Momentum transfer for momentum transfer-free which-path experiments

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We analyze the origin of interference disappearance in which-path double-aperture experiments. We show that we can unambiguously define an observable momentum transfer between the quantum particle and the path detector and we prove in particular that the so-called "momentum transfer free" experiments can be, in fact, logically interpreted in term of momentum transfer.

I. INTRODUCTION

Bohr's principle of complementarity constitutes the hallmark and one of the most intriguing features of quantum mechanics. On the basis of this principle [1,2], it is indeed universally accepted that any devices capable of determining the path taken by a particle through a Young-like double-aperture must destroy the interference. The justification usually presented is based on Heisenberg's uncertainty principle [3,4] and involves an irremediable exchange of momentum between the system considered and the measuring apparatus. Over the last decades, however, the primacy of such recoil arguments has been contested in favor of a more general decoherence mechanism considering the entanglement of the observed system with its environment [5]. In particular, Scully *et al.* emphasized [6] that an atom, after emitting a long-wavelength photon in a "micromaser-cavity," located close to one of the two pinholes, can generate recordable which-path information without transfer of significant momentum. The conclusions of [6] seem actually verified in experiments using entanglement with an internal degree of freedom to label the path with either neutrons [7], photons [8,9], or atoms [10,11].

This stirred up considerable controversy and a debate [12–22] on the genuine meaning of momentum transfer in which-path experiments. The paradox comes from the fact that any far-field interference observed with a Youngs-like double-aperture experiment is a direct map of the transverse momentum distribution of the diffracted particle in the aperture plane. Any processes able to erase the interference should be then interpretable in terms of a perturbation done on this momentum distribution. Such a proposition was formally done [12–14], but the answer is far from being universally accepted [15,16]. We found two reasons for that: Firstly, the momentum distribution of the particle in the single-aperture experiment is not affected by the detector (this intuitively implies a momentum transfer equal to zero [6]). Secondly, the momentum transfer defined in [12,13] is in general "hidden" in the sense that it is not always connected with an experimentally recordable momentum (like the photon momentum in the Feynman light microscope [23]).

In this article we revive this controversy by proposing a consistent definition of momentum transfer based only on the concept of quantum observables. We study the generality of the recoil mechanism and show in particular why the scheme proposed in [6] is not recoil free. We analyze in this context the roles played by both Heisenberg's relation and entanglement.

II. TWO FUNDAMENTAL EXAMPLES OF WHICH-PATH EXPERIMENTS

We consider at first the scheme suggested in [6] and sketched in Fig. 1. In the absence of any photon emission [see Fig. 1(a)] the atomic state immediately behind the holes evolves into a sum of two diffracted waves, i.e., $\Psi_I(\mathbf{r}) = \Psi_A(\mathbf{r}) + \Psi_B(\mathbf{r})$, where $\Psi_{A,B}$ are single-aperture wave
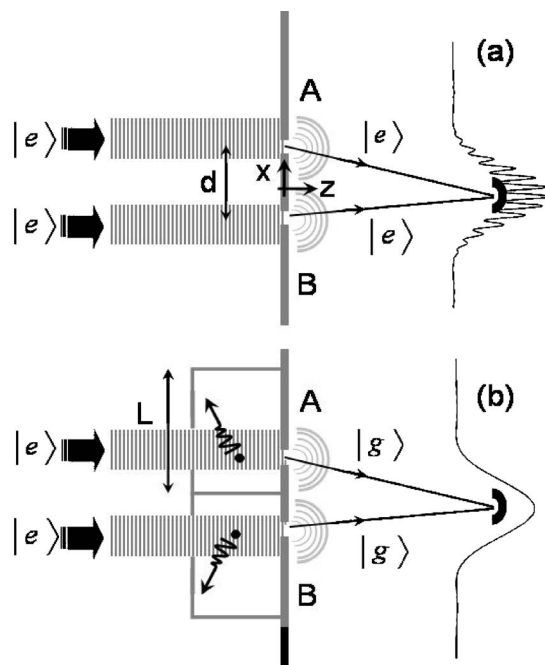

FIG. 1. Sketch of Young's double-hole experiment for one atom as discussed in Ref. [6] and in the present article. The separation between the pinholes A and B is d and the aperture screen is located in the plane $x=0$. (a) Without which-path detectors we observe interference fringes with high contrast. (b) Oppositely, if the two micromaser cavities of length $L \gg d$ are introduced the atom radiates a photon giving the which-path information. As a consequence of Bohr's complementarity, the fringes must disappear.

