Electron-positron pair creation by powerful laser-ion impact

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In view of recently achieved extreme laser powers for which the ponderomotive energy $U_p \gg 2mc^2$, it has become of interest to reconsider fundamental processes of quantum electrodynamics in such powerful laser fields. In the present work, we evaluate the rates of producing electron-positron pairs by the impact of laser pulses of extreme power on highly charged ions at rest. Contrary to previous investigations, we shall consider a linearly polarized laser pulse, since at extreme laser powers the contribution of the A^2 part of the electromagnetic interaction becomes significant. As long as the duration of the laser pulses considered is in the femtosecond domain, the description of the laser beam by a powerful, monochromatic electromagnetic plane wave will still be justified, which simplifies considerably the calculations required.

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I. INTRODUCTION

In the early days of quantum electrodynamics several fundamental processes of electromagnetic interactions at relativistic energies were considered. Reviews of these early investigations can be found in the by now historic books of Heitler [1] and of Jauch and Rohrlich [2]. More recent accounts of these investigations can be found in the book by Itzykson and Zuber [3]. With the advent of the laser, theoreticians immediately started investigating the above fundamental processes in powerful laser fields, even though at that time such investigations appeared highly academic. Surveys of such early research can be found in reviews by Eberly [4], Mitter [5], and Bunkin *et al.* [6]. Early works on pair creation in a laser field were published by Nikishov and Ritus [7], Narozhnyi *et al.* [8], Reiss [9,10], Yakovlev [11], and Lyulka [12]. A more recent investigation can be found in the work of Mittleman [13], who concludes from his analysis that the probabilities of pair creation in a powerful laser field will be considerably larger for linearly polarized laser light than for a circularly polarized radiation pulse. But this author also estimates that the probabilities of pair creation will be very small.

Since today laser pulses of extreme power have become available [14], renewed efforts have been made recently, to experimentally verify laser induced nonlinear processes. In particular, Compton scattering and pair creation by the collision of γ rays with powerful laser pulses were investigated in the work of Burke *et al.* [15] and of Bamber *et al.* [16]. The process of pair creation by a powerful laser pulse interacting with some target particle was reconsidered in recent work by Liang *et al.* [17], Dietz and Pröbsting [18], and Müller *et al.* [19–22]. In the first two of these investigations, a target ion is taken at rest, on which powerful, linearly polarized laser pulses are impinging. However, in the second of these papers, the laser field is described in the dipole approximation, which seems to be a too crude simplification for the laser powers considered by these authors. In the calculations of the second authors, the collision of a beam of high energy protons with high power laser pulses is considered and circular polarization of the laser pulses is used [20], since this choice considerably simplifies the treatment of the problem. A general elliptic polarization was also considered in Refs. [21,22], but only for such parameters for which the number of absorbed laser photons is not very large. The aim of our contribution is to analyze the laser-induced pair production process in a linearly polarized plane wave and for energies (in the laboratory frame) of highly charged ions such that the number of absorbed laser photons is very large. As in the previous studies, we assume that the recoil effects due to the finite mass of the ions can be neglected in a first approximation.

In view of the conclusions drawn in the work of Mittleman [13] and similar results we have obtained in our earlier work on Compton scattering, Mott scattering, and Möller scattering at very high laser powers [23–26], we shall consider in the present work laser-induced pair creation by the collision of a powerful, linearly polarized laser beam with a heavy and highly charged ion at rest. In that case the ion can be described to a very good approximation by a static Coulomb potential of charge $-Ze$. Consequently, to a first order of approximation we neglect the recoil effects, imparted on the ion during the pair creation. As long as the duration of the laser pulses is in the fs-regime, it will still be permitted to describe the laser field by a monochromatic, linearly polarized plane wave of infinite extent. Thus we can use for the calculations the Volkov states [27] for representing the laser-dressed electrons and positrons generated during the process of pair creation. The matrix element of pair creation is in lowest order of approximation very similar to the matrix element of Mott scattering, we only have to replace the laser-dressed ingoing electron by a corresponding outgoing laser-dressed positron, as depicted in Fig. 1. We can therefore use in our following calculations the same procedures and calculational techniques we have employed earlier for the treatment of Compton scattering and Mott scattering.

Our paper will be organized as follows. In Sec. II we shall outline the derivation of the matrix element of pair creation

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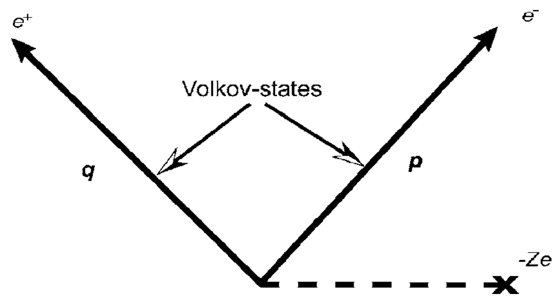

FIG. 1. The Feynman diagram of electron-positron pair creation during the collision of a laser pulse with an ion of charge $-Ze$ in the lowest order of approximation. The boldface lines represent the generated electron-positron pair described by the corresponding Volkov wave functions for a Dirac-particle embedded in an electromagnetic plane wave of arbitrary field strength.

in a laser field and derive the expressions for the probabilities and rates of pair production for a particular number N of absorbed laser photons. In Sec. III we shall then present some numerical examples for demonstrating the orders of magnitude of the efficiency of pair creation for a particular number N of absorbed laser quanta at a particular laser power I. In Sec. IV we show results for the total rates of pair creation and we estimate the number of created pairs by presently available laser-field intensities. The final Sec. V will then be devoted to a summary of our results and to some concluding remarks.

