Interaction of Rydberg atoms with two contrapropagating ultrashort laser pulses

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In this paper we investigate how Rydberg atoms respond to perturbation by two contrapropagating ultrashort laser pulses. We consider the case where the durations of both pulses τ_1 and τ_2 are shorter than the inverse of the initial-state energy ε_i^{-1}. When acting alone such a pulse passes through the atom without noticeable alteration in the atomic state. The situation is different if two such pulses interfere in the region of atom localization. In this case the atomic response is significantly enhanced. This is due to the nonzero momentum transferred to the electron by the interplay of the electric field of one pulse and the magnetic field of the other. The sudden perturbation approximation is used to evaluate the transition probabilities. They are shown to depend on the atom position with respect to the pulse interference region. This dependence is determined by the relationship between the atomic diameter d_i and the interference-region size $l=c(\tau_1+\tau_2)$ (c is the speed of light). If $d_i \ll l$ this dependence is sensitive to the function form of the pulses. For sufficiently strong fields the atoms can be ionized completely. In the opposite case of $d_i \gg l$ the transition probabilities are sensitive to the electron density distribution along the propagation direction. The probabilities of the initial-state destruction and atom ionization drop as l/d_i irrespective of the characteristics of the pulses.

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I. INTRODUCTION

For the last two decades Rydberg atoms interacting with ultrashort electromagnetic pulses have received increasing experimental [1–9] and theoretical [10–13] attention. For example, Rydberg atoms kicked by trains of half-cycle pulses have been extensively studied to gain a deeper insight into nonlinear dynamics in Hamiltonian systems and quantum-classical correspondence [8,13–15]. Another example is Rydberg atom stabilization against ionization in strong laser fields [16,17]. Apart from being of fundamental interest Rydberg atoms are also attractive for technological applications. Recently, such atoms collected in mesoscopic atomic ensembles have been proposed as a tool for quantum-information processing. To do it one needs the methods for Rydberg-atom control and manipulation. In particular this can be accomplished with the use of light fields [18].

Electrons in Rydberg states are loosely bound. One could expect that the stronger is the electromagnetic pulse the easier it can affect the atomic state. Indeed, this is what happens to Rydberg atoms subjected to half-cycle pulses of durations τ_s shorter than the characteristic time $T_{n_i}=2\pi n_i^3$ of electron oscillation in the initial state $|i\rangle$ [19]. When the interaction is complete the electron acquires the nonzero drift momentum

$$\boldsymbol{p} = \int_0^{\tau_s} \boldsymbol{E}(t)dt, \tag{1}$$

where $\boldsymbol{E}$ is the electric component of the field. Then, the atom can be ionized if $p=|\boldsymbol{p}|$ is sufficiently large. The ionization probability grows with increase in fluence (time-integrated intensity). No ionization suppression has been detected for this case [2,20,21].

In contrast, for Rydberg atoms subjected to laser pulses shorter than the period T_{n_i} complete photoionization is prohibited irrespective of the fluence [4]. The probabilities of bound-bound transitions are also very small, even for moderate fields [22]. This is due to the fact that, for a short period of time $\tau \ll T_{n_i}$, the atomic electron moving on some Kepler orbit, can be considered as a free particle. It can gain neither drift momentum nor displacement from the electric field when the pulse action is complete [22–25]. The laser field can affect the electron only near the atomic core where the momentum and energy conservation laws let the electron pick up field photons. In the initial Rydberg state the wave function has appreciable amplitude both near the core and far away. Laser-induced processes are limited only by depletion of the wave function near the core [4].

The apparent contradiction between the two cases arises due to the fact that laser pulses and half-cycle pulses interact with Rydberg atoms in different regimes. An ultrashort laser pulse comprises many or at least a few complete oscillations for the time τ. In turn, a half-cycle pulse consists of a very short main half cycle of duration $\tau_s \ll T_{n_i}$ delivering nonzero momentum $\boldsymbol{p}$. This part is followed by a long low-amplitude half-cycle pulse of opposite polarity. The duration of the long part τ_l is typically larger than the Kepler period T_{n_i}. Its action on the atom is usually negligible and is not taken into account in calculations. However, when $\tau_s+\tau_l \ll T_{n_i}$ the atomic stabilization can also be observed [26].

Recently, attosecond pulses became available for experimental use [27–36]. The minimal pulse duration is now $\sim$0.1 fs. For such pulses the regime $\tau < T_{n_l}$ is realized even if atoms are in the ground state. As follows from the previous discussion, we have come to the limit where there is no benefit from getting ever-shorter durations to manipulate or control the quantum state of a Rydberg atom with a single laser pulse. Under condition $\tau \ll T_{n_i}$ one needs more than a single pulse or even a train of single pulses for effective quantum "state management." In this paper we study the

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case of a Rydberg atom interacting with two contrapropagating ultrashort laser pulses. The idea is to make two pulses work simultaneously.

