Relativistic Ps⁻ and Ps

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Relativistic positronium (Ps) is of potential use to address fundamental questions in QED—e.g., through direct lifetime measurements of the "para" state of Ps, severe magnetic quenching of the ortho-Ps lifetime, so-called "superpenetration," and possibly to measure Ps-atom cross sections. Existing schemes for the production of relativistic positronium have relied on pion production through, e.g., nuclear interactions. This yields a low-intensity beam with relatively poor characteristics in both longitudinal and transverse emittance. By use of positrons impinging on a thin carbon foil inside a high-frequency rf cavity, it is proposed to generate relativistic positronium ions (Ps^-) by rapid acceleration. Relativistic Ps can be derived from the positronium negative ion by subsequent Lorentz stripping or photodetachment. Intensities of the order 100 per second and Lorentz factors $\simeq 20$ are feasible with present technology.

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I. INTRODUCTION

With the recent developments of high-density positron beams from extracted plasmas in Penning traps of the Surko type [1], it now becomes possible to generate relativistic positronium from acceleration of the positronium negative ion (Ps^-). Hitherto, it has proven possible to measure the binding energy and lifetime of this—perhaps the most simple imaginable—three-body system [2–4]. Measurements of the characteristics of Ps^- are important as they can test an understanding of bound three-body systems, in this case an essentially isolated system consisting of three pointlike particles bound together by forces that are known to very high precision. An analog to this QED system can be found in QCD where the proton to a first approximation consists of two up quarks and a down quark—i.e., three entities with two different charges and flavors, bound together by forces that are much less tractable than in the QED case. A similar analogy exists between, e.g., charmonium ($c\bar{c}$) and positronium [5]. It is thus tempting to call Ps^- the "QED proton," even though the proton is not a combination of particles and their antiparticles.

Second, relativistic positronium (Ps) may enable a test of the accuracy of QED calculations through direct precision measurements of the "para" state of Ps instead of through state mixing in a magnetic field [6] or test the alleged "superpenetration" of relativistic positronium beams in solid targets [7–10], related to the King-Perkins-Chudakov effect of reduced ionization in the vicinity of a pair-creation vertex: see [11], Refs. [1–3]. Similarly, it will be possible to extend the energy range of measurements of Ps-atom cross sections [12]. In general, being simple electrically bound systems, Ps atoms may be used to address a number of fundamental questions [13–15].

Photodetachment of the positronium negative ion (Ps^-) was suggested as a means to obtain "high-velocity Ps" in connection with the first observation of Ps^- [2]. However, since then, to our knowledge no realistic schemes have been proposed using this reaction. Other schemes are under development—e.g., in the LEPTA project at the Joint Institute for Nuclear Research in Russia [16–19] and proposed mechanisms like photo production or electroproduction [20–25] on nuclei or as by-products in the operation of relativistic heavy-ion colliders [26]. A different scheme is the so-called REFER facility [27] (see also [28]), which produces at best a few hundred singlet Ps per hour produced by a $10^{13} e^-/s$ beam. Nonrelativistic, but "fast" Ps has also been proposed to be generated by electrostatic acceleration [29]. Here, an alternative scheme is proposed with the main advantages being its comparatively low cost, high efficiency, and simplicity.

II. PROPOSED SCHEME

In short, the proposed scheme consists of five parts: first a bunched positron beam, e.g., obtained from a trap of the Surko type or a high-power microtron/linac with electrons impinging on a high-Z target and subsequent selection and moderation of the produced positrons. These are well-proven methods that have been used to generate relatively intense positron beams of bunch lengths in the microsecond range and below; for an example, see [1,30].

The second part is a bunch compression in, e.g., a pulsed parabolic potential. This can provide nanosecond pulse widths of positrons at the expense of energy broadening [31].

The third part is the conversion of positrons to Ps^- ions. This double-electron-capture process happens with a probability of up to 3×10^{-4} in a thin carbon foil [2,32,33]. The transport of positrons up to the target takes place in a solenoidal field of about 50 G which is then increased at the target to provide a small target region (sub-mm). The carbon foil must be situated at the entrance of a rf cavity.