II. THEORY

We start our derivation of the matrix element of pair creation with the presentation of the Volkov waves for an electron and a positron. The Volkov wave for an electron fulfills

$$[\gamma^\mu(i\partial_\mu - eA_\mu(k\cdot x)) - mc]\psi_p^{(e)}(x) = 0 \tag{1}$$

and has the general form

$$\psi_p^{(e)}(x) = \sqrt{\frac{mc^2}{VE_{\vec{p}}}}\left(1 - \frac{e\gamma^\mu A_\mu(k\cdot x)\gamma^\nu k_\nu}{2k\cdot p}\right)u_p e^{-i[p\cdot x + S_p(k\cdot x)]}, \tag{2}$$

where $k^2=0$, $k\cdot A=0$, and $(\gamma^\mu p_\mu - mc)u_p=0$, while V is the normalization volume. In order to find the yet unknown function $S_p(k\cdot x)$, we insert the ansatz Eq. (2) into the Dirac equation, Eq. (1), and we obtain after some algebra

$$S_p(k\cdot x) = \int^{k\cdot x}\left[\frac{eA(\phi)\cdot p}{k\cdot p} - \frac{e^2A^2(\phi)}{2k\cdot p}\right]d\phi. \tag{3}$$

In a similar fashion we can find the corresponding Volkov wave for a positron. This wave fulfills

$$[\gamma^\mu(i\partial_\mu + eA_\mu(k\cdot x)) - mc]\psi_p^{(p)}(x) = 0 \tag{4}$$

and with the ansatz

$$\psi_p^{(p)}(x) = \sqrt{\frac{mc^2}{VE_{\vec{p}}}}\left(1 - \frac{e\gamma^\mu A_\mu(k\cdot x)\gamma^\nu k_\nu}{2k\cdot p}\right)v_p e^{i[p\cdot x + S_p(k\cdot x)]}, \tag{5}$$

in which $(\gamma^\mu p_\mu + mc)v_p=0$, we derive from Eq. (4) the same function $S_p(k\cdot x)$ as in Eq. (3) for an electron.

The S-matrix element for pair production in a laser field, as depicted in Fig. 1, can now be easily written down

$$S_{fi} = -i\int dx\bar{\psi}_p^{(e)}(x)e\gamma^\nu V_\nu(\vec{x})\psi_q^{(p)}(x), \tag{6}$$

where

$$V_\nu(\vec{x}) = -\frac{1}{c}\frac{Ze}{4\pi\varepsilon_0|\vec{x}|}\delta_{\nu 0},$$

$$e\gamma^\nu V_\nu(\vec{x}) = -\frac{e^2}{4\pi\varepsilon_0 c}\frac{Z}{|\vec{x}|}\gamma^0 = -\frac{\alpha Z}{r}\gamma^0. \tag{7}$$

Thus we find after some algebra

$$\begin{aligned} S_{fi} = {} & iZ\alpha\frac{mc^2}{V\sqrt{E_{\vec{p}}E_{\vec{q}}}} \\ & \times\int dx\bar{u}_p\left(1 + \frac{e\gamma^\mu A_\mu\gamma^\nu k_\nu}{2k\cdot p}\right)\gamma^0\left(1 - \frac{e\gamma^\sigma A_\sigma\gamma^\tau k_\tau}{2k\cdot q}\right)v_q \\ & \times\frac{1}{r}\exp\{i[(p+q)\cdot x + S_p(k\cdot x) + S_q(k\cdot x)]\}. \end{aligned} \tag{8}$$

For a monochromatic, linearly polarized electromagnetic plane wave field, described by the vector potential

$$\vec{A}(k\cdot x) = \vec{\varepsilon}a_0\cos(k\cdot x), \quad A^0(k\cdot x) = 0, \tag{9}$$

which will be used later on for the description of the laser pulse, we can define the renormalized four momenta

$$\bar{p} = p + \frac{e^2a_0^2}{4k\cdot p}k, \quad \bar{q} = q + \frac{e^2a_0^2}{4k\cdot q}k, \quad \mu = -\frac{ea_0}{mc} \tag{10}$$

of the generated electron and positron, respectively. In that case, we can rewrite the exponential function in Eq. (8) in the form

$$\exp\{\cdots\} = \exp\{i[(\bar{p}+\bar{q})\cdot x + Q_p(k\cdot x) + Q_q(k\cdot x)]\}, \tag{11}$$

where Q_p and Q_q are now periodic functions in $k\cdot x$. Consequently, the following part of S_{fi} in Eq. (8) can be decomposed into a Fourier series

$$\begin{aligned} & \bar{u}_p\left(1 + \frac{e\gamma^\mu A_\mu\gamma^\nu k_\nu}{2k\cdot p}\right)\gamma^0\left(1 - \frac{e\gamma^\sigma A_\sigma\gamma^\tau k_\tau}{2k\cdot q}\right)v_q e^{i[Q_p(k\cdot x)+Q_q(k\cdot x)]} \\ & = \sum_{N=-\infty}^{+\infty} M_N e^{-iNk\cdot x}, \end{aligned} \tag{12}$$

defining at the same time the Fourier coefficients M_N. Introducing the decomposition Eq. (12) into S_{fi}, we can perform the integrations over x and we thus derive for the probability rates of pair creation by the absorption of N laser photons the expression

$$w_{fi}^{N}=\frac{|S_{fi}^{N}|^{2}}{T}=\frac{32\pi^{3}Z^{2}\alpha^{2}m^{2}c^{5}}{V^{2}E_{\vec{p}}E_{\vec{q}}}\frac{|M_{N}|^{2}}{(\vec{\bar{p}}+\vec{\bar{q}}-N\vec{k})^{4}}\delta\left(\bar{p}^{0}+\bar{q}^{0}-N\frac{\omega}{c}\right). \tag{13}$$

