Effects of spatial transverse correlations in second-harmonic generation

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Second-harmonic generation is studied for the case where the fundamental field is light produced in a spontaneous parametric down-conversion process. We show that second-harmonic generation is sensitive to the transverse correlations between signal and idler fields. In particular, when the fundamental is prepared in a state exhibitting spatial antibunching, the second-harmonic intensity may be zero, independent of the intensity of the fundamental field.

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I. INTRODUCTION

Second-harmonic generation (SHG) has been widely investigated in the context of nonlinear optics since its first observation [1]. Quantum effects in SHG have also been investigated [2,3], but in most cases not taking into account the transverse properties of the fields involved. On the one hand, in the classical optics framework, the transverse properties of the fields in SHG have been investigated and it has been demonstrated the conditions for image transfer [4] and angular spectrum transfer [5]. On the other hand, there has been little research interest on certain aspects of SGH processes concerning the quantum spatial (transverse) properties of the fundamental on the second-harmonic fields [6,7].

Recently, quantum spatial properties of light have received a great deal of attention from the quantum optics community, since promising applications for so-called quantum images have been proposed. Spontaneous parametric down-conversion (SPDC), has always been in the heart of this research. It has been used for demonstrating the existence of quantum spatial correlations [8–10] and for understanding how to prepare states presenting this kind of entanglement [11]. When the quantum states are properly prepared, it is possible to observe the de Broglie wavelength of the twin photon pair [12], which has been considered for applications to so-called quantum lithography [13,14]. Though the quantum character of the spatial correlations between twin photons from the down-conversion has been a subject of debate [15–17], it has been proven that they can violate classical inequalities [18–20], therefore qualifying them as quantum correlations.

Among the possibilities of preparing and observing the quantum spatial correlations between twin photons from SPDC, states presenting spatial antibunching can be prepared in at least two different configurations [18,19]. In one of these configurations [19], a Dove prism is inserted in the path of one of the two propagating twin beams before they are combined in a single beam. Though extremely simple, this operation is crucial to make the combined beam homogeneous in fourth order. Further manipulation of the pump beam allows for the preparation of a spatial antibunched state in the combined beam. In the present work, we will show that the insertion of the Dove prism in the path of one of the twin beams has further physical consequences.

We calculate the quantum state of the second-harmonic field produced in a SHG process, where the fundamental field is composed of twin beams originating from parametric down-conversion. We take into account the angular spectrum of all fields involved, the fundamental and the second harmonic. We compare the results for two cases: when the combined twin beams are directly used as the fundamental field or when one of the twin beams is manipulated using the Dove prism before being combined with the other beam to form the fundamental field. It is demonstrated that the SHG intensity is different for the two cases. In particular, when the fundamental field is prepared in an antibunched state, the SHG intensity may be zero, independently of the fundamental field intensity. This shows that transverse quantum correlations are crucial in the SHG process.

From the experimental point of view, the main difficulty for performing this kind of experiment is the low SHG efficiency, which combined with the low intensity of the twin beams, reduces the probable signal-to-noise ratio to the dark count level of the currently available photon counters. Periodically poled nonlinear crystals are good candidates for such experiments, as they achieve higher conversion efficiencies for the up- and down-conversion processes. The use of twin beams as fundamental fields in the SHG process has already been demonstrated [21]. Therefore, the implementation of an experiment for testing our theoretical results is possible with current technology.

II. THEORY

Let us consider the situation sketched in Fig. 1. A nonlinear crystal is placed at the origin of a Cartesian coordinate system. A monochromatic and linearly polarized pump beam interacts with the crystal giving rise to spontaneous parametric down-conversion. The pump field can be described by the scalar function $E(\mathbf{r},t)=W_b(\boldsymbol{\rho},z)e^{-i\omega_0 t}$, where $W_b(\boldsymbol{\rho},z)$ represents the beam profile at the position $\boldsymbol{\rho}$ in plane z [22]. In the