The fourth step is a rapid acceleration that gives enough kinetic energy to the Ps^- within its proper lifetime $\tau_0 = 1/\lambda_0 = 1/2.092\,797\ \mathrm{ns}^{-1}$ [34], making use of the fact that time dilation as observed from the laboratory prolongs the lifetime of the ion from the value at rest. In a uniform accelerating field of 25 MV/m, an acceleration to $\gamma \simeq 25$ yields a proper time of 0.8 ns, only slightly larger than the intrinsic lifetime. Uniform electric fields of this order of magnitude cannot be produced, but one viable option is acceleration in an rf cavity.

The fifth and final step to generate the relativistic Ps from Ps^- is photodetachment by means of a laser pulse of sufficient time duration, power, and effective area to strip the ions efficiently. Alternatively, Lorentz stripping of the ion in a permanent magnet gap, employing the Lorentz-transformed magnetic field resulting in a strong electric field sufficient for effective detachment, may be used.

Since the first experiments that proved the existence of Ps^- [2,3], the accuracy of calculations of its binding energy and decay rate have increased enormously to an ultimate accuracy of about 10^{-12} [35]; see [36] for references. Only recently have relativistic and QED energy corrections to the binding energy been calculated [33].

III. ACCELERATION

As discussed by Mills [2], field stripping "does not appear to preclude accelerating Ps^- to relativistic energies." In the following we discuss this aspect a little more in detail. The proper time of the uniformly accelerated particle is given as

$$\tau = \int_0^t \frac{1}{\beta\gamma} \mathrm{d}t = \frac{c}{a} \sinh^{-1}\left(\frac{at}{c}\right), \tag{1}$$

where $a = E_0 e / m_0$ in a uniform electric field [37]. From the velocity

$$v = \frac{at}{\sqrt{1 + a^2t^2/c^2}}, \tag{2}$$

the relativistic Lorentz factor $\gamma = \mathcal{E}/mc^2 = 1/\sqrt{1-v^2/c^2}$ can be derived where $\mathcal{E}$ is the total energy of the particle. However, as noted above, such uniform fields are unrealistic and it is necessary to use, e.g., rf fields.

Following [38], the electric field along the z axis of an rf cavity can be assumed to be a sum of a forward- and a backward-propagating wave:

$$E_z = E_0 \cos(kz)\sin(\omega t + \phi_0), \tag{3}$$

where E_0 is the peak electric field, λ the rf wavelength, $k = 2\pi/\lambda$, $\omega = ck$, and ϕ_0 the rf phase. From Eq. (3) and $\mathrm{d}\gamma/\mathrm{d}z = eE_z/3mc^2$ we get

$$\frac{\mathrm{d}\gamma}{\mathrm{d}z} = \frac{eE_0}{6mc^2}[\sin(\phi) + \sin(\phi + 2kz)], \tag{4}$$

where

$$\phi = \omega t - kz + \phi_0 = k\int_0^z \left(\frac{1}{\beta} - 1\right)\mathrm{d}z + \phi_0 \tag{5}$$

and $\beta = \sqrt{1-\gamma^{-2}}$. As shown in [38], for the first few wavelengths and for $\phi_0 \simeq \pi/2$ a good approximation to γ is given by

$$\gamma = 1 + \alpha_{3m}\{kz\sin(\phi) + [\cos(\phi) - \cos(\phi + 2kz)]/2\}, \tag{6}$$

with $\alpha_{3m} = eE_0/6mc^2k$ as a dimensionless field strength parameter, $\tilde{\gamma} = 1 + 2\alpha_{3m}\sin(\phi_0)kz$, and $\phi = [(\sqrt{\tilde{\gamma}^2 - 1} - (\tilde{\gamma} - 1) + \phi_0)]/2\alpha_{3m}\sin(\phi_0)$. The first few wavelengths compose an important region close to the Ps^- target where the ion is

