Manipulating spectral anomalies of focused pulses in a medium with electromagnetically induced transparency

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We study the possibility of manipulating the focusing properties of a medium with electromagnetically induced transparency. In the focal region of focused ultraslow light pulses, the spectral anomalous behaviors can be actively modified by varying the control field intensity. Unlike the case in free space, we find in slow light focusing that the spectrum bandwidth of the incident field needed to produce observable spectral changes can be reduced by several orders. Numerical simulations with accessible parameters clearly show that spectral anomalies of focused μs pulses are observable.

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Electromagnetically induced transparency (EIT) is a very hot research field with both theoretical importance and practical applicability [1]. The value of EIT lies in its ability to coherently control the optical properties of a medium. With the help of EIT, we can actively manipulate the light velocity [2–5], coherently store and retrieve light pulses [6–9], transfer quantum states between photons and atoms [10–15], and realize quantum information processing [16–19]. So far, most investigations on EIT are concerned with pulse propagation. Focusing properties of light waves are very important and play fundamental roles in optical science, such as imaging, microscopy, and pulse shaping [20]. Thus the ability to actively manipulate the pulse focusing properties is of great importance. There are some works discussed regarding the induced focusing of light beams in EIT media [21,22]. In this paper, we will study the pulse focusing properties of a medium with EIT. As an example we show that, by varying the control field intensity, the spectral anomalous behavior of focused pulses in an EIT medium can be dramatically modified.

Spectral anomaly [23,24] is a new topic in singular optics [25]. Singular optics study the structure of wave fields in the neighborhood of points where the intensity has zero value. It is well known that the phase of the wave field is singular when its amplitude is zero. Thus there exists very complicated topological singularities in the neighborhood of these phase singular points, such as wave front dislocations and optical vortices [26,27]. In literatures, most of the publications on singular optics deal with monochromatic waves. But very recently, in 2002, Gbur, Visser, and Wolf studied the spectral anomalous behaviors in the neighborhood of intensity zeros near the focus of a polychromatic wave diffracted at an aperture [23,24]. Drastic spectral changes near phase singularities were predicted. At different points, the spectrum can be redshifted, blueshifted, or split into two parts. Soon after, this prediction was experimentally verified by Popescu and Dogariu [28]. Since then, there are increasing research interests in spectral anomalies and its link to spectral switches [29–32].

The purpose of this work is to give an example of actively manipulating the focusing properties of light pulses. We found that pulse focusing properties could be controlled by using EIT. Especially, we show that slow light focusing can significantly affect a well-known optical phenomenon, spectral anomalies. Also, to the best of our knowledge, this paper is the first theoretical research on spectral anomalies in dispersive media. Due to the steep dispersion of an EIT medium, the group velocity can be reduced to several orders smaller than the light speed in vacuum. For example, ultraslow propagation of the signal pulse with a group velocity 17 m/s has been observed in cold atoms [2]. Using both analytical and numerical methods, we find that the spectral anomalies also occur in the focal region of ultraslow pulses focused in EIT media. In particular, the spectrum bandwidth of the incident pulse needed to produce observable anomalous behavior can be several orders narrow compared with the case in free space.

Let us consider that a signal pulse propagates in the positive Z direction and is incident on a focusing lens. The optical setup is shown in Fig. 1. The left side of the lens is free space, while the right side of the lens is a dispersive medium. In this paper, the medium is constituted of three-level Λ type atoms with EIT properties as shown in the left lower part of Fig. 1. The atomic density is ρ. The ground state $|0\rangle$ and the metastable state $|1\rangle$ are coupled individually with the excited