Recently, the arrangement with two contrapropagating pulses has been considered by Va'zquez de Aldana *et al.* [37] and Krylstra *et al.* [38] for a two-dimensional ground-state atom in relation to the atomic stabilization breakdown. They studied the case of two identical linear-polarized femtosecond laser pulses forming a standing wave. For the atom placed in a magnetic-field node they found that the detrimental effects of the Lorenz force is not negated and the ionization probability is even higher than in the case of a single laser pulse of the same intensity. Also, Milosevic *et al.* [39] suggested recently a new technique for imaging attosecond dynamics of nuclear processes with the use of two equal-handed, contrapropagating, circularly polarized laser pulses. In this case the magnetic-field effect was shown to be eliminated over the whole focal region.

In contrast to Refs. [37] and [38] we study the case where atoms are initially in highly excited states. The distinctive feature of our case is that the pulse sizes can be comparable or even less than the characteristic size of the atom localization area. So the theoretical description has to take into account the nondipole corrections. This regime of laser-atom interaction has been relatively unexplored. Also, we consider the more general case where the atom position with respect to the laser pulses can be arbitrary.

In this work we employ the sudden perturbation approximation (SPA) [40] used previously for description of atom interaction with a singe ultrashort laser pulse [22]. The structure of the paper is as follows. In the next section we give the required theoretical details. Then, in Sec. III we report the results on atom perturbation by two contrapropagating pulses. We consider the case where the pulses are of the same duration. In Sec. IV we give the physical explanation of our results and compare the used approach with the other theories for description of atom interaction with ultrashort pulses. The major conclusions are summarized in Sec. V. Atomic units are used throughout unless specified otherwise.

II. THEORY

Consider a hydrogen atom being initially in a state $|i\rangle$. The atom is subjected to two contrapropagating ultrashort laser pulses (see Fig. 1). To be specific we assume that the propagation direction of the first pulse determines the z axis and the coordinate origin O coincides with the position of atomic nucleus. The electronic state of the atom is perturbed by the potential

$$\hat{V}(z,t) = -\boldsymbol{A}(z,t)\hat{\boldsymbol{p}} + |\boldsymbol{A}(z,t)|^2/2, \tag{2}$$

where

$$\boldsymbol{A}(z,t) = \boldsymbol{A}_1(t-\alpha z) + \boldsymbol{A}_2(t+\alpha z - t_0) \tag{3}$$

is the vector potential of the electromagnetic field, $\hat{\boldsymbol{p}}$ is the momentum operator, $\alpha = 1/c \approx 1/137$, c is the speed of light, and t_0 characterizes the delay in action between the pulses at $z=0$. The vector potential $\boldsymbol{A}_i$ of the ith pulse is related to the corresponding electric fields $\boldsymbol{E}_i$ by

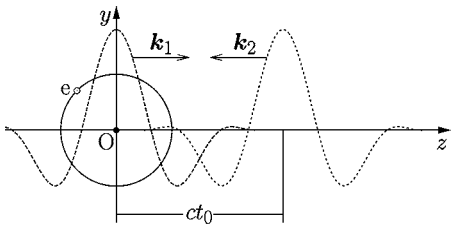

FIG. 1. Diagram for an atom in the field of two contrapropagating ultrashort laser pulses at $t=0$ when the first pulse center passes through the atomic nucleus. The atom localization area is indicated with the solid circle. The pulses are shown with the broken lines. The z axis is directed along the wave vectors $\boldsymbol{k}_1$ of the first pulse. The interval t_0 characterizes the "delay" between the pulses.

$$\boldsymbol{E}_i = -\frac{\partial \boldsymbol{A}_i}{\partial t}. \tag{4}$$

In this paper we assume that both pulses comprise only few field oscillations. Brabec and Krausz [41] showed that the concept of the envelope can be extended for such pulses to durations equal to the carrier oscillation period $T_0 = 2\pi/\omega$. We assume that each pulse is of the form

$$\boldsymbol{E}_i(\eta) = E_0^{(i)} f_i(\eta)[\hat{\boldsymbol{x}}\cos(\omega_i\eta + \varphi_i)\cos\xi_i + \hat{\boldsymbol{y}}\sin(\omega_i\eta + \varphi_i)\sin\xi_i], \tag{5}$$

where $E_0^{(i)}$, $f_i(\eta)$, ω_i, φ_i, and ξ_i are, respectively, the pulse magnitude, envelope, carrier frequency, carrier-envelope phase, and polarization index which can be different for different parameters. The form (5) is often used in the literature for ultrashort laser pulses [42–46].