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functions reducing to two narrow peaks located at $\mathbf{r}_{A,B} = \pm d/2\hat{\mathbf{x}}$ in the aperture plane (d being the distance between the holes). The which-path detectors consist of two micromaser empty cavities, one placed at each aperture A and B of a Young interferometer. Before passing through the double aperture, an atom initially in a long-lived Rydberg exited state $|e\rangle$, will radiate a photon in one or the other cavity and will finish its journey in the ground state $|g\rangle$ [see Fig. 1(b)]. The joint wave function atom-photon is actually an entangled state that carries the position ambiguity of the molecule over to an ambiguity of the photon state:

$$|\Psi_J\rangle \simeq \int d^3\mathbf{r}[\Psi_A(\mathbf{r})|\gamma_A\rangle + \Psi_B(\mathbf{r})|\gamma_B\rangle]|\mathbf{r}\rangle. \qquad (1)$$

Here, $|\gamma_{A,B}\rangle$ are single photon states well localized in one or the other cavity. Such condition can create a distinguishability since the photon states are orthogonal, i.e., $\langle\gamma_B|\gamma_A\rangle = 0$. In the far field of the apertures, the intensity collected on the screen is proportional to $G^{(1)}(\mathbf{r} = Tr[\hat{\rho}|\mathbf{r}\rangle\langle\mathbf{r}|]$ (with the density operator $\hat{\rho} = |\Psi_J\rangle\langle\Psi_J|$), and we deduce

$$G_S^{(1)}(\mathbf{r},t) \propto 1 + \mathcal{V}\cos[p_x d/\hbar + \phi], \qquad (2)$$

where the visibility $\mathcal{V} = |\langle\gamma_B|\gamma_A\rangle|$ and phase shift $\phi = \arg(\langle\gamma_B|\gamma_A\rangle)$ are identically equal to zero and where $p_x = \hat{\mathbf{x}}\cdot\hat{\mathbf{r}}h/\lambda_{dB}$ is the transverse atomic momentum for a particle with de Broglie wavelength λ_{dB}. There is clearly a one-to-one relation between $G_S^{(1)}(\mathbf{r},t)$ and the momentum distribution $P(p_x)$ of the atom in the aperture plane, and we will accept the generality of this relationship in the following.

It is remarkable, however, that equations similar to Eqs. (1) and (2) can be written in the case of an atom spontaneously emitting a photon while it is still in the vicinity of the double pinhole. Supposing the nonrelativistic approximation and that the de-excitation occurs sufficiently fast behind the pinhole, one obtains in this Heisenberg-like which-path experiment $\langle\gamma_B|\gamma_A\rangle = F_0(k_\gamma d)$ [24–26], which tends to zero for photon wavelength $\lambda_\gamma = 2\pi/k_\gamma$ much smaller than d. This is indeed in agreement with Heisenberg's back-action argument. We justify the recoil mechanism at work during the spontaneous emission process by expanding the photon states in the wave-vector/polarization-vector basis:

$$|\gamma_{A,B}\rangle = \sum_{\mathbf{k},\epsilon}\gamma_{\mathbf{k},\epsilon}^{A,B}|\mathbf{k},\epsilon\rangle = \sum_{\mathbf{k},\epsilon}\gamma_{\mathbf{k},\epsilon}^{(0)}e^{-i\mathbf{k}\cdot\mathbf{r}_{A,B}}|\mathbf{k},\epsilon\rangle, \qquad (3)$$

where $\gamma_{\mathbf{k},\epsilon}^{(0)}$ is the photon amplitude for a hole centered at $x=y=z=0$ [27]. In this context, Eq. (1) becomes

$$|\Psi_J\rangle \simeq \int d^3\mathbf{r}\sum_{\mathbf{k},\epsilon}[\Psi_A(\mathbf{r},t)\gamma_{\mathbf{k},\epsilon}^A + \Psi_B(\mathbf{r},t)\gamma_{\mathbf{k},\epsilon}^B]|\mathbf{k},\epsilon,\mathbf{r}\rangle, \quad (4)$$