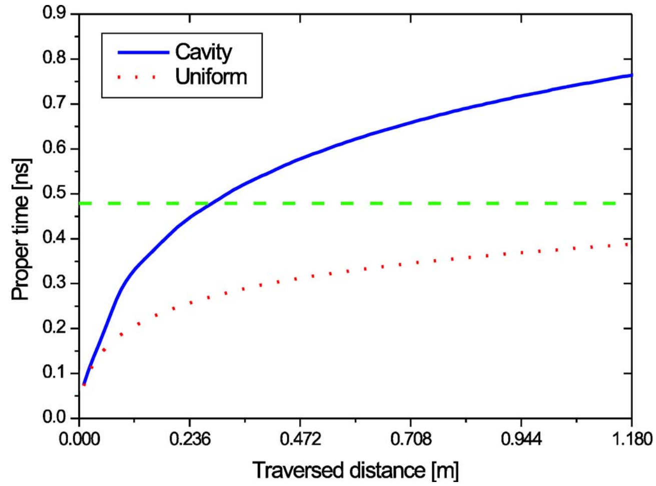

FIG. 1. (Color online) The proper time of Ps^- during acceleration in a $E_0 = 60$ MV/m, 1269 MHz ($\lambda = 0.236$ m) rf cavity (solid line) and in a uniform field of 25 MV/m (dotted line), both as a function of distance traversed. The horizontal dashed line shows the proper lifetime of Ps^-.

nonrelativistic and the proper time τ is close to the laboratory time t. The proper time as a function of laboratory time or length traversed is calculated as

$$\tau = \int_0^t \frac{1}{\beta\gamma}\mathrm{d}t = \int_0^z \frac{1}{c\gamma\sqrt{1-\gamma^{-2}}}\mathrm{d}z, \tag{7}$$

with γ from Eq. (6).

As expected and seen from Fig. 1, a high gradient for acceleration is desirable to compete with the decay rate of the Ps^- ion. In the following, a $E_0 = 60$ MV/m, 1269 MHz ($\lambda = 0.236$ m) rf cavity is chosen as an example, inspired by the use of high-gradient photoinjectors [39]. As shown below, much higher gradients lead to field stripping during acceleration and smaller gradients lead to decay and smaller resulting values of the Lorentz factor γ. The phenomenon as such, however, does not depend critically on the value of the accelerating gradient within about a factor of 2.

In Fig. 2 is shown the surviving fraction and Lorentz factor of Ps^- during acceleration in the mentioned rf cavity, compared to a (hypothetical) uniform field of 25 MV/m. The oscillations in the rf cavity Lorentz factor within the first few wavelengths result in the "kinked" behavior of the slope seen in Fig. 1.

The bunch length of the primary positron beam (about 10 ns), and therefore of the Ps^- beam, is typically much larger than the rf period (about 1 ns) and therefore much larger than the so-called rf bucket for acceleration. Thus, a significant fraction (about 70%) of the Ps^- ions are lost due to mismatching of the rf phase; i.e., some ions are initially non accelerated or accelerated in the wrong direction. On the other hand, this removes the complication of phase-matching the rf and positron ejection system on the sub-ns time scale. Reducing the rf frequency to ease such a phase matching results in a lower maximum gradient following the rough scaling law $E_{\max} \propto \lambda^{-7/8}$ [40]; i.e., there is a trade-off between intensity and final energy. The nondecayed fraction