The vector potential $\boldsymbol{A}_i$ reads

$$\boldsymbol{A}_i(\eta) = -\int_{-\infty}^{\eta} \boldsymbol{E}_i(t)dt. \tag{6}$$

This expression ensures that $\boldsymbol{A}_i$ vanishes in the area of atom localization in the infinite past. We will consider the fields without the residual dc component in the postinteraction regions defined by the conditions $t-\alpha z > \tau_1/2$ and $t+\alpha z - t_0 > \tau_2/2$ for the first and second pulses, respectively. This requires additional constrains on the laser pulse parameters. They will be specified in Sec. III.

For the field of Eq. (3) the interaction potential $\hat{V}$ of Eq. (2) can be presented as a sum

$$\hat{V}(z,t) = \hat{V}_1(\eta) + \hat{V}_2(\eta') + \hat{W}(z,t) \tag{7}$$

of the electron interaction potentials with each wave

$$\hat{V}_i(\eta) = -\boldsymbol{A}_i(\eta)\cdot\hat{\boldsymbol{p}} + A_i^2(\eta)/2 \tag{8}$$

and the interference term

$$\hat{W}(z,t) = [\boldsymbol{A}_1(t-\alpha z)\cdot\boldsymbol{A}_2(t+\alpha z - t_0)]. \tag{9}$$

For fields $\boldsymbol{E}_1$ and $\boldsymbol{E}_2$ defined by Eq. (5) one can see that $\hat{W} = 0$ if

$$|\eta_0| = |t_0 - 2\alpha z| > \tau, \tag{10}$$

where

$$\tau = (\tau_1 + \tau_2)/2. \tag{11}$$

The pulses never overlap at z specified by this inequality and their action on the electron with coordinate z is separated in time.

For given τ_1, τ_2, and t_0 the space-time area where the pulses overlap is determined by the conditions

$$\frac{c[t_0 - \tau]}{2} < z < \frac{c[t_0 + \tau]}{2}, \tag{12}$$

$$\frac{t_0 - \tau}{2} < t < \frac{t_0 + \tau}{2}. \tag{13}$$

As follows from the inequality (12) the size of this area is equal to $l = c\tau$. For electromagnetic pulses comprising only a few field oscillations this value can be considered as a characteristic size of the field variation. For available attosecond pulses ($\tau \sim 10^2$ as), $l \sim 10^3$ a.u., which is comparable with or smaller than the atomic characteristic size $d_i \sim 2n_i^2$ for Rydberg states with $n_i \gtrsim 17$. For femtosecond pulses $l \lesssim d_i$ if $n_i \gtrsim 50$.

If the spatial area defined by condition (12) does not intersect with the area of atom localization ($|z| \lesssim d_i$) one can neglect $\hat{W}$ in $\hat{V}$. In this case atom interaction with two contrapropagating pulses can be viewed as sequential perturbation with two single pulses. In the zeroth order of the SPA each pulse results only in a spatially independent phase shift causing no transitions [22]. So, if the higher-order SPA corrections are small, two contrapropagating laser pulses pass through the atom without significant alteration of its electronic state.

In this paper we consider the opposite case where the contrapropagating pulses overlap in the atom localization area. The transition amplitude from the initial state $|i\rangle$ to some final state $|f\rangle$ induced by perturbation $\hat{V}$ can be approximated in the zeroth order of the SPA by [22]

$$a_{i,f}^{(0)} = \langle f|\exp\left(-i\int_{-\infty}^{\infty} \hat{V}(z,t)dt\right)|i\rangle. \tag{14}$$

For $\hat{V}$ of Eq. (7) this expression can be reduced to the form

$$a_{i,f}^{(0)} = e^{-i\Phi}\langle f|\exp[-iF(\eta_0)]|i\rangle, \tag{15}$$

where

$$\Phi = (1/2)\int_{-\infty}^{\infty} [A_1^2(t) + A_2^2(t)]dt \tag{16}$$

and

$$F(\eta_0) = \int_{-\infty}^{\infty} \hat{W}(z,t)dt = \int_{-\infty}^{\infty} [\boldsymbol{A}_1(\eta)\cdot\boldsymbol{A}_2(\eta - \eta_0)]d\eta. \tag{17}$$

Generally, function F is not equal to zero. So, in contrast to the single-pulse case, the effect of two contrapropagating pulses interfering in the atom localization region appears in the zeroth-order of the SPA.

The SPA is applicable if the following two conditions are met. First, it is required that

$$\varepsilon\tau_{\text{int}} \ll 1, \tag{18}$$

where $\tau_{\text{int}} \lesssim 2[\tau + d_i/c]$ is the total laser-atom interaction time [47]. In Eq. (18) ε corresponds to any eigenenergy of the unperturbed Hamiltonian for the states involved in the processes of laser-atom interaction. These states include the initial state, the final state, and all intermediate states which can be excited during the pulse action with sufficiently large probability. Inequality (18) is a formal requirement ensuring convergence of the SPA expansion for the $\hat{S}$ operator [40].