By integrating this rate over the density of final states of the electron and positron, we finally obtain the differential rates of pair creation

$$\frac{dW_{N}}{dE_{\vec{p}}d\Omega_{\vec{p}}d\Omega_{\vec{q}}}=Z^{2}R_{N} \tag{14}$$

in which

$$R_{N}=\frac{\alpha^{2}m^{2}c^{2}}{2\pi^{3}}\frac{|\vec{p}|\cdot|\vec{q}|}{(\vec{\bar{p}}+\vec{\bar{q}}-N\vec{k})^{4}}|M_{N}|^{2}F \tag{15}$$

and the parameter F results from the integration

$$F=\int dE_{\vec{q}}\delta(\bar{E}_{\vec{q}}+\bar{E}_{\vec{p}}-N\omega)=\frac{1}{\left|\dfrac{\partial\bar{E}_{\vec{q}}}{\partial E_{\vec{q}}}\right|}. \tag{16}$$

Equation (14) represents the probability rates that an electron of kinetic energy $E_{\vec{p}}$ will be created in the direction of the momentum $\vec{p}$ and a positron in the direction of the momentum $\vec{q}$ during the absorption of N laser photons. The energy $E_{\vec{q}}$ of the positron will be determined by the energy conservation relation $\bar{E}_{\vec{q}}+\bar{E}_{\vec{p}}-N\omega=0$.

Choosing now for the laser field the monochromatic, linearly polarized plane wave, given by the vector potential Eq. (9), the function $S_{p}(k\cdot x)$ in Eq. (3) can be explicitly evaluated to yield

$$\begin{aligned}S_{p}(k\cdot x)&=\int^{k\cdot x}d\phi\left(-\frac{ea_{0}\vec{\varepsilon}\cdot\vec{p}}{k\cdot p}\cos\phi+\frac{e^{2}a_{0}^{2}}{2k\cdot p}\cos^{2}\phi\right)\\&=\frac{e^{2}a_{0}^{2}}{4k\cdot p}k\cdot x-\frac{ea_{0}}{k\cdot p}\vec{\varepsilon}\cdot\vec{p}\sin(k\cdot x)+\frac{e^{2}a_{0}^{2}}{8k\cdot p}\sin(2k\cdot x).\end{aligned} \tag{17}$$

With reference to Eqs. (8), (11), and (16), we can now evaluate

$$\frac{\partial\bar{E}_{\vec{q}}}{\partial E_{\vec{q}}}=1-\frac{\mu^{2}}{4}(mc^{2})^{2}\frac{1}{(E_{q}-c\vec{q}\cdot\vec{n})^{2}}\frac{c^{2}\vec{q}^{2}-E_{\vec{q}}c\vec{q}\cdot\vec{n}}{c^{2}\vec{q}^{2}} \tag{18}$$

and rewrite the argument of the exponential in Eq. (11) in the explicit form

$$\begin{aligned}Q_{p}(k\cdot x)+Q_{q}(k\cdot x)&=mc\mu\left(\frac{\vec{\varepsilon}\cdot\vec{p}}{k\cdot p}+\frac{\vec{\varepsilon}\cdot\vec{q}}{k\cdot q}\right)\sin(k\cdot x)\\&\quad+\frac{1}{8}m^{2}c^{2}\mu^{2}\left(\frac{1}{k\cdot p}+\frac{1}{k\cdot q}\right)\sin(2k\cdot x).\end{aligned} \tag{19}$$

Introducing the abbreviations

$$a=-mc\mu\left(\frac{\vec{\varepsilon}\cdot\vec{p}}{k\cdot p}+\frac{\vec{\varepsilon}\cdot\vec{q}}{k\cdot q}\right),\quad b=-\frac{1}{8}m^{2}c^{2}\mu^{2}\left(\frac{1}{k\cdot p}+\frac{1}{k\cdot q}\right), \tag{20}$$

we then find from Eq. (19)

$$Q_{p}(k\cdot x)+Q_{q}(k\cdot x)=-a\sin(k\cdot x)-b\sin(2k\cdot x) \tag{21}$$

and can now define the generalized Bessel functions $B_{N}(a,b)$ by means of the generating function

$$e^{-ia\sin(k\cdot x)-ib\sin(2k\cdot x)}=\sum_{N}B_{N}(a,b)e^{-iN(k\cdot x)}. \tag{22}$$

For the functions B_{N} different notations were used in earlier work [28–30], see, however, Ref. [31]. This finally permits us to express the matrix elements M_{N} in Eqs. (12) and (14) in the following form:

$$\begin{aligned}M_{N}&=B_{N}(a,b)\bar{u}_{p}\gamma^{0}v_{q}-\frac{1}{4}mc\mu[B_{N+1}(a,b)+B_{N-1}(a,b)]\\&\quad\times\left(\frac{1}{k\cdot p}\bar{u}_{p}\gamma^{\mu}\varepsilon_{\mu}\gamma^{\nu}k_{\nu}\gamma^{0}v_{q}-\frac{1}{k\cdot q}\bar{u}_{p}\gamma^{0}\gamma^{\mu}\varepsilon_{\mu}\gamma^{\nu}k_{\nu}v_{q}\right)\\&\quad-\frac{1}{8}m^{2}c^{2}\mu^{2}\frac{1}{(k\cdot p)(k\cdot q)}\\&\quad\times\left[\frac{1}{2}B_{N+2}(a,b)+B_{N}(a,b)+\frac{1}{2}B_{N-2}(a,b)\right]\\&\quad\times\bar{u}_{p}\gamma^{\mu}\varepsilon_{\mu}\gamma^{\nu}k_{\nu}\gamma^{0}\gamma^{\sigma}\varepsilon_{\sigma}\gamma^{\tau}k_{\tau}v_{q}.\end{aligned} \tag{23}$$

The traces over the Dirac matrices in the above formula are performed numerically.