and the atomic intensity recorded on the screen is consequently

$$G_H^{(1)}(\mathbf{r}) \propto P_H(p_x) = \sum_{\mathbf{k},\epsilon}P(p_x,\mathbf{k},\epsilon), \qquad (5)$$

with

$$P(p_x,\mathbf{k},\epsilon) = |\widetilde{\Psi}^0(p_x + \hbar k_x)|^2 \cdot |\gamma_{\mathbf{k},\epsilon}^{(0)}|^2[1 + \cos(p_x d/\hbar + \mathbf{k}\cdot\mathbf{d})], \qquad (6)$$

and where $\widetilde{\Psi}^0(p_x)$ is the Fourier transform of the single-aperture wave function $\Psi^0(\mathbf{r})$ centered at the origin. The tiny broadening of the single-aperture wave function is not fundamental, and we can make the approximation $\widetilde{\Psi}^0(p_x+\hbar k_x) \simeq \widetilde{\Psi}^0(p_x)$ valid for narrow apertures.

We can see that the momentum transferred to the photon by the atom or molecule affects the coherence of the recorded signal. This is clearly visible from the fact that $G_H^{(1)}$ is a sum of patterns with unit visibility shifted by an amount $\phi_{\mathbf{k}} = \mathbf{k}\cdot\mathbf{d} = k_x d$. Each individual pattern is unable in itself to erase the fringes, but the sum of all these patterns can do it. Indeed, from the values of the coefficients $|\gamma_{\mathbf{k},\epsilon}^{(0)}|^2$ [27–29], we deduce that the uncertainty δk_x on the x component of the photon wave vector is sufficient to account for the disappearance of fringes (this is effectively true because $\delta k_x \sim 2\pi/\lambda_\gamma \gg 2\pi/d$ and $\delta\phi_{\mathbf{k}} \gg 1$). The detailed calculation is straightforward. The coefficients

$$|\gamma_{\mathbf{k},\epsilon}^{(0)}|^2 \propto \frac{|\boldsymbol{\mu}_{ge}\cdot\boldsymbol{\epsilon}|^2}{(\omega - \omega_\gamma)^2 + \Gamma^2/4}, \qquad (7)$$

are obtained for a two-level atom with life time Γ^{-1} and transition energy $\hbar\omega_\gamma$. After summing on photon polarization ϵ, one deduces

$$\sum_\epsilon |\gamma_{\mathbf{k},\epsilon}^{(0)}|^2 \propto \frac{(|\boldsymbol{\mu}_{ge}|^2 - |\boldsymbol{\mu}_{ge}\cdot\mathbf{k}|^2/k^2)}{(\omega - \omega_\gamma)^2 + \Gamma^2/4}. \qquad (8)$$

A numerical calculation of Eq. (5) requires evaluation of the sum $\mathcal{F}(k_\gamma d) = \Sigma_{\mathbf{k}}\frac{(|\boldsymbol{\mu}_{ge}|^2 - |\boldsymbol{\mu}_{ge}\cdot\mathbf{k}|^2/k^2)}{(\omega - \omega_\gamma)^2 + \Gamma^2/4}e^{i\mathbf{k}\cdot\mathbf{d}}$. One obtains [27]

$$\mathcal{F}(k_\gamma d) = \left[\sum_{\mathbf{k}}\frac{(|\boldsymbol{\mu}_{ge}|^2 - |\boldsymbol{\mu}_{ge}\cdot\mathbf{k}|^2/k^2)}{(\omega - \omega_\gamma)^2 + \Gamma^2/4}\right]F_0(k_\gamma d) \qquad (9)$$

and

$$G_H^{(1)}(\mathbf{r},t) \propto 1 + F_0(k_\gamma d)\cos[p_x d/\hbar], \qquad (10)$$

where $F_0(k_\gamma d)$ is calculated in [30–33]. In particular, for an isotropic distribution of transition dipoles, we have