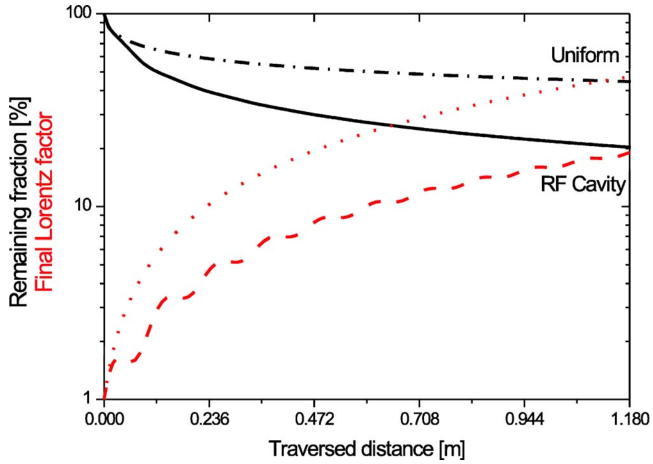

FIG. 2. (Color online) The surviving fraction of Ps^- during acceleration in a E_0=60 MV/m, 1269 MHz (λ=0.236 m) rf cavity (solid line) with $\phi=\pi/2$, after acceleration in a 25-MV/m uniform field (dash-dotted line) and the respective Lorentz factors (rf, dashed line; uniform, dotted line), all as a function of cavity length.

$\exp(-\tau/\tau_0)$ and resulting Lorentz factor γ as a function of phase ϕ are shown in Fig. 3. On the average 26% of the ions with phases in the interval $\phi/\pi \in [0.12;0.48]$ survive and appear with Lorentz factors $\gamma \in [20.0;23.8]$. Thus, of the total produced Ps^- ions, 4.8% are accelerated to $\gamma \simeq 22$.

As for the emittance, a rough estimate can be based on the following few steps: first, the extraction of positrons from a Surko-type trap to zero magnetic field and a kinetic energy of 100 eV, which results in an approximate emittance of 60 mm mrad [41]. Second, the emittance increase from the multiple scattering $\Delta\varepsilon \simeq \pi\beta_t\Delta\theta^2/2$, $\Delta\theta$ being the multiple-scattering angle, and from the double-electron capture in the carbon foil can be reduced to a negligible level by applying a focus at the foil—i.e., by minimizing the betatron amplitude to $\beta_t \lesssim 0.1$ m. Finally, the transport matrix of a pure π-mode cavity with $\phi_0=\pi/2$ [42] shows a reduction of the emittance by a factor of $\simeq 40$, mainly due to adiabatic damping [43]. The emerging beam thus has an emittance of approximately a few mm mrad, comparable to the best of other proposed schemes [27].

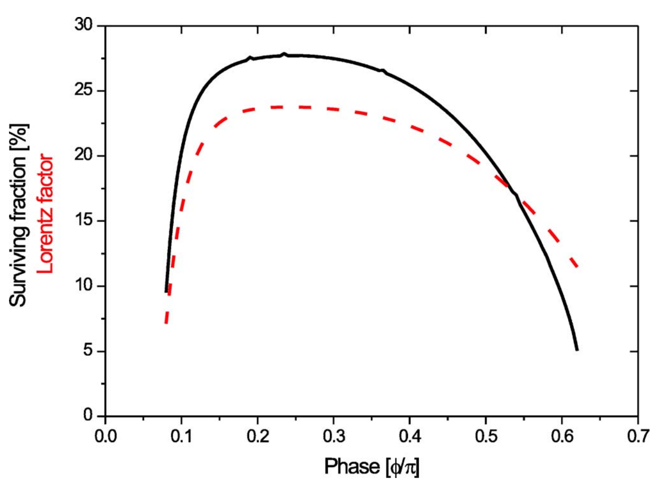

FIG. 3. (Color online) The surviving fraction (solid line) and the resulting Lorentz factor (dashed line) of Ps^- after acceleration in a E_0=60 MV/m, 1269 MHz (λ=0.236 m) rf cavity as a function of phase.