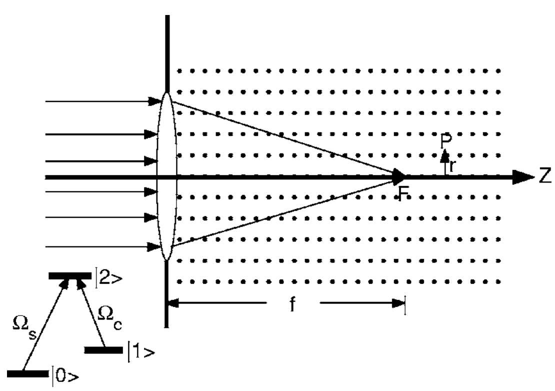

FIG. 1. Schematic diagram of the optical setup. The right side of the lens is occupied by a dispersive medium, which is constituted of three-level Λ type atoms shown in the left lower corner.

state $|2\rangle$ via a weak signal pulse (with Rabi frequency Ω_s) and an intense control laser field (with Rabi frequency Ω_c), respectively. The continuous control field is in resonant with the transition $|1\rangle \to |2\rangle$, and the frequency ω_s of the signal pulse is assumed to be close to the transition frequency of $|0\rangle \to |2\rangle$. Under the condition of EIT, the linear susceptibility of the signal field has been obtained in many literatures, such as Ref. [33],

$$\chi(\omega_s) = \frac{\rho\mu^2}{\epsilon_0\hbar}\frac{4\Delta_s(\Omega_c^2 - 4\Delta_s^2) - 4\Delta_s\Gamma_1^2 + 8i\Delta_s^2\Gamma_2 + 2i\Gamma_1(\Omega_c^2 + \Gamma_1\Gamma_2)}{[\Omega_c^2 + \Gamma_1\Gamma_2 - 4\Delta_s^2]^2 + 4\Delta_s^2(\Gamma_1 + \Gamma_2)^2}, \tag{1}$$

where μ is the electric dipole moment of transition $|0\rangle \to |2\rangle$, $\Delta_s = \omega_s - \omega_{20}$ is the signal frequency detuning, and Γ_1 and Γ_2 are decay rates of atomic states $\rangle 1\rangle$ and $\rangle 2\rangle$, respectively. Generally in an EIT medium, the value of χ is very small but the dispersion is very large, thus the wave number $k(\omega_s)$ is

$$k(\omega_s) = \frac{\omega_s\sqrt{1+\chi(\omega_s)}}{c} \approx \frac{\omega_s}{c}\left(1 + \frac{\chi(\omega_s)}{2}\right). \tag{2}$$

For a monochromatic plane wave with frequency ω and amplitude $A(\omega)$, the optical field at a point $P(x,y,z)$ in the focal region is given by $A(\omega)H(x,y,z,\omega)$, which can be calculated from the Fresnel diffraction integral

$$H(x,y,z,\omega) = \frac{k(\omega)}{j2\pi z}e^{jk(\omega)z}e^{jk(\omega)/2z(x^2+y^2)} \times \int_S t(x',y') \times e^{jk(\omega)/2z(x'^2+y'^2)}e^{-jk(\omega)/z(xx'+yy')}dx'dy', \tag{3}$$

where $k(\omega)$ is given by Eq. (2) and S is a circular aperture with radius a. The lens is assumed to be no dispersive and its transmission function is given by

$$t(r) = e^{-j\pi/\lambda f r^2}, \tag{4}$$

where $r = \sqrt{x^2+y^2}$, f is the focal distance and $\lambda = 2\pi c/\omega$ is the wavelength in free space. From Eqs (3) and (4), the spectral intensity at a point $P(x,y,z)$ in the focal region is

$$S(x,y,z,\omega) = S^{(0)}(\omega)M(x,y,z,\omega), \tag{5}$$

where $S^{(0)}(\omega) = |A(\omega)|^2$ is the spectral intensity of the incident field and the *spectral modifier* $M(x,y,z,\omega)$ has the formula

$$M(x,y,z,\omega) = |H(x,y,z,\omega)|^2 = \left|\frac{k(\omega)}{z}\right|^2 \left|\int_0^a e^{j\alpha r'^2}J_0(\beta r')r'dr'\right|^2, \tag{6}$$

where $\alpha = k(\omega)/2z - \pi/\lambda f$, $\beta = k(\omega)r/z$, and $J_0(x)$ is the Bessel function of the first kind and order zero. Due to the dispersion relation given in Eq. (2), the spectral anomalies in an EIT medium will differ from the case in free space.