Another condition that should be satisfied is imposed on the perturbation potential $\hat{V}$. It is

$$[\hat{V}(z,t), \hat{V}(z,t')] = 0, \tag{19}$$

for any t and t', where $[\hat{A},\hat{B}] = \hat{A}\hat{B} - \hat{B}\hat{A}$. One can show that this condition is met for the copropagating or contrapropagating transversely polarized electromagnetic fields $\boldsymbol{A}_1$ and $\boldsymbol{A}_2$. For any other arrangements the SPA is inapplicable.

III. RESULTS

In this section we study numerically the response of a hydrogen atom to two ultrashort electromagnetic pulses interfering in the atom localization region. This response is determined by function F of Eq. (17) which depends on the scalar product $(\boldsymbol{A}_1\cdot\boldsymbol{A}_2)$. This dependence excludes the selection rules for electron transitions related to the laser pulse polarizations in the xy plane. Nevertheless, the choice of pulse polarizations allows control, to some extent, the shape and magnitude of function F.

To be specific we assume in this paper that the fields are linearly polarized and their vector potentials $\boldsymbol{A}_1$ and $\boldsymbol{A}_2$ are of the form

$$\boldsymbol{A}(\eta) = \hat{\boldsymbol{x}}A_0 \sin(\omega\eta + \varphi) \times \begin{cases} \cos^2(\pi\eta/\tau) & \text{if } |\eta| \le \tau/2, \\ 0 & \text{otherwise.} \end{cases} \tag{20}$$

The pulses can have different parameters A_0, ω, τ, and ϕ which will be specified with a superscript. The vector potential magnitude A_0 is related to the magnitude of the pulse's electric-field component as $A_0 = E_0/\omega$.

We require that the electromagnetic field (20) satisfies the conditions for real physical pulses. As mentioned in the Introduction free electrons gain neither drift momentum

$$\Delta\boldsymbol{P} = \int_{-\tau/2}^{\tau/2} \boldsymbol{E}(t)dt = -\boldsymbol{A}(\tau/2) \tag{21}$$

nor displacement

$$\Delta\boldsymbol{R} = -\int_{-\tau/2}^{\tau/2} \boldsymbol{A}(t)dt \tag{22}$$

when the action of a single realistic electromagnetic pulse $\boldsymbol{E}(t)$ of duration τ is complete. Expression (20) ensures that

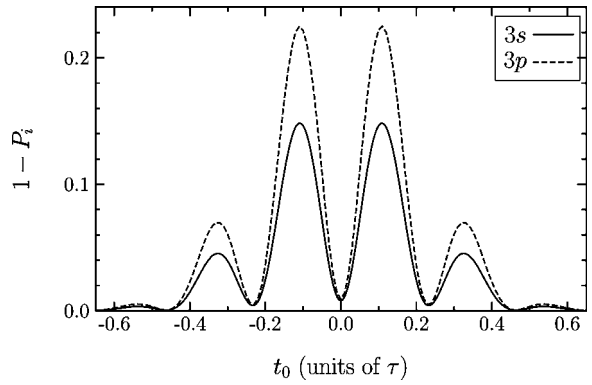

FIG. 2. The probability for an electron to leave its initial state $|i\rangle$ due to the action of two contrapropagating laser pulses as a function of the parameter t_0. This parameter characterizes the delay between the pulses (see Fig. 1). The used parameters are $F_0=E_0^{(1)}E_0^{(2)}=10$, $\tau=\tau_1=\tau_2=5$, $n_1=n_2=2$, and $\phi_1=\phi_2=0$. The solid (broken) line corresponds to the depopulation probability for the atom being initially in the $3s$ ($3p$) state.

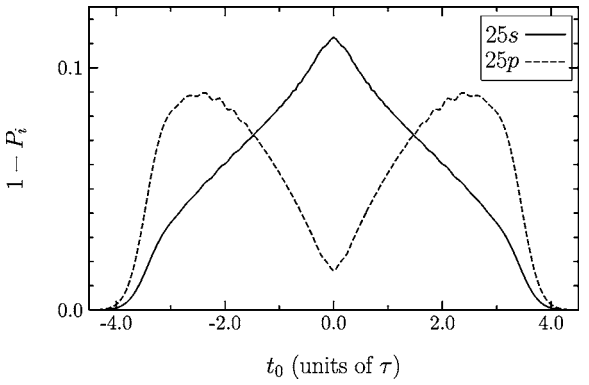

FIG. 3. Same as Fig. 2 but for the initial states $25s$ (solid line) and $25p$ (broken line).