IV. FIELD STRIPPING IN THE CAVITY

Following [44,45], the probability per unit time of detaching an electron in an electric field E is given as

$$W_{lm_l}(E) = A_{lm_l}\left(\frac{F(E)}{\gamma_b^3}\right)^{m_l+1}\exp\left(\frac{-2\gamma_b^3}{3F(E)}\right), \tag{8}$$

where

$$F(E) = \frac{meE}{\hbar^2} \tag{9}$$

expresses the field strength and

$$\gamma_b^2 = \frac{2m\mathcal{E}_b}{\hbar^2} \tag{10}$$

depends on the binding energy $\mathcal{E}_b$=0.3267 eV. The coefficient in Eq. (8) is given as

$$A_{lm_l} = B_l^2\frac{\hbar\gamma_b^2}{m}\frac{2l+1}{2^{2m_l+2}}\frac{(l+m_l)!}{(l-m_l)!\,m_l!}, \tag{11}$$

with m being the magnitude of the projection of the orbital quantum number l and

$$B_l^2 = 2^{2l+1}\frac{(l-1)!\,(l+1)!}{(2l-2)!\,(2l+2)!}\exp(\gamma_b r_0)(\gamma_b r_0)^{2l-1}, \quad B_0^2 = 2, \tag{12}$$

for $\gamma_b r_0$, r_0 being the "range" of the potential well. For $l = m_l = 0$ the exact value of r_0 is immaterial.

Finally, the survival probability in the case of an inhomogeneous electric field is found from

$$I = \exp\left(-\int_0^\tau W_{lm_l}(E(t))\mathrm{d}t\right), \tag{13}$$

where τ is the proper time of the ion spent in the electric field $E(t)$ calculated in its rest frame. The general Lorentz transformation of the fields $\vec{E}$ and $\vec{B}$ to a frame with velocity $\vec{\beta}c$ is given as [46]

$$\vec{E}' = \gamma(\vec{E} + \vec{\beta}\times\vec{B}) - \frac{\gamma^2}{\gamma+1}\vec{\beta}(\vec{\beta}\cdot\vec{E}) \tag{14}$$

and

$$\vec{B}' = \gamma(\vec{B} - \vec{\beta}\times\vec{E}) - \frac{\gamma^2}{\gamma+1}\vec{\beta}(\vec{\beta}\cdot\vec{B}). \tag{15}$$

Assuming cylindrical symmetry of the rf fields, using Maxwell's equations (see [38]) and Eq. (3), the radial component of the electric field becomes $E_r = \pi E_0 \sin(kz)\sin(\omega t+\phi_0)r/\lambda$; i.e., as long as radial coordinates $r \ll \lambda/\gamma\pi = 75$ mm$/\gamma$ are considered, it is sufficient to use E_z to calculate the field stripping. Identical conditions (in the limit β=1) apply for

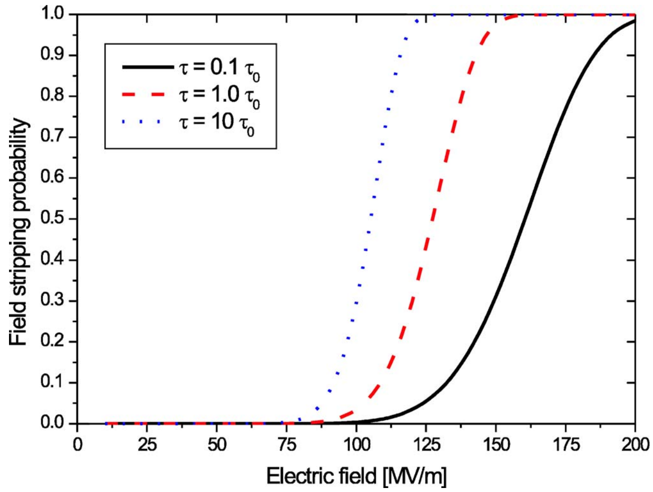

FIG. 4. (Color online) The field stripping probability as a function of electric field. The curves are calculated for $\tau = 0.1\tau_0$ ns (solid line), $\tau = \tau_0$ ns (dashed line), and $\tau = 10\tau_0$ ns (dotted line).