To give an approximately theoretical analysis, we suppose that the incident field is centered at frequency ω_0 and the dispersive relation can be expanded as $k(\omega) = k_0 + (\omega - \omega_0)/V_g$ with V_g the group velocity in the medium. According to Eqs. (1) and (2), in an EIT medium, V_g is actively controllable. On the Z axis, $\beta = 0$,

$$\alpha = \frac{\omega_0}{2cz} + \frac{\Delta}{2V_g z} - \frac{\omega_0 + \Delta}{2cf}, \tag{7}$$

where $\Delta = \omega - \omega_0$ is the frequency detuning. The *spectral modifier* on the Z axis can be integrated

$$M(0,z,\omega) = \left|\frac{k(\omega)}{z}\right|^2 \left[\frac{\sin(\alpha a^2/2)}{\alpha a^2/2}\right]^2, \tag{8}$$

where $\alpha = \alpha(z,\Delta)$ is given by Eq. (7). The modifier function at frequency ω_0 is zero when $\alpha(z_n,0)a^2 = 2n\pi$, where $z_n = fa^2/(2n\lambda_0 f + a^2)$ ($n \neq 0$). When the observation point on the Z axis changes to $z = z_n + \delta z$, the frequency corresponding to the zero modifier function also changes. Consider at the neighborhood of z_1, from Eq. (8), the zero modifier function requires

$$\alpha(z_1 + \delta z, \Delta(\delta z))a^2 = 2\pi. \tag{9}$$

Thus we are able to solve $\Delta(\delta z)$ at which the modifier function is zero as the function of δz

$$\Delta(\delta z) = \frac{4\pi V_g cf(z_1 + \delta z)/a^2 - \omega_0 V_g(f - z_1 - \delta z)}{cf - V_g z_1 - V_g\delta z}, \tag{10}$$

If δz is small, approximately we have

$$\Delta(\delta z) \approx \frac{V_g(4\pi cf/a^2 + \omega_0)}{cf - V_g z_1}\delta z \approx \begin{cases} \dfrac{\omega_0 a^2}{\lambda_0 f^2}\delta z, & V_g = c \\ \dfrac{V_g}{cN_0}\dfrac{\omega_0 a^2}{\lambda_0 f^2}\delta z, & V_g \ll c \end{cases} \tag{11}$$

where $N_0 = a^2/\lambda_0 f$ is the Fresnel number and is assumed to be very large as done in Refs. [23,24].

Equation (11) is one of the key results obtained in this work. For an incident field with a Gaussian shape spectrum centered at ω_0, the spectrum will be split into two parts at $z = z_1$. At a distance a little larger than z_1, $\delta z > 0$, the modifier function will be zero at a frequency larger than ω_0, so the spectrum will be redshifted. On the other hand, at a distance a little smaller than z_1, $\delta z < 0$, the modifier function will be zero at a frequency smaller than ω_0, so the spectrum will be blueshifted. The spectrum changes is determined by $\Delta(\delta z)$.

As an example, suppose $N_0=100$ and $f=10a$; let us observe the spectrum changes at point $z_1\pm\lambda_0$. In the free space, $\Delta(\lambda_0)/\omega_0=a^2/f^2=0.01$. Thus to observer spectral anomalies in free space, the bandwidth of the incident field should be very broad, like the experiment done in Ref. [28]. However, in an EIT medium, the bandwidth of the incident field can be reduced by a factor V_g/cN_0. At present, it is not difficult to realize $V_g=10^{-6}$ c in an EIT medium, so the spectral anomalies of an incident field with bandwidth about $10^{-10}\omega_0$ should be observable in slow light focusing experiments. Since the group velocity in an EIT medium is controllable by varying the control field intensity, we can manipulate the spectral anomalous behaviors actively. In fact, there exists a simple physical interpretation in Eq. (11). From Eq. (3), we can see that the optical field in the focal region is explicitly related to k, but implicitly related to the ω. Thus, the same field distribution means same k. Since the dispersion in free space is much smaller than in a slow light medium, to get the same changes of k, the frequency changes in the slow light medium are much smaller than in free space.