$\Delta \boldsymbol{P}=\boldsymbol{0}$ for any laser pulse parameters A_0, τ, ω, and φ. To satisfy the second condition we choose the laser frequency ω to be equal to $2\pi n/\tau$ where n is a positive integer corresponding to the number of field oscillations during the pulse. Also the condition $\Delta \boldsymbol{R}=\boldsymbol{0}$ requires that $n\geq 2$ for the field given by Eq. (20).

We would like to emphasize that Eq. (20) is a standard theoretical form used to simulate the effect of an ultrashort laser pulse on an atomic electron. For pulses involving only a few oscillations this form should be tailored to produce the same effect on an electron, in the absence of any other fields, as would a realistic laser pulse [23]. This is achieved presently by specifying the conditions (21) and (22). They would not be necessary if a more precise description of the electromagnetic field is used.

Our choice of ultrashort laser pulses is restricted with pulses of femtosecond and attosecond durations. So one can have two different situations which are (i) $\tau_1\approx\tau_2\approx\tau$ and (ii) $\tau_1\ll\tau_2$. In the first case the information of the pulse shape is stored in the transformed wave function through the t_0 dependence of F [see Eq. (15)]. For pulses of significantly different durations ($\tau_1\ll\tau_2$) one can have access only to the spatial behavior of the long pulse. It can be seen from the following estimation:

$$F(\eta_0)\approx -\left(\boldsymbol{E}_2(-\eta_0)\cdot\int_{-\tau_1/2}^{\tau_1/2}\eta\boldsymbol{A}_1(\eta)d\eta\right). \tag{23}$$

To derive Eq. (23) we took into account that $\Delta R=0$ for realistic pulses.

Now we study the t_0 dependence of the probability $1-P_i$ for the atom to leave the initial state $|i\rangle$ when the interaction with the pulses is complete ($P_i=|a_{i,i}^{(0)}|^2$). We consider only the case $\tau_1=\tau_2$ since no additional physically meaningful information can be obtained if $\tau_1\ll\tau_2$. Figures 2 and 3 depict the depopulation probabilities $1-P_i$ for the $3s$, $3p$, $25s$, and $25p$ states. These states were chosen to demonstrate the most characteristic properties of the t_0 dependence of $1-P_i$. Both laser pulses have the same parameters for both figures. They are specified in the caption of Fig. 2.

From inequality (12) one can see that the interference area overlaps with the atom localization for

$$|t_0|/\tau\lesssim 1+d_i/l. \tag{24}$$

The interference effect we study can occur only for t_0 satisfying this condition. Taking into account relation (24) one can expect that there are two different limiting cases depending on the ratio d_i/l. If $d_i/l\ll 1$ we see that the area of t_0 defined by Eq. (24) is independent of the atomic size. In the other limiting case ($d_i/l\gg 1$) this area increases linearly with d_i. Figure 2 illustrates the case where the atomic size is small in comparison with the size of the interference region. It shows the depopulation probabilities $1-P_i$ of the states with the principal quantum number $n_i=3$ versus t_0.

To study how this probability depends on the initial state symmetry we conducted calculations for the states of different orbital momenta l_i. These dependencies are typical for all values of l_i so only the probabilities for l_i equals 0 and 1 are shown in Fig. 2. We see that these probabilities are of the same qualitative behavior. Moreover, one can show that the ratio $(1-P_{3p})/(1-P_{3s})\approx 1.5$ for the whole region.

Figure 3 illustrates the opposite case $d_i>l$ for $n_i=25$. It shows the t_0 dependence of $1-P_{25s}$ (solid line) and $1-P_{25p}$ (broken line) calculated for the same set of laser parameters. We see that the depopulation probabilities for both initial states are comparatively large in a wider region $|t_0/\tau|<3.5$ than for the case of Fig. 2. This is in agreement with the estimate (24) which gives $|t_0/\tau|\lesssim 2.8$. For $|t_0/\tau|>3.5$ the probabilities decrease rapidly with t_0. Also, Fig. 3 shows that the t_0 dependencies of $1-P_{25s}$ and $1-P_{25p}$ are qualitatively different. This is in contrast to similar interference like behavior of $1-P_{3s}$ and $1-P_{3p}$ shown in Fig. 2. The peak value of $1-P_{25s}$ is slightly larger than the maximum value of $1-P_{25p}$.