III. NUMERICAL EXAMPLES

Since we assume that the very heavy and highly charged target ion can be approximated by a static Coulomb potential of charge $-Ze$, there will be no momentum conservation requirement in our process and, consequently, we can consider in the following the case in which the electrons and positrons are created with equal energies and move in opposite directions. Introducing the variables

$$y=\frac{N\omega}{mc^{2}},\quad w=\frac{E_{\vec{p}}}{mc^{2}}=\frac{E_{\vec{q}}}{mc^{2}},\quad \xi=\frac{\vec{p}\cdot\vec{n}}{|\vec{p}|}=-\frac{\vec{q}\cdot\vec{n}}{|\vec{q}|}, \tag{24}$$

we can rewrite the conservation of energy in Eq. (16) as follows:

$$y=2w+\frac{\mu^{2}}{4}\left(\frac{1}{w-\xi\sqrt{w^{2}-1}}+\frac{1}{w+\xi\sqrt{w^{2}-1}}\right) \tag{25}$$

which, after some algebraic manipulations, adopts the form of a cubic equation for w,

$$2(1-\xi^{2})w^{3}-y(1-\xi^{2})w^{2}+\left(2\xi^{2}+\frac{\mu^{2}}{2}\right)w-y\xi^{2}=0. \tag{26}$$

This equation possesses in general three real solutions. For a very strong laser field N is not a proper parameter because it might happen that for a given N there are up to three possible energies. We overcome this problem, if we label the process by the energy. In this case the correspondence "energy $\rightarrow$ number of absorbed laser photons" is unique. In the following the laser field will be characterized by its frequency ω and by the parameter μ, defined in Eq. (10), which determines the intensity of the laser field. For the frequency $\omega_0 = 1.5498$ eV of a Ti:sapphire laser the parameter $\mu = 1$ for the intensity $I = 2.14 \times 10^{18}$ W cm^{-2}. Let us also mention that the parameter μ is very important for the relativistic invariant description of laser-assisted or laser-induced processes at very high radiation power. The renormalized energy $\bar{E}$ and momentum $\vec{\bar{p}}$ of a particle fulfill the conservation relation $\bar{E}^2 = \bar{m}^2 c^4 + \vec{\bar{p}}^2 c^2$ which corresponds to an on-shell particle of effective mass $\bar{m} = m\left(1+\frac{\mu^2}{2}\right)^{1/2}$ [29,9,32]. The characteristic intensity parameter μ, introduced in Eq. (10), can also be written as $\mu = \sqrt{I/I_c}$, where I is the average intensity of the laser radiation and $I_c = \alpha\omega^2/8\pi r_0^2$ (with α being the fine-structure constant and $r_0 = \alpha/mc$) is for a given frequency ω the critical laser intensity at which $\mu^2 = 1$, in which case the problem has to be treated relativistically. The parameter μ is a relativistic invariant. A more detailed analysis can be found in Ref. [33], where it is shown that relativistic effects become important at even lower laser field intensities.

In our discussions to follow, we shall consider two cases of the creation of electron-positron pairs by the impact of a laser pulse on an ion at rest, namely the particles are moving either parallel or perpendicular to the direction of laser field propagation. The first case, in which the created particles move in the direction of the laser field propagation, is characterized by the condition $\xi^2 = 1$, for which Eq. (26) has one unique solution,

$$w = \frac{y}{2 + \mu^2/2}. \tag{27}$$

Taking into account that $w \geqslant 1$, we find from this formula the minimum number of absorbed photons required to create an electron-positron pair to be given by

$$N_0 = 2\frac{mc^2}{\omega}\left(1 + \frac{\mu^2}{4}\right). \tag{28}$$

Moreover we infer from Eq. (27) that the energy of the created particles increases with the increase of the number of absorbed laser photons N, and $N \geqslant N_0$. Also, in this case the arguments of the generalized Bessel functions in Eq. (23) become equal to

$$a = 0, \quad b = -\frac{\mu^2}{\mu^2 + 4}\frac{N}{2}, \tag{29}$$

from which it is readily seen that for large values of μ ($\mu \gg 1$)the second argument b tends to $-N/2$. This indicates that in the limit of large μ, in which we are interested, we can expect significant rates for the pair creation. To our knowledge, this geometry has not been studied before in the context of pair creation (see also a related analysis in Ref. [28]).

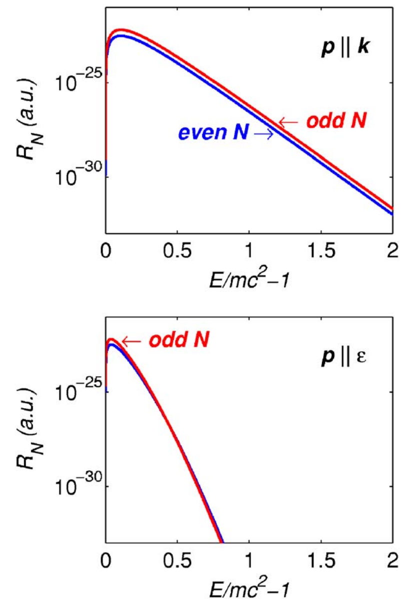

FIG. 2. (Color online) For a Ti:sapphire laser source with $\omega_0 = 1.5498$ eV are shown the rates of pair production R_N in atomic units for "odd" (red lines) and "even" (blue lines) values of N as functions of the electron's kinetic energy in units of its rest energy for the two geometries, denoted by $(\vec{p} \| \vec{k})$ and $(\vec{p} \| \vec{\varepsilon})$, respectively, and discussed in the text. We chose the parameter $\gamma_D = 10^3$ and $\mu = 10^2$ such that the intensity of the laser beam is of the order of 10^{22} W cm^{-2}. For small kinetic energies the pairs are created predominantly in the configuration $(\vec{p} \| \vec{\varepsilon})$, while for larger energies pair creation takes place preferentially in the geometry $(\vec{p} \| \vec{k})$.