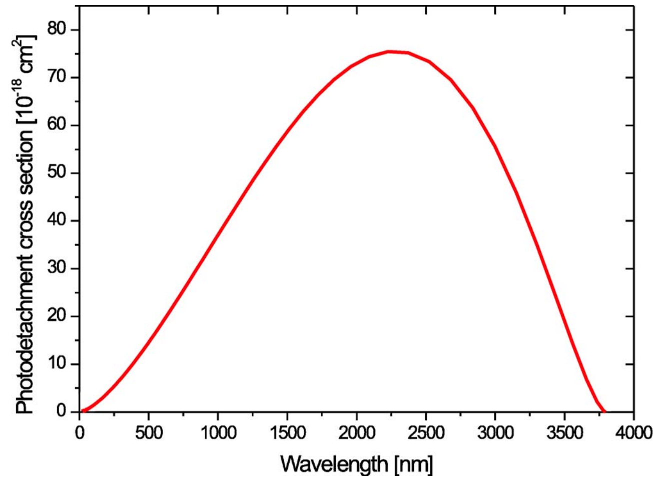

FIG. 5. (Color online) The photodetachment cross section as a function of wavelength according to [60] [Eq. (28)].

the azimuthal component of the rf magnetic field B_θ.

As seen from Fig. 4, the field stripping probability for the chosen rf cavity of 60 MV/m is negligible. Doubling the accelerating field, however, would lead to stripping of an appreciable fraction of the beam.

Interestingly, although Ps^- is the lightest charged species with internal structure, rapid acceleration in stripping limited fields of the order 100 MV/m only leads to a shift of 2×10^{-10} eV through the Unruh mechanism: see [47] for a recent review. Even the hyperfine interval in Ps is more than six orders of magnitude higher in energy [48–50] which seems to preclude a measurement of the Unruh effect using Ps^-. Furthermore, it is unlikely that relativistic Ps will prove advantageous compared to the use of thermal Ps in a microwave cavity, to address the possible positronium hyperfine puzzle [51,52].

On the other hand, passing relativistic positronium or its negative ion through a weak magnetic field enables an investigation of the synchrotron radiation emitted from a nonelementary “particle.” Whether such “particles” radiate at all (Ps) or radiate like three or one elementary charges (Ps^-) is likely to depend upon the observed frequencies—i.e., the formation time for the emitted radiation. Such questions are presently under investigation.

V. PHOTO DETACHMENT

Cross sections for photodetachment as a function of, e.g., photon energy have been calculated [53–58] but have hitherto not been measured. By photodetachment, the Ps^- beam can be used to generate a beam of Ps.

One complication arises due to the proposed photodetachment if taking place in a strong electric field: complications such as modulations imposed by quantum interference between different detachment channels [59].

The binding energy can possibly be measured to a reasonable accuracy by a least-squares fit of the function $\sigma = \sigma_0(\lambda/\lambda_0)^{3/2}(1-\lambda/\lambda_0)^{3/2}$ [56–58] [or [60] Eq. (28)] shown in Fig. 5 to the experimental photodetachment cross section and thus extracting the threshold wavelength λ_0, which gives the binding energy. As seen in Fig. 5, the photodetachment cross section peaks at a wavelength of $\simeq 2250$ nm with a magnitude of $\simeq 75 \times 10^{-18}$ cm^2, about 10% higher than a previous calculation [62], which, however, does not provide an analytic expression. At 2 μm wavelength, $\gtrsim 25$ mJ, $\simeq 10$ ns pulses with a repetition rate of >10 Hz may be obtained from diode-pumped Tm,Ho:YLF laser systems used for, e.g., atmospheric measurements. Alternatively, by using the Doppler shift of the laser frequency to the frame of the ion $\omega' = \gamma\omega[1-\beta\cos(\theta)]$ a CO_2 laser of 10.6 μm at a central angle of 38° will operate around the maximum of the cross section. The achievable intensities of such lasers are sufficient to photodetach a substantial fraction of the Ps^- ions and even to study multiphoton detachment [61]. Moreover, by applying a Nd:YAG laser of 1064 nm at 38°, it is possible to populate the $Ps(n=2)$ state by exploiting the Doppler shift and the resonance at 5.4287 eV in the photodetachment spectrum [62].