To explain these regularities we transform expression (15) for $a_{i,f}^{(0)}$ as follows:

$$a_{i,f}^{(0)}=\delta_{i,f}-\int_{-\infty}^{\infty}dx\int_{-\infty}^{\infty}dy\int_{c(t_0-\tau)/2}^{c(t_0+\tau)/2}dz \times\psi_f^*(\boldsymbol{r})[1-e^{-iF(t_0-2\alpha z)}]\psi_i(\boldsymbol{r}), \tag{25}$$

where we dropped the insignificant factor $e^{-i\Phi}$ and used in-

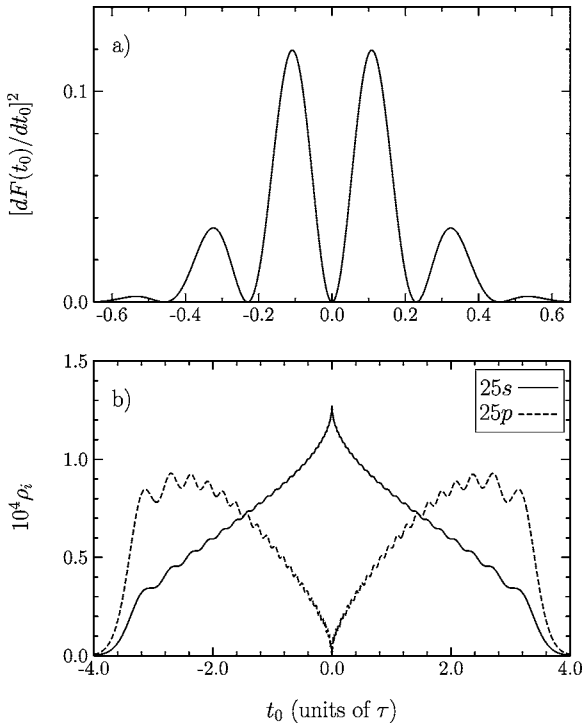

FIG. 4. (a) $|F'|^2$ versus t_0. The laser pulse parameters are the same as for Fig. 2. (b) The electron density distribution $\rho_i(ct_0/2)$ versus t_0 for states $25s$ (solid line) and $25p$ (broken line). The laser pulse parameters are the same as for Fig. 2 for both figures.

equality (12). This equation can be significantly simplified if the dipole approximation condition $d_i \ll l$ is applicable. In this case $F(t_0-2\alpha z)$ is a slowly varying function of z in the area of atom localization. This area provides the dominant contribution to the integral in Eq. (25). So, by expanding the exponential function in the Taylor series at $z=0$ one can derive, up to the second order in d_i/l,

$$a_{i,f}^{(0)} \approx \langle f|\exp[(i\Delta p_z z)]|i\rangle \tag{26}$$

$$\approx [\delta_{i,f} + i\Delta p_z \langle f|z|i\rangle - (\Delta p_z^2/2)\langle f|z^2|i\rangle], \tag{27}$$

where

$$\Delta p_z = 2\alpha F'(t_0) \tag{28}$$

and F' is the first derivative of F. In Eq. (27) we omitted the factor $\exp[-iF(t_0)]$ which is canceled when the transition probabilities are calculated. Taking into account that $\langle i|z|i\rangle = 0$ for any $|i\rangle$ one can get from Eq. (27)

$$1 - P_i = 1 - |a_{i,i}^{(0)}|^2 \approx \Delta p_z^2 \langle i|z^2|i\rangle. \tag{29}$$

One should note that this approximation is valid if the depopulation probability $1-P_i$ is small. We see from Eqs. (28) and (29) that the t_0 dependence of $1-P_i$ comes from the first derivative of F. This function is shown in Fig. 4(a) for the laser parameters used in calculations for Fig. 2. Comparing these two figures one can see that the probability of the $3s$-state depopulation is well described with Eq. (29). Also, from Eq. (29) it follows that the ratio $(1-P_{3p})/(1-P_{3p}) = \langle 3p|z^2|3p\rangle/\langle 3s|z^2|3s\rangle = 1.56$.

For a sufficiently high-energy state $|i\rangle$ one has $d_i \gg l$ and the dipole approximation is inapplicable. Assuming that the same condition is met for the final state $|j\rangle$ one can derive from (25)

$$a_{i,f}^{(0)} = \delta_{i,f} - (c/2) I_L \rho_{i,f}(ct_0/2), \tag{30}$$

where

$$I_L = \int_{-\tau}^{\tau} d\eta_0 [1 - e^{-iF(\eta_0)}] \tag{31}$$

and

$$\rho_{i,f}(z) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \psi_f^*(\boldsymbol{r})\psi_i(\boldsymbol{r}) dx\, dy. \tag{32}$$

So, one has

$$P_i = |1 - (c/2) I_L \rho_i(ct_0/2)|^2, \tag{33}$$

where $\rho_i = \rho_{i,i}$. If the depopulation probability $1-P_i$ is small one can derive from Eq. (33)

$$1 - P_i \approx c\,\mathrm{Re}(I_L)\rho_i(ct_0/2), \tag{34}$$

where $\mathrm{Re}(I_L) = (I_L + I_L^*)/2$ is the real part of I_L.