$$\langle F_0(k_\gamma d)\rangle = \sin(k_\gamma d)/(k_\gamma d)e^{-\Gamma d/(2c)} \simeq \sin(k_\gamma d)/(k_\gamma d), \qquad (11)$$

which indeed tends to zero for $\lambda_\gamma \ll d$. This result is in agreement with the complementarity principle since the fringe visibility depends on the amount of information that is, in principle, available to an outside observer. It is consistent with Heisenberg's back-action argument, since those photons that provide a better path-information also impart a stronger recoil. Finally, it is derived by decoherence theory, which means it is based on the entanglement between the atom and the emitted photon. Equivalently, we can write

$$P_H(p_x) = \sum_{\mathbf{k},\epsilon} |\gamma_{\mathbf{k},\epsilon}^{(0)}|^2 P_0(p_x + \hbar k_x), \tag{12}$$

with $P_0(p_x) = |\tilde{\Psi}_I(p_x)|^2 = |\tilde{\Psi}_A(p_x) + \tilde{\Psi}_B(p_x)|^2$. This is equivalent to Eq. (5) and shows that we can define a momentum transfer having the effect of a correlation function. The problem can be then analyzed semiclassically only by using intuitive arguments based on Heisenberg's relation.

At this point it is relevant to repeat that the transfer of momentum does not disturb the single-aperture pattern, which is not broadened significantly! This is a central issue here. Indeed, in both the Heisenberg and the Scully *et al.* examples, the spatial wave functions $\Psi_{A,B}(\mathbf{r},t)$ associated with the center of mass are not affected since they factorize from the photon states. However, our treatment of the Heisenberg which-path experiment considers explicitly the momentum transfer. This prompts the question as to whether there is or there is not a recoil mechanism in the experiment proposed by Scully *et al.*

One should now logically observe that the momentum analysis of Heisenberg's experiment can be extended to the proposal made in [6]. The photon states in the micromaser can indeed be expanded as in Eq. (3) but with different coefficients $\gamma_{\mathbf{k},\epsilon}^{A,B}$, taking into account the specifical structure of the photon field confined by the cavity walls. In the micromaser setup, we consider the electric field being oriented along x and ideally constant inside of the cavity of width L [6,19]. In order to prohibit overlap between the two cavities, we suppose additionally that they are centered on $x = \pm L/2$. After straightforward calculations, we deduce consequently

$$\gamma_{\mathbf{k},\epsilon}^{A,B} \propto \mathrm{sinc}\left(\frac{k_x L}{2}\right) e^{\pm i k_x L/2} = \gamma_{k_x}^{(0)} e^{\pm i k_x L/2}, \tag{13}$$

and we write $G_S^{(1)}(\mathbf{r}) \propto P_S(p_x)$ with

$$P_S(p_x) = \sum_{k_x} |\gamma_{k_x}^{(0)}|^2 [1 + \cos(p_x d/\hbar + k_x L)] |\tilde{\Psi}_0(p_x)|^2. \tag{14}$$

Again, we have a sum of oscillation terms with unit visibility as in Eq. (5), but now with a phase shift $\phi_{\mathbf{k}} = k_x L$ instead of $k_x d$. Considering this difference we deduce that in the Scully *et al.* scheme one only needs a variation of the wave vector in the interval $\delta k_x \sim 2\pi/L \ll 2\pi/d$ in order to destroy the fringes. Such variation is made possible because the typical dispersion δk_x in the Fourier space of the photon is $\sim 2\pi/L$. The result can be rigorously proven by using the Wiener-Khintchine theorem. Indeed, the total visibility of $G_S^{(1)}(\mathbf{r})$ is equivalently defined by $\mathcal{V} = |\int |\gamma_{k_x}^0|^2 e^{ik_x L} dk_x|$. From Wiener-Khintchine's theorem, we deduce $\int |\gamma_{k_x}^0|^2 e^{ik_x L} dk_x = \int dx f(x+L) f^*(x) = 0 = \mathcal{V}$ (autocorrelation function), where $f(x) = \mathcal{F}^{-1}[\gamma_{k_x}^0]$ is the unit rectangular function of width L centered on the origin $x = 0$.

It is then not true to say that the photon momentum is not involved in a recoil mechanism since it is only by summing all the oscillating contribution in Eq. (14) that we can account for the decoherence effect.