As a second possibility, let us consider the standard case where an electron-positron pair is created in the direction of the polarization vector $\vec{\varepsilon}$, such that $\xi = 0$. Now, the cubic equation, Eq. (26), has three real solutions,

$$w = 0, \quad w = \frac{y \pm \sqrt{y^2 - 4\mu^2}}{4}, \tag{30}$$

provided that $y \geqslant 2\mu$. Let us note that the solution $w = 0$ has to be rejected since $w \geqslant 1$. In Eq. (30) the solution with the minus sign in front of the square root must also be larger than 1, which determines the upper limit for the number of absorbed photons, $N \leqslant N_0$, with N_0 defined in Eq. (28). By analyzing this solution in the (y, w) plane, one can also find the lower limit of y. We find that the lower limit of the number of laser photons above which pair creation can take place is given by

$$N_1 = 2\mu. \tag{31}$$

Surprisingly, for this solution the energy of created particles grows monotonically, in the range of values between mc^2 and $mc^2\mu/2$, while the number of absorbed photons decreases from N_0 to N_1 [Eqs. (28) and (31), respectively]. Such a dependence of the energy of produced (e^-, e^+) pairs on the number of absorbed laser photons has been observed previously by Mittleman [13]. The last solution of the cubic equation, Eq. (26), that has not been discussed yet, is the one with the plus sign in front of the square root in Eq. (30) that

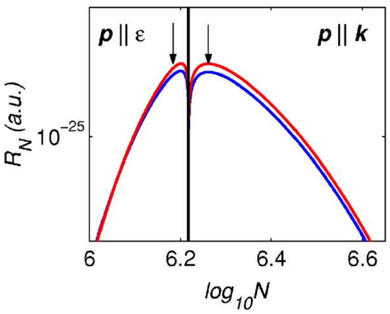

FIG. 3. (Color online) The same data as in Fig. 2, but as a function of absorbed laser photons. The vertical line marks the border at $N_0=1\,649\,172$ between the two geometries $(\vec{p}\|\vec{k})$ and $(\vec{p}\|\vec{\varepsilon})$. Arrows indicate odd N.

does not show this feature. In this case, w is a monotonically growing function of its argument y, and approaches asymptotically the value $y/2$ for large y $(y \gg 1)$.

Before we start presenting our numerical results, a brief comment is necessary. As has been noted previously by Müller *et al.* [19–22] and anticipated by Mittleman [13], the production rate for laser-induced pairs is negligibly small if nowadays available powerful laser pulses interact with target ions at rest. Therefore Müller *et al.* [19–22] have proposed to consider the head-on collision of a laser beam with a beam of target ions. This idea is derived from the following fact. First of all, if an ion is counter-moving with relativistic velocity toward a laser pulse of linear polarization, then the laser field remains linearly polarized in the rest frame of the ion, but the direction of linear polarization has changed. But, on the other hand, the photon frequency will be Doppler up-shifted at the same time and, consequently, the number of laser photons required to produce electron-positron pairs will be decreased. Note that the calculations we have performed until now will remain valid in the rest frame of the ions of the beam, if we

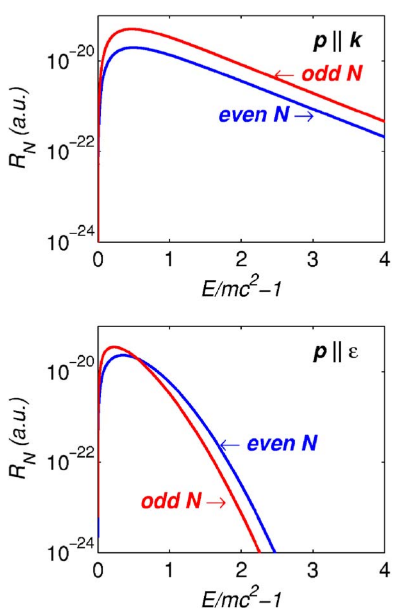

FIG. 4. (Color online) Similar data as in Fig. 2 but for a larger laser intensity with $\mu=10^3$.

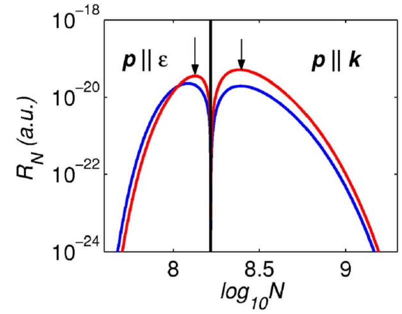

FIG. 5. (Color online) The same data as in Fig. 4 but as a function of N with the transition value at $N_0=164\,851\,836$ between the $(\vec{p}\|\vec{k})$ and $(\vec{p}\|\vec{\varepsilon})$ geometries, respectively. Arrows indicate odd N.

scale the frequency of the laser field by the factor [34]

$$\gamma_D = \gamma(1 - \beta \cos \theta), \tag{32}$$

depending on the angle θ between the laser and the ion beam, the reduced velocity of the ion in the laboratory frame β, and the Lorentz factor γ. For the figures, discussed in the following, we therefore consider the collision of highly ionized atoms with laser pulses, for which the frequency in the reference frame of the ions at rest is equal to $\omega=\gamma_D\omega_0$, where ω_0 is the laser frequency in the laboratory frame. We consider, as mentioned before, a Ti:sapphire laser source with $\omega_0=1.5498$ eV. Since the parameter μ is a relativistic invariant, it has in both reference frames the same parameter value.

In Fig. 2 we show the rates of pair production R_N in atomic units for odd (red lines) and even (blue lines) values of N as functions of the electron's kinetic energy in units of its rest energy for the two geometries considered above, with $\gamma_D=10^3$ and $\mu=10^2$ in which case the intensity of the laser beam in the laboratory frame is of the order of magnitude 10^{22} W cm^{-2}. We observe that for small kinetic energies the pairs are created predominantly in the direction of the electric field $(\vec{p}\|\vec{\varepsilon})$. However, for larger energies the rates for the second geometry $(\vec{p}\|\vec{k})$ become by many orders of magnitude larger. In Fig. 3 the same data are shown as in Fig. 2, but as a function of absorbed laser photons. The vertical line marks the transition value $N_0=1\,649\,172$ between the two geometries considered.