The relation (33) shows that the t_0 dependence of $1-P_i$ is sensitive to the electron density distribution along the z axis. The electron densities $\rho_{25s}(ct_0/2)$ and $\rho_{25p}(ct_0/2)$ for the states $25s$ and $25p$ are shown in Fig. 4(b). Comparison of Figs. 3 and 4(b) shows indeed similar qualitative behavior of the probabilities $1-P_{25s}$ and $1-P_{25p}$ and the corresponding electron densities ρ_{25s} and ρ_{25p}.

To estimate the depopulation probability $1-P_i$ we notice that $\mathrm{Re}(I_L)$ tends to 2τ with increase in magnitude of the function F due to the increase in F_0. In this case the exponential function is highly oscillatory and its contribution to the integral tends to zero. In the case of s-states the maximum value of the density ρ_i is at $z=0$. One can show that $\rho_i(0) = 1/2n_i^2$ for any s state. Thus we see that

$$1 - P_{n_i s} \lesssim l/d_i \ll 1. \tag{35}$$

For some fixed l this expression shows that the probability to remain in the initial state is higher for higher energy states. For states with $l_i \neq 0$ the maximum value of the electron density ρ_i is of the same order and the estimate Eq. (35) is also applicable.

The extent of wave-function transformation and, hence, the transition probabilities depend on the values of laser pulse parameters. This dependence is different for the limiting cases mentioned above. We show this by the example of the ionization probabilities P_{ion}. We define P_{ion} as follows:

$$P_{\mathrm{ion}} = \int |a_{i,\boldsymbol{k}}^{(0)}|^2 d\boldsymbol{k} \tag{36}$$

where $\boldsymbol{k}$ is a wave vector of the continuum state $|\boldsymbol{k}\rangle$.

In the limit $d_i \ll l$ the amplitude $a_{i,\boldsymbol{k}}^{(0)}$ is equal to the form factor with a transferred momentum $\Delta p_z \sim F_0$ [see Eq. (26)]. If F_0 is sufficiently large the atom can be completely ionized [20]. The amplitude $a_{i,\boldsymbol{k}}^{(0)}$ has a known analytical form for

both bound-bound and bound-free transitions [20,48]. So we will not discuss this case further.

Now we get a simple estimate for the ionization probability in the opposite case ($d_i \gg l$). To do so we replace the true continuum state $\psi_{\mathbf{k}}^{(-)}$ in Eq. (36) with a plane wave $\exp[i\mathbf{k}\cdot\mathbf{r}]/(2\pi)^{3/2}$. After integration in Eq. (36) one can get

$$P_{\text{ion}} \approx c\,\text{Re}(I_{\text{L}})\rho_i(ct_0/2) \tag{37}$$

and

$$P_{n_i s} \lesssim l/d_i \ll 1. \tag{38}$$

This expression shows, in agreement with Eq. (35), that the atom cannot be ionized in this case irrespective of the value of F_0. From Eq. (38) we see that the ionization probability decreases as n_i^{-2}.

IV. DISCUSSION

To specify the physical mechanism responsible for the enhanced atomic response to the simultaneous action of two laser pulses one can employ the classical description. Then, up to the first order in v/c, one can derive the following expression for the electron velocity components:

$$v_x \approx v_x^{(1)} + v_x^{(2)}, \tag{39}$$

$$\frac{dv_z}{dt} = v_x^{(1)}(t)H_y^{(2)}(\eta - \eta_0) + v_x^{(2)}(t)H_y^{(1)}(\eta - \eta_0), \tag{40}$$

where $v_x^{(i)} = -A_i$, the electron velocity components due to the action of the electric fields E_i. Also, $H_y^{(i)} = \partial A_i/\partial z$ is the magnetic field of the ith pulse. From Eqs. (39) and (41) one can get

$$v_z \approx \alpha \int_{-\infty}^{\infty} d\eta [A_1'(\eta)A_2(\eta - \eta_0) - A_1(\eta)A_2'(\eta - \eta_0)]. \tag{41}$$

If $d_i \ll l$ the z dependence in Eq. (41) can be neglected. Then, one can show with the use of integration by parts that this equation leads to the same expression for Δp_z defined by Eq. (28). Thus we see that the electron gains the momentum Δp_z due to the action of the magnetic field of each wave on the electron motion induced by the other wave.

If the spatial extent of the field is less than the atomic size the integral of Eq. (41) is different from zero in the very narrow interval around $z = ct_0/2$. Only in this interval does the electron have a chance to interact simultaneously with the two pulses. The probability for the electron to be in this region is determined by the electron density $\rho_i(ct_0/2)$. This explains the t_0 dependence for the second limiting case.