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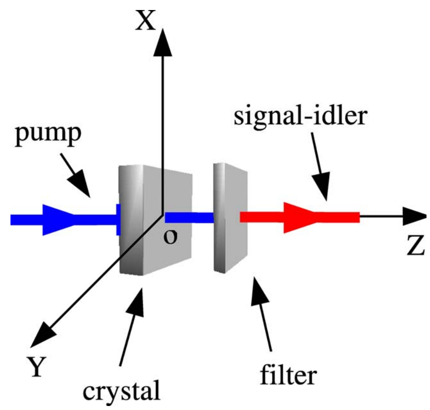

FIG. 1. (Color online) Signal-idler beams filtered from the pump in degenerate SPDC.

paraxial and thin crystal approximations, it is possible to show that the quantum state of the signal and idler down-converted fields, for the degenerate case, is given by [11,23]

$$|\psi_I\rangle = |\text{vac}\rangle + \zeta \int d^2q' \int d^2q'' v(\mathbf{q}' + \mathbf{q}'';0) \times \left| \mathbf{q}' + \frac{k_0}{2}\left(1 - \frac{2\mathbf{q}'^2}{\mathbf{k}_0^2}\right)\hat{z} s_0' \right\rangle_s \times \left| \mathbf{q}'' + \frac{k_0}{2}\left(1 - \frac{2\mathbf{q}''^2}{\mathbf{k}_0^2}\right)\hat{z} s_0'' \right\rangle_i |\text{vac}\rangle_0 + O(\zeta^2), \quad (3)$$

where

$$\zeta \equiv -\frac{iL_z\epsilon_0 t}{2\hbar(2\pi)^3} l^*\left(\frac{\omega_0}{2}, s_{10}'\right) l^*\left(\frac{\omega_0}{2}, s_{10}''\right) \tilde{\chi}_{ijk}^{(2)}\left(\omega_0; \frac{\omega_0}{2}, \frac{\omega_0}{2}\right) \times [\boldsymbol{\epsilon}(\mathbf{k}_0, s_0)]_i [\boldsymbol{\epsilon}^*(\mathbf{k}_0', s_0')]_j [\boldsymbol{\epsilon}^*(\mathbf{k}_0'', s_0'')]_k. \quad (4)$$

In this expression, L_z is the length of the crystal in the z direction, t is the interaction time, $\boldsymbol{\epsilon}(\mathbf{k},s)$ is a polarization vector, $l(\omega,s)=i[\hbar\omega(\mathbf{k},s)/2\epsilon_0 n^2(\mathbf{k},s)]^{1/2}$, and $\tilde{\chi}_{ijk}^{(2)}(\omega;\omega',\omega'') \equiv \int_{-\infty}^{\infty}dt' \int_{-\infty}^{\infty}dt'' \chi_{ijk}^{(2)}(t',t'') e^{i(\omega' t' + \omega'' t'')}$, where $\chi_{ijk}^{(2)}$ are the components of the nonlinear susceptibility tensor.

This state is a perturbative series in the parameter ζ. The first term is the vacuum state and the second is an entangled state with two photons, with the same frequency $\omega_0/2$, propagating along the z direction and having wave vectors $\mathbf{k}'=\mathbf{q}'+(k_0/2)(1-2\mathbf{q}'^2/\mathbf{k}_0^2)\hat{z}$ and $\mathbf{k}''=\mathbf{q}''+(k_0/2)(1-2\mathbf{q}''^2/\mathbf{k}_0^2)\hat{z}$. Here $\mathbf{q}'$ and $\mathbf{q}''$ are transverse wave vectors, $\hat{z}$ is the unit vector in the z direction, and $|\text{vac}\rangle_0$ denotes the unoccupied field modes. The signal and idler beams are also linearly polarized with polarizations s_0' and s_0'', respectively. In this expression, $v(\mathbf{q}'+\mathbf{q}'';0)$ is the angular spectrum [22] of the pump beam calculated in $\mathbf{q}'+\mathbf{q}''$ at the plane $z=0$. As usual, terms of the order of $O(\zeta^2)$, corresponding to states with more than two photons, are neglected.

