Proposal for a probabilistic "dial-up" generator of Fock states in a traveling-wave optical field

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We discuss a method for generating Fock (number) states in a single-mode traveling-wave optical field, based on a method we recently proposed for performing a quantum nondemolition measurement of parity and for the generation of parity eigenstates [C. C. Gerry, A. Benmoussa, and R. A. Campos, Phys. Rev. A **72**, 053818 (2005)]. The approach is a kind of "dial-up" scheme that is probabilistic but also depends partially on directed state reductive measurements.

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An important goal of quantum optics is the generation of Fock states, or number states, $|n\rangle$ for arbitrary n, for a quantized electromagnetic field. The existence of such states is fundamental to the most accurate physical theory we have, quantum electrodynamics. But they may also be of practical importance in the field of quantum information science, including quantum metrology. For example, maximally entangled number states of the form $|n\rangle|0\rangle+|0\rangle|n\rangle$ [1] and twin Fock states $|n\rangle|n\rangle$ [2] have been shown to be of utility in performing interferometric phase shift measurements of Heisenberg-limited sensitivity, $\Delta\varphi_{\mathrm{HL}}\sim 1/n$, an improvement over the standard quantum limit $\Delta\varphi_{\mathrm{SQL}}\sim 1/\sqrt{n}$. There has been much theoretical and experimental work done towards the generation of photon number states in the context of cavity QED [3]. There has also been discussion of techniques for generating these states in optical traveling-wave fields [4] though experiments lag behind the theory. Recently, the present authors along with R. A. Campos [5], proposed a method for the quantum nondemolition measurement of parity and for the production of parity eigenstates, including higher-order parity eigenstates [6], where we indicated how, under certain conditions, photon number states could be generated. In the present paper, we systematically flesh out the details of our ideas on the generation of number states. In a sense to be discussed below, our proposed scheme has the characteristics of a probabilistic "dial-up" number state generator.

A sketch of our proposed generation scheme is given in Fig. 1. Mode c contains a coherent state $|\beta\rangle_c$ out of which we wish to extract a number state. In terms of the number states

$$|\beta\rangle_c = e^{-|\beta|^2/2}\sum_{n=0}^{\infty}\frac{\beta^n}{\sqrt{n!}}|n\rangle_c. \tag{1}$$

The cross-Kerr medium couples the c mode to the a_1 mode through the nonlinear interaction

$$\hat{H}_K = \hbar\,\chi\hat{a}_1^{\dagger}a_1\hat{c}^{\dagger}\hat{c}, \tag{2}$$

where χ is proportional to the third-order nonlinear susceptibility $\chi^{(3)}$. Mode a_1 is an internal clockwise mode of a Mach-Zehnder interferometer (MZI), the counterclockwise mode being labeled a_2. The beam splitters BS1 and BS2 are 50:50. The output modes of the BS2 are labeled b_1 and b_2. An incident coherent state $|\sqrt{2}\alpha\rangle$, with only the vacuum at the other input port, at BS1, as indicated in Fig. 1, is split into two coherent states, i.e., $|\sqrt{2}\alpha\rangle|0\rangle\rightarrow|\alpha\rangle_{a_1}|\alpha\rangle_{a_2}$. We assume that appropriate phase shifting elements can be inserted, if necessary, so that the output coherent states will have the required phases. For example, if BS1 causes the reflected beam to pick up a phase shift of $\pi/2$, a compensating $-\pi/2$ phase shifter can be inserted into that beam. The phase shift denoted θ in the counterclockwise beam takes into account any other required phase shifts for the coherent state entering BS2 in order to obtain certain outputs, as will be explained below. This phase shift is the "dial" in our scheme.

We assume then that the state entering the Kerr medium is

$$|\Phi_{\mathrm{in}}\rangle = |\beta\rangle_c|\alpha\rangle_{a_1} = e^{-|\beta|^2/2}\sum_{n=0}^{\infty}\frac{\beta^n}{\sqrt{n!}}|n\rangle_c|\alpha\rangle_{a_1}, \tag{3}$$

where

$$|\alpha\rangle_{a_1} = \exp(-|\alpha|^2/2)\sum_{m=0}^{\infty}\frac{\alpha^m}{\sqrt{m!}}|m\rangle_{a_1}. \tag{4}$$