The comparison becomes more evident if we work in the limit of very narrow apertures neglecting the modification of the single hole diffraction pattern:

$$P_S(p_x) \simeq \sum_{\mathbf{k},\epsilon} |\gamma_{k_x}^{(0)}|^2 P_0(p_x + \hbar k_x L/d). \tag{15}$$

The approximation $|\tilde{\Psi}_0(p_x + \hbar k_x L/d)|^2 \simeq |\tilde{\Psi}_0(p_x)|^2$ is here justified since $\delta k_x L/d \sim 1/L \cdot L/d = 1/d$ and since we work in the limit $\delta p_x^{\text{single hole}} \simeq h/a \gg h/d$ for the single hole of width a. As in Eq. (12), we find a correlation function, but here the result is not intuitive because we need an effective momentum transfer

$$\hbar k_x L/d = \hbar \phi_{\mathbf{k},\epsilon}/d \tag{16}$$

instead of $\hbar k_x$. Since $\hbar \phi_{\mathbf{k},\epsilon}/d \gg \hbar k_x$ this proves that the momentum transferred is much higher that the intuitive and semiclassical expectation $\sim h/L$. This analysis, however, does not constitute a disproof but actually a confirmation of Heisenberg's mechanism in terms of recoil transfer because the effective momentum is typically $\sim h/d$. Both the mathematical treatment leading to Eqs. (12) and (15) are based on a purely quantum analysis of momentum transfer. In the case of Eq. (12), the calculations confirm the intuitive semiclassical reasoning. However, in the case of Eq. (15), only a quantum treatment can justify the result when the semiclassical analogy fails.

III. DISCUSSION: WHICH-PATH INFORMATION AND QUANTUM ERASER

A. Which-path information and momentum transfer

The precedent result, which can seem rather surprising, is in fact very general and not limited to the particular examples of Eqs. (12) and (15). Consider the general which-path state given by Eq. (1), but where the detector states are not necessarily associated with photons and can be expanded in a arbitrary orthonormal basis $|\xi\rangle$ as $|\gamma_{A,B}\rangle = \Sigma_\xi \gamma_\xi^{A,B} |\xi\rangle$. An analysis similar to the previous one leads to

$$P(p_x) = \sum_\xi (|\gamma_\xi^A|^2 + |\gamma_\xi^B|^2) \left[1 + \mathcal{V}_\xi \cdot \cos\left(\frac{p_x d}{\hbar} + \phi_\xi\right)\right] |\tilde{\Psi}_0(p_x)|^2, \tag{17}$$

where $\phi_\xi = \arg \gamma_\xi^A - \arg \gamma_\xi^B$ and where $\mathcal{V}_\xi$ is the fringe visibility given by

$$\mathcal{V}_\xi = \frac{2|\gamma_\xi^A| \cdot |\gamma_\xi^B|}{(|\gamma_\xi^A|^2 + |\gamma_\xi^B|^2)}. \tag{18}$$

From the discussion of the Scully *et al.* and Heisenberg-like examples, we conclude that if (i) $\mathcal{V}_\xi$ equals unity for those terms of the sum Eq. (17) for which $|\gamma_\xi^A|^2 + |\gamma_\xi^B|^2$ has significant values, and if (ii) in the mean time the phase ϕ_ξ changes significantly in an interval $\delta\phi_\xi \sim 1$, then decoherence will occur and a momentum transfer can be invoked in the basis ξ. To be more precise and still in analogy with Eqs. (12) and (15), we can write in the narrow-aperture limit

$$P(p_x) \simeq \sum_\xi (|\gamma_\xi^A|^2 + |\gamma_\xi^B|^2)[(1-\mathcal{V}_\xi) \cdot |\tilde{\Psi}_0(p_x)|^2$$

$$+ \mathcal{V}_\xi P_0(p_x + \hbar \, \phi_\xi/d)] \tag{19}$$

because we suppose $\tilde{\Psi}_0(p_x + \hbar \, \phi_\xi/d) \simeq \tilde{\Psi}_0(p_x)$. The regime $\mathcal{V}_\xi = 1$ was the one considered in both Eqs. (12) and (15). It leads to a simple interpretation in terms of a correlation function with a momentum transfer