Figure 4 presents similar data as in Fig. 2 but for a larger laser intensity with $\mu=10^3$ and in Fig. 5 are again shown the same data as in Fig. 4 but as a function of the absorbed laser photons N. Here the transition value is $N_0=164\,851\,836$. Now pairs are produced predominantly along the propagation vector, and we can expect that for larger laser field intensities this tendency will prevail. In order to check this statement, for instance for the Schwinger intensity [35], for which the amplitude of the electric field is equal to $m^2c^3/|e|\hbar=1.32\times10^{16}$ V cm^{-1}, one has to perform the calculation at least with quadruple precision. This problem is presently under consideration.

Up to now we considered two geometries in which pairs are produced, either in the directions of $\vec{\varepsilon}$ or along $\vec{k}$. Now we shall analyze the angular dependence of the rates R_N still

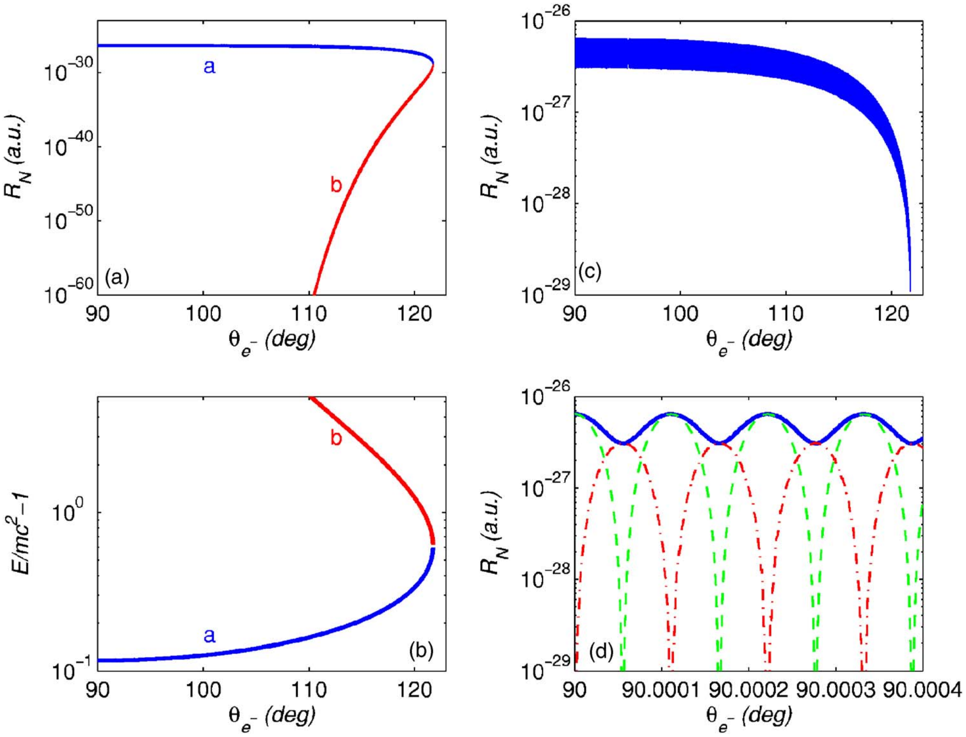

FIG. 6. (Color online) The angular dependence of R_N as a function of θ_{e^-} between the electron momentum $\vec{p}$ and the field polarization $\vec{\varepsilon}$ in the plane $(\vec{\varepsilon}\vec{k})$, for $\gamma_D=10^2$, $\mu=10^3$, and $N=1\,840\,496\,533$. The dependence of the electron energy on θ_{e^-} is not unique as shown in frame (b). At smaller energies (labeled "a") the rates are much larger than for larger energies shown in frame (a). The rates R_N oscillate as a function of θ_{e^-} as seen in frames (c) and (d). In (d) the green line (dashed) marks the contribution for parallel spins of the pairs, whereas the red line (dash-dotted) is for anti-parallel spins. The sum of both contributions leads to the regular sinusoidal dependence of R_N on θ_{e^-}.

assuming that in the reference frame of the ions at rest, the produced electrons and positrons are emitted in opposite directions. We consider the dependence of the rates R_N on the azimuthal angle θ_{e^-} between the electron momentum vector $\vec{p}$ and the laser field polarization vector $\vec{\varepsilon}$ in the plane determined by the vectors $\vec{\varepsilon}$ and $\vec{k}$. We keep the polar angle of the emitted electron fixed at $\varphi_{e^-}=0^\circ$, so that for positrons $\theta_{e^+}=180^\circ-\theta_{e^-}$ and $\varphi_{e^+}=180^\circ+\varphi_{e^-}$.