As expected, the intensity of the idler (it is the same for the signal) beam taken individually, at position $\mathbf{r}_1=\boldsymbol{\rho}_1+z_1\hat{z}$, with $\boldsymbol{\rho}_1$ being a vector perpendicular to $\hat{z}$, is given by

$$\Gamma^{(1,1)}(\mathbf{r}_1;\mathbf{r}_1) \equiv \langle \hat{I}(\mathbf{r}_1,t)\rangle_\psi = |\zeta|^2 \frac{k_0^2}{4\pi\epsilon\nu z_1^2} P_b(0), \quad (5)$$

where

$$\hat{I}(\mathbf{r},t) \equiv \hat{\mathbf{V}}^\dagger(\mathbf{r},t) \cdot \hat{\mathbf{V}}(\mathbf{r},t) \quad (6)$$

is the intensity operator,

$$\hat{\mathbf{V}}(\mathbf{r},t) \equiv \frac{1}{(2\pi)^{3/2}} \sum_s \int d^3k\, \hat{a}(\mathbf{k},s) e^{i(\mathbf{k}\cdot\mathbf{r}-\omega t)} \boldsymbol{\epsilon}(\mathbf{k},s) \quad (7)$$

is the photoeletric operator, and

$$P_b(z) = \frac{1}{2}\epsilon\nu \int d^2\rho |W_b(\boldsymbol{\rho},z)|^2 \quad (8)$$

is the power of the pump beam in the plane z. Here ϵ and ν are the electric permittivity constant and the speed of light inside the crystal, respectively.

The intensity correlations between signal and idler beams are given by the normally and time-ordered fourth-order correlation function $\Gamma^{(2,2)}(\mathbf{r}_1,\mathbf{r}_2;\mathbf{r}_2,\mathbf{r}_1)=\langle \mathcal{T}{:}\hat{I}(\mathbf{r}_1,t_1)\hat{I}(\mathbf{r}_2,t_2){:}\rangle_\psi$ which is proportional to the coincidence counting rates in experiments where signal and idler are detected with single-photon counters. From Eq. (3), we calculate this fourth-order correlation function at signal and idler detector positions $\mathbf{r}_1=\boldsymbol{\rho}_1+z_1\hat{z}$ and $\mathbf{r}_2=\boldsymbol{\rho}_2+z_2\hat{z}$, and find

$$\Gamma^{(2,2)}(\mathbf{r}_1,\mathbf{r}_2;\mathbf{r}_2,\mathbf{r}_1) = \Gamma^{(2,2)}(\boldsymbol{\rho},z) = |\zeta|^2 \frac{k_0^2}{2\epsilon\nu(z_1+z_2)^2} I_b(\boldsymbol{\rho},z), \quad (9)$$

where

$$z = \frac{2z_1z_2}{z_1+z_2},$$

$$\boldsymbol{\rho} = \frac{z_2\boldsymbol{\rho}_1 + z_1\boldsymbol{\rho}_2}{z_1+z_2} \quad (10)$$

are positions related to $(\boldsymbol{\rho}_1,z_1)$ and $(\boldsymbol{\rho}_2,z_2)$, and

$$I_b(\boldsymbol{\rho},z) = \frac{1}{2}\epsilon\nu |W_b(\boldsymbol{\rho},z)|^2 \quad (11)$$

is the intensity of the pump beam at the position $r=\boldsymbol{\rho}+z\,\hat{z}$.

On the one hand, it is seen that the intensity of the signal or the idler beam considered separately is proportional to the power of the pump beam. It does not have any information about the pump beam profile [different beam profiles may give rise to the same value of integral in Eq. (8)]. On the other hand, the fourth-order correlation function is proportional to the intensity of the pump beam at the position $\boldsymbol{\rho}$ in the plane z and depends on the detection positions of both signal and idler fields. According to Ref. [11], this dependence is a consequence of the transfer of the angular spectrum from the pump field to the correlations between signal and idler photons. This provides us with a way of preparing quantum states where the transverse correlations between signal and idler beams are controlled by the shape of the pump beam.

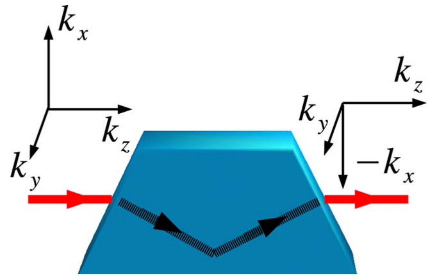

FIG. 2. (Color online) A Dove prism performs the operation $k_x \rightarrow -k_x$ in one of the beams.

