators of the cavity mode, ω_{21} is the energy separation between the excited sublevels, g_i is the atom-cavity coupling constant, δ is the detuning of the cavity frequency from ω_o, and $A_{ij}=|i\rangle\langle j|$ is the atomic population (the dipole transition) operator for $i=j(i\neq j)$. In addition to the atom-cavity interaction, the cavity field is damped by a finite temperature reservoir, characterized by the decay constant κ and the mean number of thermal photons N.

In the bad cavity limit $\kappa\gg g_i$, the density matrix elements of the atom obey

$$\dot{\rho}_{11}=-[F(\omega_{21})+F^{*}(\omega_{21})]|g_1|^2[(N+1)\rho_{11}-N\rho_{00}] - F(-\omega_{21})g_1^{*}g_2(N+1)\rho_{21}-F^{*}(-\omega_{21})g_1g_2^{*}(N+1)\rho_{12},$$

$$\dot{\rho}_{22}=-[F(-\omega_{21})+F^{*}(-\omega_{21})]|g_2|^2[(N+1)\rho_{22}-N\rho_{00}] - F^{*}(\omega_{21})g_1^{*}g_2(N+1)\rho_{21}-F(\omega_{21})g_1g_2^{*}(N+1)\rho_{12},$$

$$\dot{\rho}_{21}=-F(\omega_{21})g_1g_2^{*}(N+1)\rho_{11}-F^{*}(-\omega_{21})g_1g_2^{*}(N+1)\rho_{22} +[F(\omega_{21})+F^{*}(-\omega_{21})]g_1g_2^{*}N\rho_{00}-[F^{*}(\omega_{21})|g_1|^2(N+1) +F(-\omega_{21})|g_2|^2(N+1)+i\omega_{21}]\rho_{21},$$

$$\dot{\rho}_{10}=-\left[F(\omega_{21})|g_1|^2(2N+1)+F(-\omega_{21})|g_2|^2N-i\frac{\omega_{21}}{2}\right] \times\rho_{10}-F(-\omega_{21})g_1^{*}g_2(N+1)\rho_{20},$$

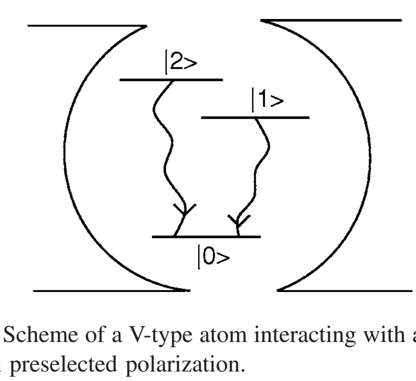

FIG. 1. Scheme of a V-type atom interacting with a single-mode cavity with preselected polarization.

$$\dot{\rho}_{20} = -\left[F(\omega_{21})|g_1|^2 N + F(-\omega_{21})|g_2|^2(2N+1) + i\frac{\omega_{21}}{2}\right] \times \rho_{20} - F(\omega_{21}) g_1 g_2^*(N+1)\rho_{10}, \quad (2)$$

where $F(\pm\omega_{21}) = [\kappa + i(\delta \pm \omega_{21}/2)]^{-1}$.

As mentioned in [1], we insert a factor $\eta(=0,1)$ in the cross-transition terms $g_i g_j^*$ to monitor the effects of quantum interference. When $\eta=0$, the cross-transition terms are cut, so quantum interference is absent. Otherwise, the interference effects are maximal.

II. STEADY STATE RESULTS

When the quantum interference $(\eta=1)$ is present, the V system does not behave like two independent sets of two-level systems. In this case, we obtain the steady state solutions of the density matrix elements by setting $\dot{\rho}_{ij}=0$ in the Eqs. (2) with the inclusion of cross-transition terms. The equations for ρ_{10}, ρ_{20} yield the trivial solutions $\rho_{10}^S = \rho_{20}^S = 0$, as they are completely decoupled from the rest of density matrix elements. The equations for ρ_{11}, ρ_{22}, and ρ_{21} can be coupled to give the steady state solutions as

$$\rho_{11}^S = \rho_{22}^S = \frac{N}{3N+1}, \quad \rho_{21}^S = 0. \quad (3)$$

Note that we obtain the same results (3) in steady state, which are independent of the cavity frequency (or the detuning δ), even in the absence $(\eta=0)$ of interference. It means that the transient response of the atom may be sensitive to the presence or absence of cross-transition terms in the master equation. But the atomic system in a cavity will reach the same steady state, obeying the thermodynamic equilibrium, irrespective of the quantum interference in decay processes. This condition is exactly identical to that reported by Agarwal and Menon [3] for V-type atoms in free space.

Surprisingly, the results of Zhou, Swain, and You [1] contradict the above conclusion. Their analysis showed the dependence of the coherence ρ_{21}^S and populations ρ_{11}^S, ρ_{22}^S on the cavity frequency due to the cavity-induced quantum interference [see Ref. [1], Fig. 4]. As a result, it was discussed that the Autler-Townes spectrum exhibits gain features due to population inversion and nonzero coherences. We believe the discrepancy with our results (3) as due to errors in numerical computation in [1].

III. CONCLUSIONS

We have shown that the decay-induced interference cannot lead to population inversion, nonzero coherence and hence probe gain in the Autler-Townes spectrum of a three-level atom inside a cavity. Though we have proved this for the specific case of V-type atoms considered in [1], our arguments should apply equally well to the Λ-type system studied in Ref. [2]. Therefore, all the numerical results reported in Refs. [1,2] are erroneous.

[1] P. Zhou, S. Swain, and L. You, Phys. Rev. A **63**, 033818 (2001).
[2] P. Zhou, Phys. Rev. A **63**, 023810 (2001).
[3] G. S. Agarwal and S. Menon, Phys. Rev. A **63**, 023818 (2001).