In this paper we used the SPA to analyze the problem of interest. There are other approximations which lead to similar expressions for the evolution operator. For example, they are the impulse approximation [49,50] and the first-order Magnus approximation (FMA) [51]. The major advantage of the SPA in comparison with the impulse approximation is that it allows exact accounting of the field spatial variation while the impulse approximation has been derived for the dipole-approximation case [22].

The FMA is very similar to the SPA which can lead to confusion [51]. The SPA derived by Dykhne and Yudin [40] yields a series expansion of the time-evolution $\hat{S}$ operator over $\varepsilon\tau_{\text{int}}$,

$$\hat{S}(t,t') = \exp\left(-i\int_{t'}^{t}\hat{V}(\tau)d\tau\right)\left(\hat{I} + \sum_{i=1}^{\infty}\hat{\Delta}_i\right) \tag{42}$$

where $\hat{\Delta}_i \sim (\varepsilon\tau_{\text{int}})^i$. If condition (19) is satisfied expansion (42) leads to the exact solution of the Schrödinger equation.

In contrast, the FMA expression

$$\hat{S}_{\text{FMA}}(t,t') = \exp\left(-i\int_{t'}^{t}\hat{\mathcal{V}}(\tau)d\tau\right) \tag{43}$$

is generally an approximation of the true $\hat{S}$ operator. The operator $\hat{\mathcal{V}}$ in Eq. (43) is the perturbation operator in the interaction representation. The FMA follows from the Magnus expansion [48]

$$\hat{S}(t,t') = \exp\left(-i\int_{t'}^{t}\hat{\mathcal{V}}(\tau)d\tau + \hat{\mathcal{M}}(t,t')\right), \tag{44}$$

when the operator $\hat{\mathcal{M}}$ can be neglected. This operator depends on various commutators of $\hat{\mathcal{V}}$ taken at different instants of time. $\hat{S}_{\text{FMA}}$ allows obtaining the exact solution of the Schrödinger equation only if $[\hat{\mathcal{V}}(t), \hat{\mathcal{V}}(t')] = 0$. This condition is not met in our case. Nevertheless, in the zeroth order both the SPA and the FMA lead to the same approximate expression for the $\hat{S}$ operator since $\hat{\mathcal{V}} \to \hat{V}$. But one can show that their first-order corrections are different.

V. SUMMARY AND CONCLUSIONS

An atomic electron in a Rydberg state behaves almost as a free particle for a pulse duration time $\tau \ll \varepsilon_i^{-1} = 2n_i^2 < T_i = 2\pi n_i^3$. It can gain neither drift momentum nor displacement due to action of the electric field of a realistic laser pulse. Such a pulse passes through the quantum system causing only a weak effect of the first order in $\tau \ll \varepsilon_i$ on the atomic states even for sufficiently high field magnitudes [22].

In contrast, two contrapropagating ultrashort pulses overlapping in the atom localization region result in a much stronger zeroth-order effect. This is in agreement with the results of Va'zquez de Aldana *et al.* [37] and Kylstra *et al.* [38] who found that the Lorenz forces of the pulses do not compensate each other and the ionization probability is higher for the case of two contrapropagating pulses than in the case of a single laser pulse of the same intensity.

The physical explanation is that the electron gains the nonzero drift momentum along the z axis when the interaction is complete. It is related to action of the magnetic field of each wave on the atomic electron driven by the electric field of the other wave. We showed that this effect is very sensitive to the atom position with respect to the region where the pulses overlap.

Without loss of generality we have studied in detail the case when the pulses are of equal durations. It has been shown that the transition probabilities depend on the atom position with respect to the interference area. The form of this dependence is determined by the ratio d_i/l. If $d_i/l \gg 1$ it is entirely dependent on the integral characteristic F of the two laser pulses. Also, no ionization suppression is possible in this case.

In the opposite case ($d_i/l \ll 1$) the dependence of the transition probabilities on t_0 reproduces the electron density distribution of the initial state along the z axis. The initial-state-depopulation probability and the probabilities of transition from the initial to some other states are of the order of l/d_i. Complete ionization of the atom is not possible irrespective of the field magnitude.

In this paper we have studied the case of two contrapropagating laser pulses. The apparent extension of this work is to consider other geometric arrangements with the pulses propagating at some angle $\alpha \neq \pi$ with respect to each other. In this case the condition (19) is violated and the SPA series is not convergent. One needs more sophisticated means for tackling this problem even at $\varepsilon\tau \to 0$ [48]. The preliminary classical analysis shows that the interplay of the electric and magnetic forces can result in more complex electron behavior such as electron dragging at relativistic intensities of the pulses. In this regime complete atom ionization is possible for any ratio of d_i/l.

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