The output state of the Kerr medium is given by

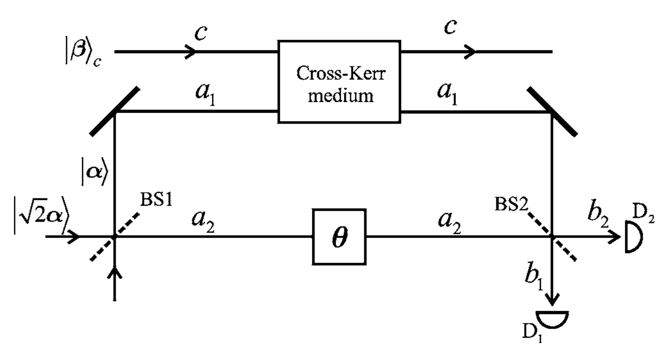

FIG. 1. Schematic of the proposed number state generation scheme. An input coherent state of small amplitude β is in the c mode and one of larger amplitude $\sqrt{2}\alpha$ is injected into the one port of the first beam splitter. The phase shift θ is adjusted to one of the values $\theta=\pi-\varphi_{L'}$ where $\varphi_{L'}=\pi L'/2^N$, $L'=0,1,\ldots 2^{N+1}-1$.

$$|\Phi_{\text{out}}\rangle = \exp(-i\hat{H}_{\text{K}}t/\hbar)|\Phi_{\text{in}}\rangle = e^{-|\beta|^2/2}\sum_{n=0}^{\infty}\frac{\beta^n}{\sqrt{n!}}|n\rangle_c|\alpha e^{-i\chi nt}\rangle_{a1}. \tag{5}$$

At this point we take $\chi t=\pi/2^N$ where N can take the values $N=0,1,2\ldots$. We now write the output state as

$$|\Phi_N\rangle = e^{-|\beta|^2/2}\sum_{n=0}^{\infty}\frac{\beta^n}{\sqrt{n!}}|n\rangle_c|\alpha e^{-i\pi n/2^N}\rangle_{a1} = \sum_{L=0}^{\lambda_N}|\psi_L\rangle_c|\alpha_L\rangle_{a1}, \tag{6}$$

where $\lambda_N=2^{N+1}-1$, $\alpha_L=\alpha e^{-i\varphi_L}$, $\varphi_L=\pi L/2^N$, and

$$|\psi_L\rangle_c = \sum_{M=0}^{\infty} C_{L+2^{N+1}M}|L+2^{N+1}M\rangle_c, \tag{7}$$

and where

$$C_{L+2^{N+1}M} = e^{-|\beta|^2/2}\frac{\beta^{L+2^{N+1}M}}{\sqrt{(L+2^{N+1}M)!}}. \tag{8}$$

For reasons that will be made clear shortly, we take the phase shift in the a_2 beam to be $\theta=\pi-\varphi_{L'}$, where $L'=0,1,\ldots,2^{N+1}-1$, so that $|\alpha\rangle_{a_2}\to|-\alpha e^{-i\varphi_{L'}}\rangle_{a_2}=|-\alpha_{L'}\rangle_{a_2}$. Then the input to the second beam splitter is

$$|\Phi_{\text{BS2}}^{L'}\rangle_{\text{in}} = \sum_{L=0}^{\lambda_N}|\psi_L\rangle_c|\alpha_L\rangle_{a_1}|-\alpha_{L'}\rangle_{a_2} \tag{9}$$

and its output state is

$$|\Phi_{\text{BS2}}^{L'}\rangle_{\text{out}} = \sum_{L=0}^{\lambda_N}|\psi_L\rangle_c|\alpha_{LL'}^{(-)}\rangle_{b_1}|\alpha_{LL'}^{+}\rangle_{b_2}, \tag{10}$$

where

$$\alpha_{LL'}^{(-)} = \frac{\alpha_L-\alpha_{L'}}{\sqrt{2}}, \quad \alpha_{LL'}^{+} = \frac{\alpha_L+\alpha_{L'}}{\sqrt{2}}. \tag{11}$$

We may rewrite this as

$$|\Phi_{\text{BS2}}^{L'}\rangle_{\text{out}} = |\psi_{L'}\rangle_c|0\rangle_{b_1}|\sqrt{2}\alpha_{L'}\rangle_{b_2} + \sum_{\substack{L=0\\L\neq L'}}^{\lambda_N}|\psi_L\rangle_c|\alpha_{LL'}^{(-)}\rangle_{b_1}|\alpha_{LL'}^{(+)}\rangle_{b_2}. \tag{12}$$