In the following we will be interested in spatial transverse properties of light fields, such as spatial photon bunching and antibunching. According to Ref. [18] one light beam presents spatial antibunching if the following Schwarz inequality is violated:

$$\Gamma^{(2,2)}(\mathbf{r};\mathbf{r}+\boldsymbol{\delta}) \leq \Gamma^{(2,2)}(\mathbf{r};\mathbf{r}), \tag{12}$$

with $\boldsymbol{\delta}$ being an arbitrary spatial displacement. However, in order to derive the above inequality, and to associate its violation with the nonclassical character of a light beam, it is necessary to start from a homogeneous field.

Let us consider the special case where signal and idler detector planes are located at the same distance from the crystal. In this case $z_1 = z_2 = Z$, and Eq. (10) reduces to

$$z = Z,$$

$$\rho = \frac{\boldsymbol{\rho}_1 + \boldsymbol{\rho}_2}{2}, \tag{13}$$

leading to

$$\Gamma^{(2,2)}(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2;z) = |\zeta|^2 \frac{k_0^2}{8\epsilon \nu z^2} I_b\left(\frac{\boldsymbol{\rho}_1 + \boldsymbol{\rho}_2}{2}, Z\right). \tag{14}$$

From Eq. (14), it is easy to see that the combined signal-idler field would not be homogeneous in the fourth-order correlation function, as the sum of the coordinates $\boldsymbol{\rho}_1 + \boldsymbol{\rho}_2$ appears in its argument. In Ref. [19], it is shown how this problem can be circumvented by simply using Dove prisms to perform the transformation $\mathbf{q}'' \rightarrow -\mathbf{q}''$ on the tranverse wave vector of one of the twin fields, keeping the other unchanged before combining them, as sketched in Fig. 2.

As a result, the quantum state that describes the fields after such manipulation is equal to Eq. (3) with the transformation

$$\left|\mathbf{q}'' + \frac{k_0}{2}\left(1 - \frac{2\mathbf{q}''^2}{\mathbf{k}_0^2}\right)\hat{z}s_0''\right\rangle_{\mathrm{i}} \rightarrow \left|-\mathbf{q}'' + \frac{k_0}{2}\left(1 - \frac{2\mathbf{q}''^2}{\mathbf{k}_0^2}\right)\hat{z}s_0''\right\rangle_{\mathrm{i}}, \tag{15}$$

and the fourth-order correlation function becomes

$$\Gamma^{(2,2)}(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2;z) = |\zeta|^2 \frac{k_0^2}{8\epsilon \nu z^2} I_b\left(\frac{\boldsymbol{\rho}_1 - \boldsymbol{\rho}_2}{2}, Z\right). \tag{16}$$

Now this function depends only on the difference between the detection coodinates so that if we combine signal and idler beams, the resulting field is now tranverse homogeneous to second and fourth order. Note that the joint probability of registering photodetections of signal and idler photons at the same point in the plane Z is proportional to the intensity of the pump beam at the origin of that plane. In Ref. [19] it is shown how this result can be used to obtain spatial antibunching. The second-order homogeneity comes from the fact that signal and idler fields are mutually incoherent and therefore their superposition will not give rise to interference. In the following, we will analyze the use of the combined signal and idler beams as the fundamental field in the SHG process.

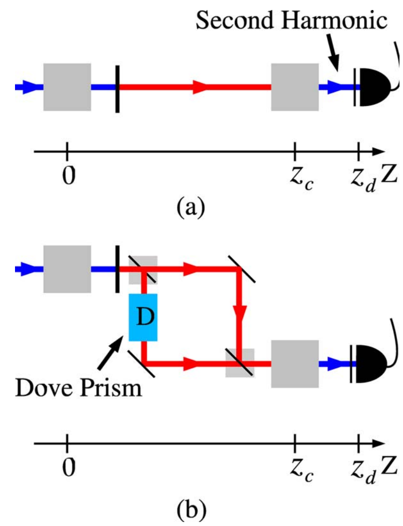

FIG. 3. (Color online) Combined signal-idler beam: (a) free propagation; (b) manipulation by the Dove prism.