In Fig. 6 we present the angular dependence of the rates R_N as a function of the angle θ_{e^-}. The parameters are: $\gamma_D=10^2$, $\mu=10^3$, and $N=1\,840\,496\,533$, for which pairs are not produced in the direction of the polarization vector. Here the dependence of the electron energy on θ_{e^-} turns out not to be unique as shown in frame (b). For smaller energies (labeled "a") the rates, as expected, are much larger than for larger energies, as presented in frame (a). The rates R_N are an oscillating function of θ_{e^-} as seen in frames (c) and (d). In frame (D) the green line (dashed) represents the contribution when the spins of the electron-positron pairs are parallel to each other, whereas the red line (dash-dotted) when the spins are anti-parallel. The sum of these two contributions leads to the regular sinusoidal dependence of R_N on θ_{e^-}. Figure 7 shows the same as in Fig. 6 but for small angles θ_{e^-} and smaller $N=1\,590\,225\,887$. For these parameters pairs are not produced in the direction of propagation of the laser beam. Here, too, regular oscillations are observed. As before, there exist angles θ_{e^-} for which the pairs are produced predominantly with spins either parallel or anti-parallel. Comparing the data in Figs. 6 and 7 we conclude that for pairs that are produced in the direction of the polarization vector $\vec{\varepsilon}$ the rates R_N are peaked around $\theta_{e^-}=0^\circ$. On the contrary, if pairs are produced in the direction of propagation of the laser field $\vec{n}$, we observe a flatter angular distribution of the rates. Therefore, if we integrate the rates R_N over all angles for those pairs generated predominantly in the direction of the laser polarization $\vec{\varepsilon}$ and compare it with the integrated rates for the pairs predominantly produced in the direction of the wave vector $\vec{k}$, we find that these integrated data become even larger for the second case, as one would expect from

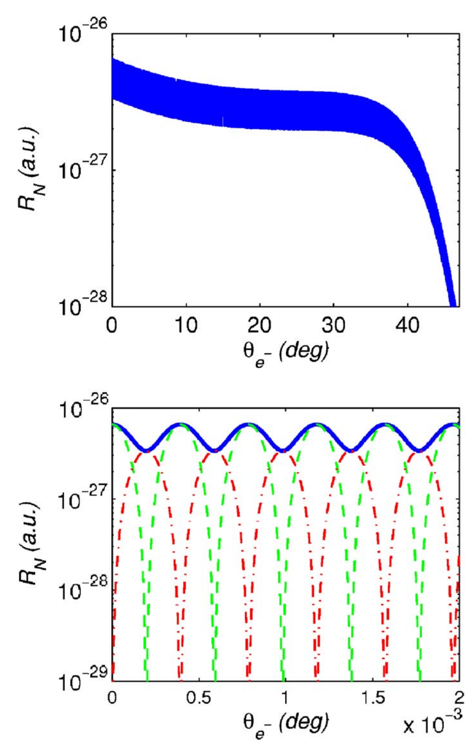

FIG. 7. (Color online) The same as in Fig. 6 but for small θ_{e^-} and $N=1\,590\,225\,887$, showing similar regular oscillations of the data. As before, there are angles θ_{e^-} for which the pairs are predominantly produced with spins either parallel or anti-parallel.

their numerical values for $\theta_{e^-}=0°$ or $\theta_{e^-}=90°$.

In closing this section, let us note that our numerical results presented above are for electrons and positrons of equal energy and moving in opposite directions in the reference frame of very heavy ions (for infinitely heavy ions this is the center of mass reference frame). In the laboratory frame the energies of electrons and positrons are no more equal and their momenta are not aligned in general as well. For light ions, and in particular for protons, the recoil effects induced by the absorption of a large number of laser photons (which transfer energy as well as momentum) will become important and will lead to a modification of the cross sections of pair creation. In order to account for these effects, one has to treat both the electron-positron pairs and the ions on equal footing. Consequently, we have to consider a Feynman diagram, similar to the one of electron-proton scattering in Bjorken and Drell's book on relativistic quantum mechanics [36], replacing the ingoing electron line by an outgoing positron line and taking care of the laser-dressing of all particles involved in the process. Calculations of this kind, which turn out to be considerably more time consuming, are presently under consideration.

IV. TOTAL RATES

In order to determine the number of pairs created during the interaction of ions with a laser pulse we have to calculate the total rate

$$W = \int d\Omega_{\vec{p}} d\Omega_{\vec{q}} \int_{mc^2}^{\infty} dE_{\vec{p}} \sum_{N=N_{\min}}^{\infty} Z^2 R_N, \qquad (33)$$

where $N_{\min}$ is the minimum number of laser photons necessary to create a pair. Since the sum extends over large numbers, we shall therefore replace it by integration and evaluate the sixfold integral

$$w = \int_{mc^2}^{\infty} dE_{\vec{p}} \int_{N_{\min}}^{\infty} dN \int_0^{\pi} d\theta_{\vec{p}} \int_0^{2\pi} d\varphi_{\vec{p}} \int_0^{\pi} d\theta_{\vec{q}} \int_0^{2\pi} d\varphi_{\vec{q}} \times \cos\theta_{\vec{p}} \cos\theta_{\vec{q}} R_N, \qquad (34)$$

by applying the Monte Carlo method. The results for two values of $\mu=10^2$ and $\mu=10^3$ and a few values for the Doppler factor γ_D are presented in Fig. 8. As expected, the total rate increases with increasing γ_D. It appears, however, that with increasing γ_D the role of the laser field intensity decreases.

Let us estimate the number of pairs created during the interaction of ions with laser fields of presently available parameter values. We shall consider the head-on collision of ion and laser beams, for which the Lorentz factor γ is one half of the Doppler factor γ_D, i.e., we take $\gamma=\gamma_D/2$. If in the laboratory frame the laser pulse duration is τ, then in the rest frame of ions the interaction time of the ions with the laser field will be equal to $\tau/\gamma=2\tau/\gamma_D$. Denoting by N_{ion} the average number of ions in the laser focus, we find the number of generated pairs to be

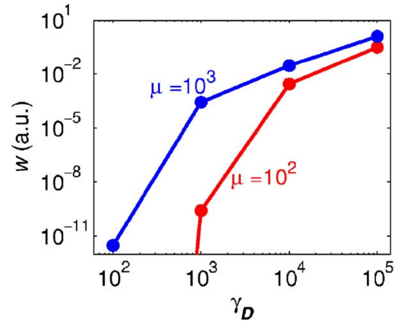

FIG. 8. (Color online) The dependence of the total rate w (in atomic units) on the Doppler factor γ_D for two laser field intensities corresponding to $\mu=10^2$ and $\mu=10^3$, respectively, for the Ti:sapphire laser of wavelength 800 nm.

$$N_{\text{pairs}} = 2Z^2 N_{\text{ion}} f \tau w/\gamma_D, \qquad (35)$$

where f is the repetition rate of the laser pulses. In particular, the number of pairs created in 1 s will then be equal to

$$N_{\text{pairs}} \approx 83 Z^2 N_{\text{ion}} f \tau w/\gamma_D, \qquad (36)$$

where the total rate w is expressed in atomic units, the repetition rate f in hertz, and the duration of the laser pulse τ in femtoseconds.