Now if detector D_1 detects no photons and D_2 any number of photons (assuming the ideal case with detectors of 100% efficiency) then the c mode is projected into the state

$$|\Phi_{L'}\rangle_c = \mathcal{N}_{L'}\sum_{M=0}^{\infty}C_{L'+2^{N+1}M}|L'+2^{N+1}M\rangle_c, \tag{13}$$

where the normalization factor is given by

$$\mathcal{N}_{L'} = \left[\sum_{M=0}^{\infty}|C_{L'+2^{N+1}M}|^2\right]^{-1/2}. \tag{14}$$

From the second term in Eq. (12) it is apparent that there is nonzero probability that some number of photons will be detected by *both* detectors. These outcomes should be discarded. We shall consider the effects of realistic, inefficient, detectors on the required projection shortly.

Before proceeding, it is perhaps worth writing out explicitly in terms of superpositions of coherent states a couple of example of the states we obtain from the ideal case of the above procedure. In the case where $N=0$ we obtain, apart from normalization factors,

$$|\Phi_0\rangle_c \sim |\beta\rangle_c + |-\beta\rangle_c,$$

$$|\Phi_1\rangle_c \sim |\beta\rangle_c - |-\beta\rangle_c, \tag{15}$$

which are the familiar even and odd quantum superposition (Schrödinger cat) states [7]. In the limit of $|\beta|$ small, $|\Phi_0\rangle_c$ goes over to the vacuum state $|0\rangle_c$ and $|\Phi_1\rangle_c$ goes over to the one photon Fock state, $|1\rangle_c$. For $N=2$ we similarly obtain the possible set of states

$$|\psi_0\rangle \sim |\beta\rangle_c + |-\beta\rangle_c + (|i\beta\rangle_c + |-i\beta\rangle_c),$$

$$|\psi_1\rangle \sim |\beta\rangle_c - |-\beta\rangle_c - i(|i\beta\rangle_c + |-i\beta\rangle_c),$$

$$|\psi_2\rangle \sim |\beta\rangle_c + |-\beta\rangle_c - (|i\beta\rangle_c + |-i\beta\rangle_c),$$

$$|\psi_3\rangle \sim |\beta\rangle_c - |-\beta\rangle_c + i(|i\beta\rangle_c - |-i\beta\rangle_c). \tag{16}$$

In the limit of small $|\beta|$, the first two states go over to the vacuum and single photon states as before, but the third and fourth states go over to the states $|2\rangle_c$ and $|3\rangle_c$, respectively. Let us consider the case of $|\psi_3\rangle$. The normalized form of this state, $|\Phi_3\rangle=[\langle\psi_3|\psi_3\rangle]^{-1/2}|\psi_3\rangle$, can be shown to be, in the limit of small $|\beta|$,

$$|\Phi_3\rangle = \left[1-\frac{1}{2}\left(\frac{3!}{7!}\right)|\beta|^8\right]\left[|3\rangle_c + \sqrt{\frac{3!}{7!}}\beta^4|7\rangle_c + \cdots\right]. \tag{17}$$

Clearly, with $|\beta|$ small, the coefficient of the number state $|7\rangle_c$ will be very small enough (as will be the coefficients of all the other number states) to be ignored, and the field mode is found to be in the state $|3\rangle_c$ with a high degree of probability. Properties of the general "four photon" coherent states of Eq. (16) were discussed some time ago [8], and a scheme for generating them, and the number states which arise from them in the low-field-strength limit, was discussed some time ago by one of us (C.C.G.) in the context of cavity QED [9]. These states have recently been of interest as they have been shown to possess "sub-Planckian" phase-space structures in their associated Wigner functions [10].

Returning to the general case, and assuming we have the outcome corresponding to the first term of Eq. (12), that is, that we have projected out the normalized state of Eq. (13), the probability that mode c is in the number state $|L'+2^{N+1}M\rangle_c$ is given by

TABLE I. Probability of obtaining the number states $|L'\rangle_c$ and $|L'+2^{N+1}\rangle_c$, respectively, for $N=1$.