A. SHG with SPDC beams

Let us consider now, the situation sketched in Fig. 3. Two identical nonlinear crystals are located at the origin and at position $\mathbf{r}_c = z_c\hat{z}$, respectively, of a Cartesian coordinate system. A monochromatic beam is used to pump the first crystal and the produced signal and idler beams filtered out from the pump are sent to the second crystal. The second-harmonic generation in the second crystal will be analyzed for two cases: (i) when the combined signal-idler beam is directly used as the fundamental and (ii) when the signal (idler) beam is manipulated via the use of Dove prisms before being sent to the second crystal.

Treating the SHG as the Hermitian conjugate process of the degenerate SPDC, we can follow [24–27], and write the second-harmonic quantum state as

$$|\psi(t)\rangle = \exp\left(-\frac{i}{\hbar}\int_0^t dt' \hat{V}_I(\mathbf{r}_c,t')\right)|\psi_F(0)\rangle, \tag{17}$$

where $|\psi_F(0)\rangle$ is the state of the fundamental field, and

$$\int_0^t dt' \hat{V}_I(\mathbf{r}_c,t') = \frac{\epsilon_0 t V}{2(2\pi)^{9/2}} \sum_{s_1'''} \int d^3k_1''' \sum_{s_1''''} \int d^3k_1'''' \times \sum_{s_2} \int d^3k_2 l(\omega_1''',s_1''') l(\omega_1'''',s_1'''') l^*(\omega_2,s_2) \times \tilde{\chi}^{(2)*}_{lmn}(\omega_2;\omega_1''',\omega_1'''') \times \hat{a}(\mathbf{k}_1''',s_1''') \times \hat{a}(\mathbf{k}_1'''',s_1'''') \hat{a}^\dagger(\mathbf{k}_2,s_2) e^{i(\omega_2-\omega_1'''-\omega_1'''')t/2} \times \mathrm{sinc}[(\omega_2-\omega_1'''-\omega_1'''')t/2] e^{-i(\mathbf{k}_2-\mathbf{k}_1'''-\mathbf{k}_1'''')\cdot \mathbf{r}_c} \times \prod_p \mathrm{sinc}[(\mathbf{k}_2-\mathbf{k}_1'''-\mathbf{k}_1'''')_p L_p/2] \times [\epsilon(\mathbf{k}_1''',s_1''')]_n [\epsilon(\mathbf{k}_1'''',s_1'''')]_m [\epsilon^*(\mathbf{k}_2,s_2)]_l + \mathrm{H.c.} \tag{18}$$

Here $\hat{V}_I(\mathbf{r}_c,t')$ is the Hamiltonian operator that describes the interaction at the position $\mathbf{r}_c$ at time t'. V is the quantization volume, t is the interaction time, $\epsilon(\mathbf{k},s)$, $\hat{a}(\mathbf{k},s)$, and $\hat{a}^\dagger(\mathbf{k},s)$ are, respectively, the polarization vector, photon annihilation operator and photon creation operator. Furthermore, $l(\omega,s)$ and $\tilde{\chi}^{(2)}_{lmn}(\omega;\omega',\omega'')$ are defined in the same way as in Eq. (4). The second-harmonic field is labeled by the index 2, and the fundamental by the index 1, where the primes $'''$ and $''''$ stand for the signal and idler fields, respectively.