In deriving Eq. (11), the absorption and high order dispersion have been neglected. In the following, we use numerical simulations to study spectral anomalies of a focused pulse in an EIT medium. In our calculations, we choose the atomic density to satisfy $\frac{\rho\mu^2}{\epsilon_0\hbar\omega_{20}}=1.2\times10^{-9}$ for conveniently. Other parameters are $\Gamma_1=10^{-3}\Gamma_2$, $\Gamma_2=3\times10^7$ s^{-1}, and $\omega_{20}=2.5\times10^{15}$ s^{-1}. Ω_c is a variable parameter. For the focusing setup, the parameters are $\lambda_0=\lambda_{20}$, $f=10a$ and $N_0=100$. All these parameters are experimentally accessible. So we can use Eqs. (1)–(3) to calculate the spectral intensities at different points in the focal region. To be in agreement with the literature, scaled coordinates $u=2\pi N_0(z-f)/f$ and $v=2\pi N_0 r/a$ are used in the following. The incident signal pulse is assumed to have a Gaussian spectral distribution

$$S^{(i)}(\omega)=S_0\exp\left[-\frac{(\omega-\omega_0)^2}{2\sigma^2}\right], \qquad (12)$$

where S_0 is a small constant to satisfy the EIT condition and $\sigma_0=2\times10^{-10}\omega_0$ characterizing the bandwidth of the incident pulse [see Fig. 2(a)]. At point $u=u_1=2\pi N_0(z_1-f)/f$ and $v=0$, the modifier function at frequency ω_0 is near zero, thus the spectrum is split into two peaks, as shown in Fig. 2(c). Unlike the case in free space, here two peaks are not symmetric since the susceptibility is asymmetric. At point $u=u_1+0.15$ and $v=0$, the spectrum depicted in Fig. 2(b) is redshifted. A smaller value of control field Ω_c makes the spectrum more redshifted. On the other hand, in Fig. 2(d), the spectrum at $u=u_1-0.15$ and $v=0$ is blueshifted. Larger Ω_c makes the spectrum less blueshifted. Thus the spectral anomalous behaviors can be manipulated by changing the control field. These figures are qualitatively in agreement with our theoretical analysis.

We use Fig. 3 to show global behaviors of the spectrum in the focal region. Mean frequency $\bar{\omega}$ is defined as [23]