L'	$P_{L',0}$	$P_{L',1}$
0	0.694444	0.166667
1	0.81	0.1
2	0.891975	0.055556
3	0.942042	0.029412

$$P_{L',M} = |{}_c\langle L' + 2^{N+1}M|\Phi_{L'}\rangle_c|^2 = |\mathcal{N}_{L'}|^2 \frac{e^{-|\beta|^2}|\beta|^{2(L'+2^{N+1}M)}}{(L' + 2^{N+1}M)!}, \tag{18}$$

where we must have $\Sigma_{M=0}^{\infty} P_{L',M}=1$ for all $L'=0,1,\ldots,2^{N+1}-1$. We are interested in obtaining, to a good approximation, the number state $|L'\rangle_c$ $(M=0)$ and the probability that the c mode is in this state is given by

$$P_{L',0} = \left| 1 - \frac{L'!}{2(L' + 2^{N+1})!}\beta^{2(L'+2^{N+1})} \right|^2 \approx 1 - \frac{L'!}{(L' + 2^{N+1})!}\beta^{2(L'+2^{N+1})}, \tag{19}$$

for β assumed to be real and small, whereas the probability of obtaining the next highest number state $|L'+2^{N+1}\rangle_c(M=1)$ is given by

$$P_{L',1} \approx \frac{L'!}{(L' + 2^{N+1})!}\beta^{2(L'+2^{N+1})}. \tag{20}$$

If β is sufficiently small the only state with significant population is the state $|L'\rangle_c$. That is, we shall have $P_{L',0} \gg P_{L',1} \gg P_{L',2}\ldots$. We tabulate $P_{L',0}$ and $P_{L',1}$ for $N=1$ and $N=2$, in Tables I and II, respectively, for the allowed ranges of L' and for $\beta=1.0$. Note that $P_{L',0}$ approaches unity as L' approaches its maximum value $2^{N+1}-1$.

Finally, we take into account the effects of detector efficiencies on our proposed scheme. Recall that to project out the state of Eq. (13) we have to obtain no photon counts from mode b_1 and any number of counts from mode b_2. This is straightforward with ideal detectors, but in the realistic case where they are not ideal, "dark" counts in the b_1 mode detector, that is, photons undetected due to limited detector efficiency, will be indistinguishable from the "No" counts resulting from the vacuum state. To account for these "dark" counts we introduce a positive operator-valued measure (POVM) for the outputs of the two detectors. Again, for detectors of 100% efficiency, the desired outcomes of the measurements after the second beam splitter is to have "No" (N) photons counted by D_1 and any number of photons, a "Yes" (Y) measurement, by detector D_2. These results are consistent with the POVM

TABLE II. Same as Table I but for $N=2$.

L'	$P_{L',0}$	$P_{L',1}$
0	0.694444	0.166667
1	0.81	0.1
2	0.891975	0.055556
3	0.942042	0.029412
4	0.969927	0.015152
5	0.984675	0.007692
6	0.992263	0.003876
7	0.996113	0.001946

$$\hat{\prod}_{\mathrm{N}_1} = |0\rangle_{b_1}\langle 0|_{b_1}, \quad \hat{\prod}_{\mathrm{Y}_2} = \hat{\mathrm{I}} - |0\rangle_{b_2}\langle 0|_{b_2}. \tag{21}$$

To incorporate detector efficiency, represented by η, where $0 \le \eta \le 1$, the above POVM is modified to

$$\hat{\prod}_{\mathrm{N}_1} = \sum_{p=0}^{\infty} (1-\eta)^p |p\rangle_{b_1}\langle p|_{b_1},$$

$$\hat{\prod}_{\mathrm{Y}_2} = \hat{\mathrm{I}}_2 - \sum_{q=0}^{\infty} (1-\eta)^q |q\rangle_{b_2}\langle q|_{b_2}, \tag{22}$$

where $\hat{\mathrm{I}}_2$ is the unit operator associated with the states of mode b_2. We have assumed the detectors to have the same efficiencies. Thus using the state of Eq. (12) we obtain the joint probability of obtaining a "No" in D_1 and a "Yes" D_2 associated with obtaining the superposition containing the states $|L'+2^{N+1}M\rangle_c$ as

$$\begin{aligned}\mathcal{P}_{\mathrm{NY}}^{L'}(\alpha,\beta,\eta) &= \langle \Phi_{BS2}^{L'}|\hat{\prod}_{\mathrm{N}_1}\hat{\prod}_{\mathrm{Y}_2}|\Phi_{BS2}^{L'}\rangle \\ &= \sum_{L=0}^{\lambda_N}\sum_{p=0}^{\infty}(1-\eta)^p\langle\psi_L|\psi_L\rangle e^{-|\alpha_{LL'}^{(-)}|^2}\frac{|\alpha_{LL'}^{(-)}|^{2p}}{p!} \\ &\quad + \sum_{L=0}^{\lambda_N}\sum_{p=0}^{\infty}\sum_{q=0}^{\infty}(1-\eta)^{p+q}\langle\psi_L|\psi_L\rangle \\ &\quad \times e^{-|\alpha_{LL'}^{(-)}|^2-|\alpha_{LL'}^{(+)}|^2}\frac{|\alpha_{LL'}^{(-)}|^{2p}}{p!}\frac{|\alpha_{LL'}^{(+)}|^{2q}}{q!},\end{aligned} \tag{23}$$