1. Free signal-idler beam as the fundamental

In this case, the twin beams propagate freely to the second crystal, as sketched in Fig. 3(a). As a result, the state of the fundamental field is given by Eq. (3). If we use that state in Eq. (17), we find

$$|\psi(t)\rangle = \left(\hat{1} - \frac{i}{\hbar}\int_0^t dt' \hat{V}_I(\mathbf{r}_c,t') + \cdots \right)|\psi_F(0)\rangle = |\mathrm{vac}\rangle + \zeta \int d^2q_1' \int d^2q_1'' v(\mathbf{q}_1'+\mathbf{q}_1'';0) \times \left|\mathbf{q}_1' + \frac{k_0}{2}\left(1-\frac{2\mathbf{q}_1'^2}{\mathbf{k}_0^2}\right)\hat{z}s_{10}'\right\rangle_s \times \left|\mathbf{q}_1'' + \frac{k_0}{2}\left(1-\frac{2\mathbf{q}_1''^2}{\mathbf{k}_0^2}\right)\hat{z}s_{10}''\right\rangle_i |\mathrm{vac}\rangle_{\mathrm{sh}} + O(\zeta^2) + |\phi\rangle + \cdots , \tag{19}$$

where

$$|\phi\rangle \equiv -\frac{i}{\hbar}\int_0^t dt' \hat{V}_I(\mathbf{r}_c,t')\left(|\mathrm{vac}\rangle + \zeta\int d^2q_1' \times \int d^2q_1'' v(\mathbf{q}_1'+\mathbf{q}_1'';0)\left|\mathbf{q}_1' + \frac{k_0}{2}\left(1-\frac{2\mathbf{q}_1'^2}{\mathbf{k}_0^2}\right)\hat{z}s_{10}'\right\rangle_s \times \left|\mathbf{q}_1'' + \frac{k_0}{2}\left(1-\frac{2\mathbf{q}_1''^2}{\mathbf{k}_0^2}\right)\hat{z}s_{10}''\right\rangle_i |\mathrm{vac}\rangle_{\mathrm{sh}} + O(\zeta^2)\right). \tag{20}$$

In the state $|\psi(t)\rangle$ are represented the various possible outcomes of the light interaction with the two crystals. We are interested in the component $|\phi\rangle$ which describes the process where two photons (signal and idler), of frequency $\omega_0/2$, are created in the first crystal and annihilated in the second crystal, giving rise to a new photon of frequency ω_0, wave vector $\mathbf{q}_2 + k_0(1-\mathbf{q}_2^2/2\mathbf{k}_0^2)\hat{z}$, and linear polarization s_{20}. This photon is named the second-harmonic photon. If we define the parameter

$$\xi \equiv -\frac{i\pi^3 k_0 \epsilon_0 t}{\hbar(2\pi)^{9/2}} l\left(\frac{\omega_0}{2},s_{10}'\right) l\left(\frac{\omega_0}{2},s_{10}''\right) l^*(\omega_0,s_{20}) \times \tilde{\chi}^{(2)*}_{lmn}\left(\omega_0;\frac{\omega_0}{2},\frac{\omega_0}{2}\right) \times [\epsilon(\mathbf{k}_1',s_{10}')]_n [\epsilon(\mathbf{k}_1'',s_{10}'')]_m [\epsilon^*(\mathbf{k}_2,s_{20})]_l, \tag{21}$$

we can, after lengthy calculations, rewrite the quantum state of the second-harmonic field as

$$|\phi\rangle = \zeta\xi|\mathrm{vac}\rangle \int d^2q_2 v(\mathbf{q}_2;0)\left|\mathbf{q}_2 + k_0\left(1-\frac{\mathbf{q}_2^2}{2\mathbf{k}_0^2}\right)\hat{z}s_{20}\right\rangle_{\mathrm{sh}}. \tag{22}$$

Note the presence, in the state $|\phi\rangle$, of the angular spectrum $v(\mathbf{q}_2;0)$ of the pump beam in the plane $z=0$. This shows that the SHG also transfers the angular spectrum, which is not surprising, if we remember that SHG is the Hermitian conjugate process of degenerate SPDC. If we now calculate the intensity of the second-harmonic field at the position $\mathbf{r}_d = \boldsymbol{\rho}_\mathbf{d} + z_d\hat{z}$, we will find

$$\langle \hat{I}(\boldsymbol{\rho}_\mathbf{d},z_d)\rangle_\phi = |\zeta\xi|^2 \frac{4\pi}{\epsilon\nu} I_b(\boldsymbol{\rho}_d,z_d). \tag{23}$$

We see that the intensity of the second-harmonic field is proportional to the intensity of the pump beam at the position $\boldsymbol{\rho}_d$ in the plane $z=z_d$. This result has the same dependence as the fourth-order correlation function (9), so we can see that the information about the angular spectrum, shared by the signal and idler photons is transferred to the second harmonic photon [5].