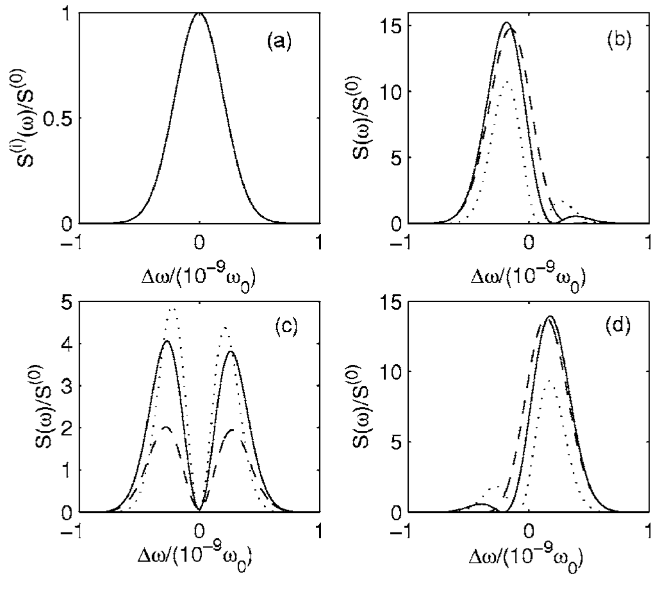

FIG. 2. (a) The spectrum of the incident field given by Eq. (12). (b) The spectrum at points $u=u_1+0.15$ and $v=0$. (c) The spectrum at points $u=u_1$ and $v=0$. (d) The spectrum at points $u=u_1-0.15$ and $v=0$. Parameters are given in the text. Solid line, $\Omega_c=1.14\times10^8$ s^{-1}; dotted line, $\Omega_c=0.76\times10^8$ s^{-1}; dashed line, $\Omega_c=1.52\times10^8$ s^{-1}.

$$\bar{\omega}(r,z)=\frac{\int\omega S(r,z,\omega)d\omega}{\int S(r,z,\omega)d\omega}. \qquad (13)$$

$\bar{\omega}>0$ and $\bar{\omega}<0$ correspond to blueshifted and redshifted spectrum, respectively. We can see there exist several singular points near which the spectrum changes drastically. On the Z axis, near points z_n, i.e., in the neighborhood of $v=0$ and $u\approx\pm4\pi,\pm8\pi,\ldots$, $\bar{\omega}$ changes very rapidly from positive to negative. These properties are similar as the results in free

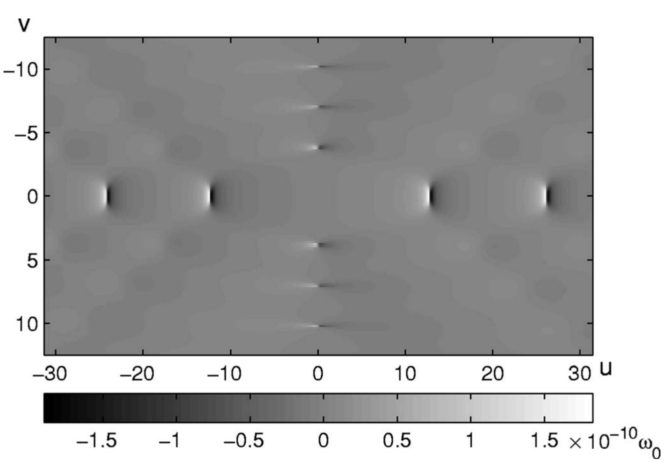

FIG. 3. Gray scaled plot of the mean frequency $\bar{\omega}$ of the spectrum in the focal region as the function of scaled coordinators u, v. The color is more bright or dark as the spectrum is more blueshifted or redshfited, respectively. $\Omega_c=1.14\times10^8$ s^{-1}.

space though there is an important difference. In free space, at points near $u \approx 4\pi, 8\pi, \ldots$, $\bar{\omega}$ changes from negative to positive when u is increased, but in an EIT medium, as shown in Fig. 3, $\bar{\omega}$ still changes from positive to negative when u is increased. In Fig. 3, changes of $\bar{\omega}$ in the focal plane ($z=0$) are smaller than those on the Z axis. Also the structure in the focal plane is quite different from the case in free space. The reason is that, in slow light focusing, the α parameter [introduced after Eq. (6)] in the focal plane is not negligible for finite frequency detuning, so the integrated results have different properties and the spectral anomalies are suppressed to some degree.

In conclusion, we have studied the spectral anomalies in the focal region of a light pulse focused in an EIT medium. Focusing properties such as spectral anomalous behaviors can be actively manipulated by varying the control field intensity. The bandwidth of the incident pulse needed to produce observable spectrum changes can be reduced by several orders compared with the case in free space. Numerical simulations suggest that the spectral anomalies of a μs pulse can be observed in an EIT medium. Differences between spectral anomalies in free space and in EIT media are discussed. Our method is very general and can be used to investigate spectral anomalies in any kind of dispersive media, such as in the medium with superluminal light propagation [34]. We believe that idea on actively manipulating pulse focusing properties should have theoretical importance and can find applications in future works.

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