$$p_\xi = \hbar \, \phi_\xi/d. \tag{20}$$

ϕ_ξ and then p_ξ are clearly experimentally defined by recording the elementary interference pattern corresponding to ξ. In this context,

$$\delta p_\xi d \simeq \hbar \, \delta\phi_\xi \simeq \hbar \tag{21}$$

plays the role of Heisenberg's relation. Yet, in general $\mathcal{V}_\xi \neq 1$, and this means that there are some terms in Eq. (19) that are proportional to $1 - \mathcal{V}_\xi$ and that cannot be interpreted in terms of correlation functions and momentum transfers in the basis ξ. At the extreme, $\mathcal{V}_\xi = 0$, and the momentum transfer discussion is completely irrelevant in the basis ξ.

However, it is not difficult using the arbitrariness in the basis choice to find a representation of the problem in which $V_\xi = 1$ and in which momentum transfer is clearly defined. This is true assuming the ideal which-path experiment

$$\langle \gamma_A | \gamma_B \rangle = 0. \tag{22}$$

Indeed, this orthogonality relation means that we can use the two states $|\gamma_{A,B}\rangle$ as a relevant basis for analyzing the problem. In this basis, $V_{A,B} = 0$. However, the equivalent choice

$$|\gamma_\pm\rangle = (|\gamma_B\rangle \pm |\gamma_A\rangle)/\sqrt{2} \tag{23}$$

is completely pertinent as well. In such basis, $V_\pm = 1$ and Eq. (20) leads to

$$p_+ = 0, \quad p_- = \hbar \, \phi_-/d = h/2d. \tag{24}$$

The momentum p_- accounts for the loss of coherence in the which-path experiment and we can indeed write

$$P(p_x) = \sum_\pm P_0(p_x + \hbar \, \phi_\pm/d). \tag{25}$$

This means that we can always interpret an ideal which-path experiment in terms of momentum transfer and correlation function in at least one basis ξ.

B. Erasing knowledge with momentum

The precedent discussion focussed on the concepts of momentum transfer and which-path information in quantum mechanics. However, we must add here a few further remarks concerning the quantum eraser experiment since the existence of a momentum transfer seems to be in contradiction with the spirit of this experiment. Quantum erasure, which was initially proposed in [6,34] and experimentally realized in [8,9,35–38], has been recently discussed in, e.g., [39,40]. Such experiments are usually thought of as "a way around the uncertainty principle" [6,29,40], and one says that in

classical physics, "this question would never come up" [6]. In order to clarify this point, one has first to recall that the philosophy of the quantum eraser proposal is based on the possibility to rewrite an entangled state as

$$\Psi_A(\mathbf{r},t)|\gamma_A\rangle + \Psi_B(\mathbf{r},t)|\gamma_B\rangle \tag{26}$$

in the equivalent form

$$\Psi_+(\mathbf{r},t)|\gamma_+\rangle + \Psi_+(\mathbf{r},t)|\gamma_+\rangle, \tag{27}$$

with

$$\Psi_\pm(\mathbf{r},t) = [\Psi_A(\mathbf{r},t) \pm \Psi_B(\mathbf{r},t)]/\sqrt{2}, \tag{28}$$

and where $|\gamma_\pm\rangle$ are defined by Eq. (23).

Clearly, if one is able to project the photon state in the orthogonal basis $|\gamma_\pm\rangle$, one would be able to retrieve interference fringes (or even anti-interference fringes if the projection is made on $|\gamma_-\rangle$). Since this projection can be delayed, the atom fringes can be rebuilt (using coincidence measurement techniques) even after that the atom reaches the screen [38].

From semiclassical physics, this behavior is prohibited [6,41]. The most famous quantum eraser proposal was given by Scully *et al.* in [6] and is based on the micromaser two-cavity setup described in Sec. II.