where

$$\langle\psi_L|\psi_L\rangle = \sum_{M=0}^{\infty}|C_{L+2^{N+1}M}|^2 = e^{-|\beta|^2}\sum_{M=0}^{\infty}\frac{|\beta|^{2(L+2^{N+1}M)}}{(L+2^{N+1}M)!}. \tag{24}$$

In Fig. 2 we plot $\mathcal{P}_{\mathrm{NY}}^{L'}(\alpha,\beta,\eta)$ vs L' for $N=2$, $\alpha=\sqrt{10}$, and $\beta=1.0$ for detector efficiencies $\eta=0.9$ and $\eta=0.1$. In Fig. 3 we repeat but with $\beta=0.5$. We include that the former detector efficiency for the sake of comparison, as it is close to the ideal, whereas the latter is more realistic. (Note that these probabilities are not required to add up to unity when summing over L'.) We notice the counterintuitive result that

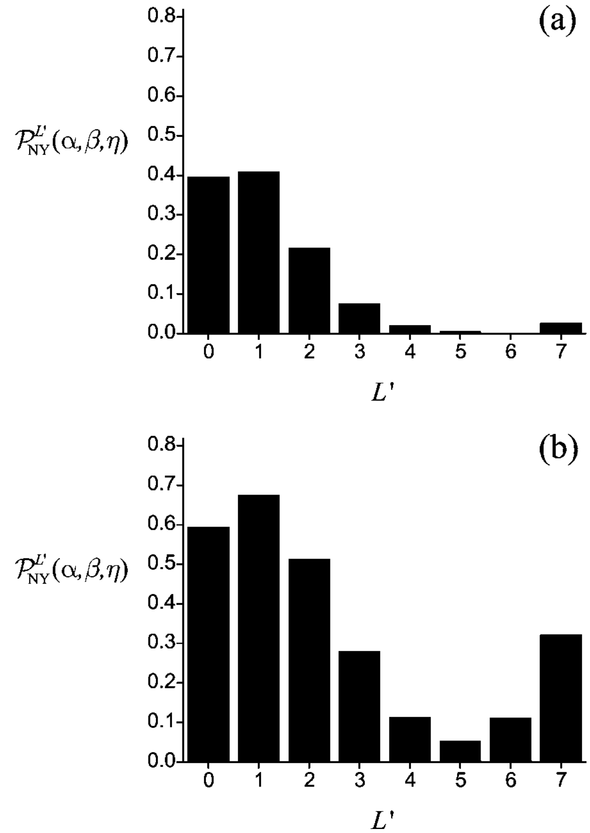

FIG. 2. Plots of the joint No-Yes probabilities $\mathcal{P}_{\mathrm{NY}}^{L'}(\alpha,\beta,\eta)$ of obtaining a No in detector D_1 and a Yes in D_2 vs L' for $N=2$, $\alpha=\sqrt{10}$, and $\beta=1.0$ for detector efficiencies (a) $\eta=0.9$ and (b) $\eta=0.1$.