2. Manipulated signal-idler beam as the fundamental

Let us suppose that the twin beams are manipulated according to the procedure described in Ref. [19] and sketched in Fig. 3(b). Now the state of the the fundamental field is given by Eq. (3) with the transformation performed in Eq. (15). The interaction between the second crystal and this field is treated in the same way as described in the previous section. As a result, in this case $|\phi\rangle$ is equal to

$$|\phi\rangle = \zeta\xi I|\mathrm{vac}\rangle \int_{|\mathbf{q}_2| \ll |\mathbf{k}_0|} d^2q_2 \left|\mathbf{q}_2 + k_0\left(1-\frac{\mathbf{q}_2^2}{2\mathbf{k}_0^2}\right)\hat{z}s_{20}\right\rangle_{\mathrm{sh}}, \tag{24}$$

where ξ is given by Eq. (21) and

$$I \equiv \frac{2}{\pi A} \exp(-ik_0 z_c) \int d^2q' v(\mathbf{q}';z_c) \mathrm{sinc}\left(\frac{L_z}{4k_0}\mathbf{q}'^2\right). \tag{25}$$

Here $A \equiv L_x L_y$, and $v(\mathbf{q}';z_c)$ is the the propagated angular spectrum in the plane z_c [22]. The condition $|\mathbf{q}_2| \ll |\mathbf{k}_0|$ assures the fulfilment of the paraxial approximation. The integrand in this expression is a product of two values which decay to zero as its arguments increase: the angular spectrum has a spectral width $\Delta \ll k_0$ that satisfies the paraxial approximation, and the cardinal sine has a spectral width Ω which is given by $\Omega = \sqrt{4\pi k_0/L_z}$ and corresponds to the first zero of that function. If the condition $\Delta \ll \Omega$ is satisfied, we can make the approximation

$$\mathrm{sinc}\left(\frac{L_z}{4k_0}\mathbf{q}'^2\right) \simeq 1, \tag{26}$$

which corresponds to taking the maximum value of the cardinal sine in the integral Eq. (25). In fact, this condition is equivalent to

$$\Delta \ll \sqrt{\frac{4\pi k_0}{L_z}}. \tag{27}$$

For typical values $L_z = 0.01$ m and $\lambda_0 = 1 \times 10^{-6}$ m, we have $\Delta \ll 10^5$ m^{-1}, which does not contradict the condition $\Delta \ll k_0 \sim 10^6$ m^{-1}. Therefore, if we substitute Eq. (26) in Eq. (25), extend its limits to $(-\infty, +\infty)$, and make some calculations, we find

$$I = \frac{8}{\pi A} \exp(-ik_0 z_c) W_b(0,z_c) \tag{28}$$

where $W_b(0,z_c)$ is the pump beam profile at the origin of the plane $z = z_c$. As a result, the quantum state of the second-harmonic field is equal to

$$|\phi\rangle = \zeta\xi' \exp(-ik_0 z_c) W_b(0,z_c)|\mathrm{vac}\rangle \times \int_{|\mathbf{q}_2| \ll |\mathbf{k}_0|} d^2q_2 \left|\mathbf{q}_2 + k_0\left(1 - \frac{\mathbf{q}_2^2}{2\mathbf{k}_0^2}\right)\hat{z}s_{20}\right\rangle_{\mathrm{sh}}, \tag{29}$$

where $\xi' = (8/\pi A)\xi$. Notice that, in this expression for the state $|\phi\rangle$, the angular spectrum $v(\mathbf{q}_2;0)$ of the pump beam is no longer present in the superposition of states of one photon of transverse wave vectors $\mathbf{q}_2$. The state $|\phi\rangle$ is now proportional to the pump beam profile $W_b(0,z_c)$ projected at the origin of the plane z_c, where is located the second nonlinear crystal. As a result, if we calculate the intensity of the second-harmonic field at the position $\mathbf{r}_d = \boldsymbol{\rho}_\mathbf{d} + z_d\hat{z}$, we find

$$\langle \hat{I}(\boldsymbol{\rho}_\mathbf{d}, z_d)\rangle_\phi = |\zeta\xi'|^2 \frac{k_0^2}{4\pi\epsilon\nu z_d^2} I_b(0,z_c). \tag{30}$$