For the top-intensity laser source reported in Ref. [37], the maximum intensity value corresponds approximately to $\mu=10^2$ for the frequency of a Ti:sapphire laser $\omega=1.5498$ eV and the wavelength $\lambda=800$ nm. Since the duration of the laser pulse equals to $\tau=30$ fs with a very small repetition rate of $f=0.1$ Hz, we consequently find for instance for lead as target nucleus (with $Z=82$)

$$\frac{N_{\text{pairs}}}{N_{\text{ion}}} = \begin{cases} 2.9\times10^{-53} & \text{for } \gamma_D=10^2 \\ 4.4\times10^{-7} & \text{for } \gamma_D=10^3 \\ 4.8\times10^{-1} & \text{for } \gamma_D=10^4 \\ 5.2 & \text{for } \gamma_D=10^5, \end{cases} \qquad (37)$$

whereas for higher intensity with $\mu=10^3$ we obtain

$$\frac{N_{\text{pairs}}}{N_{\text{ion}}} = \begin{cases} 5.1\times10^{-8} & \text{for } \gamma_D=10^2 \\ 0.4 & \text{for } \gamma_D=10^3 \\ 5.0 & \text{for } \gamma_D=10^4 \\ 20.1 & \text{for } \gamma_D=10^5. \end{cases} \qquad (38)$$

Let us note that these numbers have been calculated for a very small repetition rate f. For f of the order of a few kilohertz, as it used to be for smaller intensities, the above numbers are larger by at least four orders of magnitude. For the laser discussed in Ref. [37] the laser focus has a diameter of 0.8 μm and therefore it is justified to assume that the number of ions, N_{ion}, in the laser focus is of the order of 1.

Let us now compare the number of pairs created by the above mechanism with the direct generation of pairs by a constant electric field in vacuum, for which the probability rate per unit volume is given in atomic units by the Schwinger formula

$$W_{\mathrm{Sch}} = \frac{c\mu^2\omega^2}{4\pi^3}\sum_{n=1}^{\infty}\frac{1}{n^2}\exp\left(-\pi n\frac{c^2}{\mu\omega}\right), \tag{39}$$

in which $\mu = |e|E_0/mc\omega$ and E_0 is the amplitude of the electromagnetic plane wave, describing the laser pulse. This formula can be rewritten in a more compact form

$$W_{\mathrm{Sch}} = \frac{c\mu^2\omega^2}{4\pi^3}L_2\left[\exp\left(-\pi\frac{c^2}{\mu\omega}\right)\right] \tag{40}$$

by using Euler's dilogarithmic function [38], defined by

$$L_2(z) = -\int_0^z \frac{ds}{s}\ln(1-s). \tag{41}$$

Hence the number of pairs generated per second is equal to

$$N_{\mathrm{Sch}} = f\tau V W_{\mathrm{Sch}}, \tag{42}$$

where V is the volume of a laser focus. With the same spatial and temporal characteristics of the laser field we find that the number of created pairs per second is of the order $N_{\mathrm{Sch}} \cong 10^{-8}$, if $\mu = 1.6\times 10^4$ (taking instead $\mu = 10^2$, this rate is of the order of 10^{-4482}). This means that for the presently available laser field intensities the creation of electron-positron pairs by the laser pulse $\Rightarrow$ ion impact process is by far more efficient than the direct creation of pairs from the vacuum by the laser field. This situation may change if the intensities of a Ti:sapphire laser will be at least of the order of 10^{26} W/cm^2.

V. CONCLUSIONS

In the present work, we investigated the creation of electron-positron pairs during the impact of high power laser pulses with a relativistic beam of highly charged ions. We evaluated the rates R_N of pair creation for the absorption of N laser photons. Since the laser pulses considered are in the femtosecond regime, we were able to evaluate the matrix element of pair creation in a straightforward way by considering the Feymann diagram shown in Fig. 1, describing the generated electron-positron pair by their respective Volkov solutions. After the evaluation of the corresponding matrix element and its decomposition into a Fourier series of components of transition matrix elements for the emission or absorption of N laser photons $\hbar\omega$, the numerical evaluation of the rates R_N turned out to first require an analysis of the energy conservation relation $\bar{E}_{e^-} + \bar{E}_{e^+} = N\hbar\omega$ in order to find out the most convenient kinematical configurations of electron-positron pair creation that are more easily accessible to the numerical analysis of the rates R_N. As it turned out, the two configurations of particular interest are the creation of an electron-positron pair, if both particles are created aligned and move in opposite directions with equal kinetic energies and they are ejected from the interaction region either along (a) the direction of laser propagation or (b) perpendicular to it along the direction of linear laser polarization. The detailed numerical analysis has shown that for pair creation close to the threshold energy $2mc^2$ the emission of the particles is more efficient in the direction of the laser polarization whereas for higher particle energies the process becomes dominant in the direction of laser propagation. This finding agrees with the general conclusion that at relativistic laser intensities the contribution of the A^2 part of the electromagnetic interaction becomes equally important as the $\vec{A}\cdot\vec{\varepsilon}$ coupling of the particles with the laser field. For sufficiently dense, highly charged ion beams and laser pulses of very high power such that the relativistic intensity parameter $\mu^2 \gg 1$ the observation of pair creation in a laser field should become observable in the near future.

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