VI. LORENTZ STRIPPING

Being relativistic, the Ps^- ions passing a perpendicular magnetic field B experience an electric field in their rest frame obtained from Eq. (14) as

$$E = \beta\gamma B, \tag{16}$$

which may be inserted in $p_L = 1 - I$ with I from Eq. (13) to give the Lorentz stripping probability p_L. This has been thoroughly tested with H^- beams; see, e.g., [63–65]. With B in tesla, the electric field in Eq. (16) appears in units of 299.8 MV/m. A magnetic field of 0.7 T can be easily obtained from permanent magnets and about a factor of 3 higher for conventional electromagnets. Thus, the Lorentz-transformed electric field experienced by $\gamma = 22$ Ps^- ions in $B = 0.7$ T becomes 4.6 GV/m, sufficient to strip the ion in a few femtoseconds. Lorentz stripping in high fields is thus extremely efficient, but due to the exponential tunneling factor in Eq. (8), weak magnetic fields $\ll 0.2$ T (such as, e.g., Earth's magnetic field) are not detrimental.

The stripped Ps atom which in the field-free case appears in either the ortho or para state, will suffer state mixing while in the magnetic field as discussed above [6]. In a laboratory field of 0.7 T, a $\gamma=22$ Ps atom according to Eq. (15) experiences a field of 15.4 T in its rest frame. This leads to a significant perturbation of the decay rates for the two Ps states according to [66]

$$\lambda_T' = \frac{1}{1+y^2}(\lambda_T + y^2\lambda_S) \tag{17}$$

and

$$\lambda_S' = \frac{1}{1+y^2}(\lambda_S + y^2\lambda_T), \tag{18}$$

where λ' and λ are the decay rates with and without magnetic field, indices denoting the triplet (T) and singlet (S) states, respectively. In Eqs. (17) and (18) the magnetic field enters through $x=2g(1-5\alpha^2/24)\mu_B B/h\nu = B/3.629$ T and $y = x/[1+(1+x^2)^{1/2}]$ with the electron g factor, the Bohr magneton μ_B, the fine-structure constant α, and the Ps hyperfine energy splitting $h\nu$. In a 15.4-T rest-frame field the decay rates become of the same order of magnitude, $\lambda_T' = 3083\ \mu s^{-1}$ and $\lambda_S' = 4915\ \mu s^{-1}$, which means lifetimes in the few hundred picosecond ranges. However, time dilation as seen from the laboratory frame means that a magnetic field of 0.7 T over a distance $\gamma c/\lambda' \simeq 2$ m is required for a significant fraction to decay. On the other hand, such fields would enable a test of Eqs. (17) and (18) in a hitherto unexplored regime, $x > 1$, $y \simeq 1$.

The positrons that do not capture electrons during the passage of the carbon foil will—depending on the rf phase—either be returned through the foil or accelerated through the cavity. The returned fraction may be recycled using the parabolic bunching potential to be kept constant except at the injection of a new positron bunch; i.e., the quoted intensities are likely to be lower limits. The accelerated positrons will only utilize the opposite rf bucket (180° out of phase with the Ps^- ions) and will thus be separate from the Ps^- beam and deviate from it in the magnetic field.

VII. CONCLUSION

With a positron beam of $6\times10^6 e^+/s$ [1], a $e^+ \to Ps^-$ conversion efficiency of 3×10^{-4} and a total acceleration efficiency of $\simeq 5\%$ in a $E_0 = 60$ MV/m, 1269 MHz ($\lambda = 0.236$ m) rf cavity, a low-emittance beam of about 100 Ps^- ions per second with $\gamma \simeq 22$ can be obtained. Field stripping during acceleration is shown to be negligible for fields $\lesssim 100$ MV/m. Passing the ions through a 0.7-T magnetic field Lorentz strips essentially all the ions and a $\gamma \simeq 22$ Ps beam of 100 atoms/s can be obtained. The proposed scheme yields Ps atoms with an intensity almost as high as the LEPTA project [19], but with significantly higher energy—i.e., in the relativistic regime. Compared to other schemes for the generation of relativistic Ps, the present proposal, e.g., offers intensities more than three orders of magnitude higher.

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