Less known, however, the fact that quantum eraser could be, in principle, realized with the "Heisenberg" setup described in Sec. II and based on spontaneously emitted photons [42]. The principle of this proposal is sketched in Fig. 2(a). Detection of single photons emitted in the close vicinity of A and B is ensured by microscope objective(s) able to resolve clearly the apertures region. The photon state coming from A is oriented through an optical fiber to the first entrance of a 50-50 beam splitter (BS). The second photon state follows a similar path to the second entrance of the beam splitter. The recombination of the photon states at the exit outports D_1 and D_2 erases the which-path information since we have no way to know where the photon was coming from. One could thus retrieve fringes or antifringes by changing the phase χ between the photon paths and by detecting in coincidence the molecule arrival on a screen in M (this experiment is very close to the one realized in [38]). If the Heisenberg experiment described in Sec. II [Eqs. (4)–(12)] was effectively based on (semi-)classical momentum kicks associated with the emission of a particle, the quantum eraser discussed here could not work! This shows that the semiclassical picture is not adapted even in "canonical" experiments like those described by Einstein [1] or Feynman [23].

However, the important word in any quantum eraser experiment is *coincidence* (i.e., correlation photon-atom). Indeed, if it were possible to erase the fringes without correlating the detections of the photon and atom, then it could be possible to realize faster-than-light communication [43]. The argument against this possibility is the same as the one used by Bell in the context of Einstein Podolsky Rosen experiments [44]. If we do not compare the data coming from the two exits $D_{1,2}$ with the molecule arrival, we must find the initial molecule profile without fringes [see Fig. 2(b)]. This is particularly clear from the fact that we have [6]
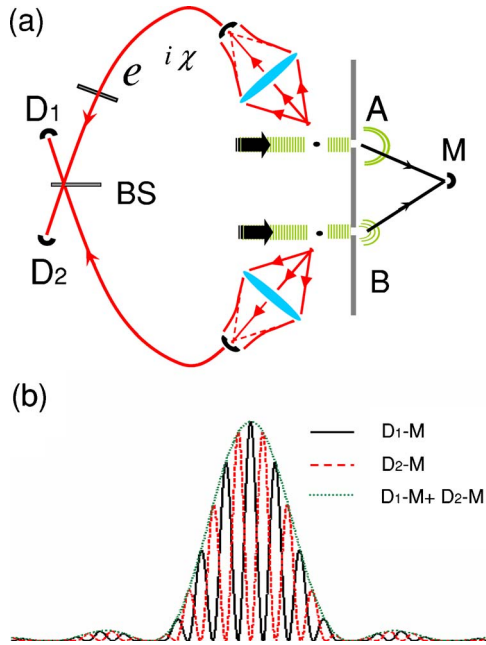
FIG. 2. (Color online) (a) Quantum eraser experiment using two microscopes for recording a photon spontaneously emitted by the atom close to the aperture A or B. The photon is oriented through optical fibers and a 50-50 beam splitter (BS) before reaching one of the two detectors D_1 or D_2. The coincidence measurement atom-photon can be used to rebuild molecular fringes. (b) Depending on the phase shift χ one can observe fringes or antifringes. If the correlation D_1-M gives us the fringe pattern $|\Psi_+|^2 = \frac{1}{2}|\Psi_A + \Psi_B|^2$ (black curve), then the correlation D_2-M gives $|\Psi_-|^2 = \frac{1}{2}|\Psi_A - \Psi_B|^2$ (red dashed curve). The sum of both patterns gives us the initial distribution $|\Psi_A|^2 + |\Psi_B|^2$ (green dotted envelope).

$$|\Psi_+(\mathbf{r},t)|^2 + |\Psi_-(\mathbf{r},t)|^2 = |\Psi_A(\mathbf{r},t)|^2 + |\Psi_B(\mathbf{r},t)|^2. \quad (29)$$

It must be remarked that this rebuttal of faster-than-light communication gives us a solution for the paradox concerning the coexistence of momentum transfer with quantum erasure. Indeed, Eq. (29) is tantamount to arguments based on decoherence theory, which say that the entanglement with the environment is responsible for the loss of coherence observed during a which-path experiment [we must sum the two quantum eraser patterns $|\Psi_+(\mathbf{r},t)|^2$ and $|\Psi_-(\mathbf{r},t)|^2$ in order to obtain the complete pattern given by $|\Psi_A(\mathbf{r},t)|^2 + |\Psi_B(\mathbf{r},t)|^2$, e.g., Eq. (12) or (15)].