This intensity is proportional to the pump beam intensity projected at the origin of the plane z_c. The more intense the projected pump beam at that point is, the stronger the intensity of the second harmonic field will be. Therefore, we can increase the second-harmonic intensity by increasing the projected pump beam intensity at the center of the second crystal. This condition could be achieved, for instance, using lenses and focusing the pump beam. However, if the projected pump beam intensity is zero at that point, the intensity of the second harmonic vanishes, even if the intensity of the fundamental is nonzero. This effect can be achieved, for example, by preparing the pump beam in a Hermite-Gaussian mode HG_{01} or HG_{10}, or a Laguerre-Gaussian mode LG_{01} or LG_{10}. It is very important to remember that the pump beam was blocked after the first crystal, so it does not reach the second crystal. Its role is indirect, through the transfer of the angular spectrum.

III. DISCUSSION

According to Eq. (16), the joint probability of registering photodetections of signal and idler photons at the same point in the plane z_c is proportional to the intensity of the pump beam at the origin of that plane. In particular, one way of preparing signal and idler photons spatially antibunched is shaping the pump beam to have null intensity at that point. In this case, Eq. (30) shows that the intensity of the second harmonic will be zero: signal and idler photons cannot be annihilated at the same place, with creation of a second-harmonic photon. Therefore, our analysis shows that SHG is sensitive to quantum transverse correlations of the fundamental field. It is also interesting to note that the insertion of the Dove prism in the path of one of the twin beams has further physical consequences: it completely changes the second-harmonic intensity.

When twin photons are produced in spontaneous parametric down-conversion, because of the phase-matching conditions, they already have quantum correlations between the transverse component of their linear momentum. The insertion of the Dove prism in one of the twin beams is a unitary operation that changes the correlations, leading to a dependence on the difference of their detection coordinates in the fourth-order correlation function. In other words, quantum correlations are always there, but they need to be shaped for a given application.

The experiment for testing our theoretical results is very simple in concept. However, there is a technical difficulty, related to the low conversion efficiency of the usual nonlinear crystals. In general, the amount of twin photons produced with a regular nonlinear crystal is of the order of millions of pairs, at best. If we send these beams to a second crystal expecting to observe second-harmonic generation from the twin pairs, we see that the up-conversion efficiency is in general smaller than 10^{-6}. Therefore, the number of up-converted photons would be smaller than the dark noise counting rate of all commercially available photon counters. In order to overcome this difficulty, one might use periodically poled nonlinear crystals. It has been demonstrated that it is possible to up-convert twin photon pairs, using two of these crystals [21]. This is the first step toward the realization of an experiment for testing our results.

IV. CONCLUSION

In conclusion, we have shown that SHG using SPDC beams as the fundamental beam depends on their transverse spatial correlations, especially the quantum ones, as a result of the transfer of the angular spectrum that also happens in SHG. If the twin beams propagate freely and are directly used as the fundamental beam, the second-harmonic field intensity is proportional to the pump beam intensity. However, if the twin beams are manipulated by Dove prisms, the intensity of the second-harmonic field will depend on the value of the pump beam at a specific point and may even be zero, for a nonzero intensity of the pump beam.

The realization of an experimental test of our results could be achieved by using periodically poled crystals.

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$$W(\boldsymbol{\rho},z) \simeq \frac{e^{ikz}}{i\lambda z}\int d^2\rho' \, W(\boldsymbol{\rho}',0)e^{i(k/2z)|\boldsymbol{\rho}-\boldsymbol{\rho}'|^2} \quad (1)$$

and

$$v(\mathbf{q};z) = v(\mathbf{q};0)e^{ik(1-\mathbf{q}^2/2k^2)z}, \quad (2)$$

where $W(\boldsymbol{\rho}',0)$ and $v(\mathbf{q};0)$ are the profile and the angular spectrum at the plane $z=0$.

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