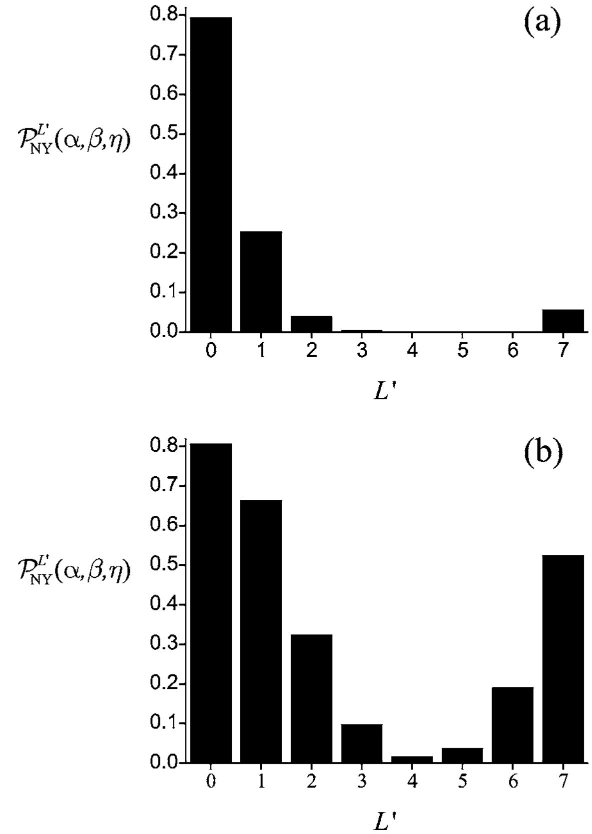

FIG. 3. Same as Fig. 2 except for $\beta=0.5$.

overall the probabilities for No-Yes detection tend to *increase* with *decreasing* detector efficiency, but not evenly. In particular, note that the probability associated with the case $L'=7$ is much higher for $\eta=0.1$ than for $\eta=0.9$. Thus for a lower efficiency detector, the probability of projecting out the state $|\Phi_7\rangle_c$, out of which we obtain the number state $|7\rangle_c$, is higher than for the case of higher detector efficiency.

In this paper we have shown how a previous proposal for a quantum nondemolition measurement of parity, and for generating eigenstates of parity, can be extend to generate number states in traveling-wave optical fields. The scheme has the character of a probabilistic dial up technique in the sense that the choice of the phase shift θ, as given by $\theta=\pi-\varphi_{L'}$, in the counterclockwise beam of the Mach-Zehnder interferometer, sets the stage for a probabilistic No-Yes measurement in the output of BS2, in the case of ideal photon detection. An essential ingredient to make this scheme workable is Kerr media of large third-order nonlinear susceptibilities $\chi^{(3)}$. Normal readily available media, such as optical fibers, have susceptibilities far too small for this proposed application. On the other hand, there is much activity being directed toward the generation of large Kerr nonlinearities through the techniques of electromagnetically induced transparency [11]. Most of the schemes likely to be useful involve four-level atoms in a double electromagnetically induced transparency regime. Recently, Munro *et al.* [12], in connection with a scheme for the quantum nondemolition measurement of photon number, found that with about 1600 atoms, fixed and stationary within a cylindrical dielectric waveguide, narrow but long compared with the optical wavelengths, a phase shift of $\chi t=0.01$ radians is possible with a residual absorption rate of less than 1%. For $\chi t=\pi/2^N \approx 0.01$ one could have N as high as $N=8$ or 9. Obviously, for smaller values of χt, higher values of N are possible. Interestingly, Munro *et al.* [12] found that the minimum number of atoms required to obtain a given phase shift *decreases* as the phase shift increases. So with different numbers of atoms contained in the dielectric waveguide, it should be possible to generate the states of Eq. (7) over a wide range of orders N. Our work is related to the work of D'Ariano *et al.* [4] but there are some important differences. Our beam splitters are all 50:50 whereas in D'Ariano *et al.* require beam splitters of very low transmissibility τ. To obtain a number state with four photons from an input coherent state of an average photon four photons requires a transmissivity of $\tau=5\times10^{-5}$. The probability of obtaining the number state $|4\rangle$ in that scheme is 0.1997. In contrast, with our scheme the probability of obtaining the state $|4\rangle$ is 0.9670 (with similar probabilities for higher number states $|5\rangle$, $|6,\rangle$ and $|7\rangle$, as shown in Table II.

We mention one other proposal for generating number state, a method proposed several years ago by Leonski [13]. This method deterministically produces number states using a Kerr medium, but only the self phase-modulation interaction for a single mode, and with parametric pumping. Unfortunately the method requires competition with the Kerr interaction with parametric nonlinearity of the kth order.

Lastly, the method we propose is not limited to the production of Fock states of a single mode field. If one mode of a two mode state, such as a two-mode squeezed vacuum or a pair coherent state, is in the input c mode and the other, which we shall call the d mode, is external to the interferometer, then it should be possible, because of the correlations inherent in such states, to generate twin Fock states, i.e., states of the form $|n\rangle_c|n\rangle_d$. As already indicated, such twin Fock states would be of utility in implementing the interferometric scheme proposed by Holland and Burnett [2].

ACKNOWLEDGMENTS

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