Now we should remember that the discussion of momentum transfer in Sec. II tells us that we have necessarily to sum all the individual patterns associated with different $|\mathbf{k}\rangle$ in order to obtain the full diffraction pattern without fringes. The present discussion is thus identical to the previous one but is done in the base $|\gamma_\pm\rangle$ instead of $|\mathbf{k}\rangle$. This is completely consistent with Sec. III A discussing the observer freedom on the choice of the detector basis $|\xi\rangle$.

In conclusion, since all the arguments presented in Sec. II are based only on decoherence theory, it means that reasoning based on quantum erasers is rigorously equivalent to that based on momentum transfer presented in this article. The quantum eraser corresponds thus precisely to procedures of

projection for measuring the momentum transfer. This is visible from Eqs. (23)–(25), which show indeed that the phase shift between the two quantum eraser patterns $|\Psi_\pm(\mathbf{r},t)|^2$ corresponds to momentum transfers of $p_+ = 0$ and $p_- = h/2d$ respectively (e.g., to phase shifts of $\phi_- = 0$ and $\phi_+ = \pi$, respectively).

It is well known that with a lens focused on the two-aperture plane, one can, in principle, realize a which-path experiment. The experimentalist has the choice to position the photon detectors at any distance behind the lens. If the detectors are located in the image plane of the lens, one can distinguish the path followed by the atom in the interferometer. However, if the photon detectors are positioned in the back focal plane of the lens (i.e., Fourier plane), then the observer can record the photon momentum distribution in the base $|\mathbf{k}\rangle$. As discussed in Sec. II [i.e., Eq. (6)] the joint distribution of probability photon-atom reveals atom fringes with a phase shift $k_x d$, which corresponds to a transverse momentum transfer $p_\mathbf{k} = \hbar k_x$. From this discussion, it is thus in principle possible to realize a quantum eraser experiment by correlating photon detection in the back focal plane of the lens with the detection of an atom on the screen. This shows once again that the quantum eraser is indeed connected to the definition of momentum transfer used in this article. The same procedure using a lens could be (in principle) realized with the setup described in [6]. Such an experiment would correspond to a measure of the momentum distribution $p_\mathbf{k} = \hbar k_x L/d$ [see Eqs. (14)–(16)]. However, the presence of the cavity walls constraints in practice such observations with a lens [45]. To realize a quantum eraser it is thus easier to consider the basis $|\gamma_\pm\rangle$ instead of the basis $|\mathbf{k}\rangle$.

The apparent contradiction between the existence of quantum erasers on one hand and the existence of momentum transfer on the other hand results from the (implicit) use of a classical definition of momentum. Once again, with such a definition, the quantum eraser would not be possible. Here, our definition of momentum transfer, e.g.,

$$p_\xi = \hbar \, \phi_\xi/d, \quad (30)$$

does not have this handicap. This definition is indeed purely quantum and represents the most direct generalization of the usual definition used for the case of the Heisenberg-like experiment in Eqs. (4)–(12). It can be applied to any double-aperture which-path or quantum eraser experiments (like the ones described in [6]).

IV. SUMMARY

Our discussion of wave-particle duality and of momentum transfer is directly based on the very basis of quantum mechanics, which involves only observable quantities associated with states $|\xi\rangle$. This means that the momentum transfer is always in principle experimentally accessible to the observer. This clearly shows the pertinence of our definition in a physical discussion concerning the meaning of momentum transfer. Our analysis shows additionally that momentum

transfer and quantum eraser are not two independent concepts and that they are in fact two formulations of the same decoherence argumentation applied two double-aperture experiments. Clearly, most of the formalism used here was already known [33]. However, the analysis in terms of momentum of experiments like the one proposed in [6] has to our knowledge never been done, consequently missing some fundamental subtleties of the interpretation.

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[45] This handicap could be in principle overcome by considering a hypothetical particle crossing the two cavities (in the transversal direction x) without interacting with the walls. If such particle absorbs the photon one could compare its momentum before and after the interaction and thus rebuild the atom interference pattern (e.g., realize a quantum eraser in the basis $|\mathbf{k}\rangle$) by correlating the information obtained with the recording of the atoms.