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Wavelength conversion via dynamic refractive index tuning of a cavity

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We demonstrate numerically that the wavelength conversion of light is possible by the simple dynamic refractive index tuning of an optical cavity in a photonic crystal. We also clarify the mechanism and conservation rule for this conversion process. In addition, we discuss the observability of this phenomenon in realistic cavities. Our results indicate that this linear adiabatic wavelength conversion process can be observed for various high-Q microcavities.

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The wavelength conversion of light is a very important phenomenon that can be employed for various optical engineering technologies; for example, information processing in wavelength-demultiplexing communication. For converting the wavelength, we generally utilize a nonlinear optical process [e.g., $\chi^{(2)}$ process], where higher-order polarization generates new frequency components [1]. This conversion requires the use of highly nonlinear crystals, and the conversion efficiency depends on the input light intensity, phase-matching condition, and light traveling distance. Thus, it is generally difficult to realize high efficiency for weak light in a tiny sample.

Recently, several theoretical papers have indicated that the wavelength properties of light can be modified by dynamic processes in photonic crystals (PCs). Reed *et al.* showed numerically that a light pulse reflected by a shock-wave front traveling in a PC exhibits a large wavelength shift and spectral compression [2]. The shift itself may appear similar to the Doppler effect, but the physical mechanism is different. Subsequently, Yanik *et al.* showed that if the refractive index of a coupled-resonator waveguide implemented in a PC is dynamically changed, light propagation can be stopped or inverted [3,4]. One of the most important aspects of their processes is that the spectral width of the pulse is dynamically compressed. These results suggest that we can dynamically control the wavelength properties of a light pulse by dynamically controlling the dispersion characteristics of the material or waveguide. However, there have been no direct or detailed investigations of such wavelength controllability. In this communication, we investigate a much simpler system—a single cavity—so as to clarify how the dynamic process can affect the wavelength properties of light. We investigate the physical mechanism of this phenomenon in comparison with conventional wavelength conversion and discuss whether we can exploit such effects practically to realize wavelength conversion in realistic structures.

Here we consider a single cavity in a two-dimensional (2D) hexagonal air-hole PC slab, which has been shown to exhibit a large quality factor (Q) and ultrasmall mode volume [5,6]. First, we study numerically the dynamic effect using a simple 2D model with the finite-difference time-domain (FDTD) method [7]. The model cavity is shown in Fig. 1(a), which has a resonant mode at $\lambda = 1623$ nm for H polarization (H is perpendicular to the 2D plane). After numerically exciting this resonant mode, the refractive index of the high-index material around the cavity [gray area in Fig. 1(a)] is tuned dynamically by $\Delta n/n = \delta = +5\times10^{-3}$ from n_1 at $t=t_1$ to $n_2 = (1+\delta)n_1$ at $t=t_2$ as a function of time as shown in Fig. 1(c). Such index tuning can be undertaken in various

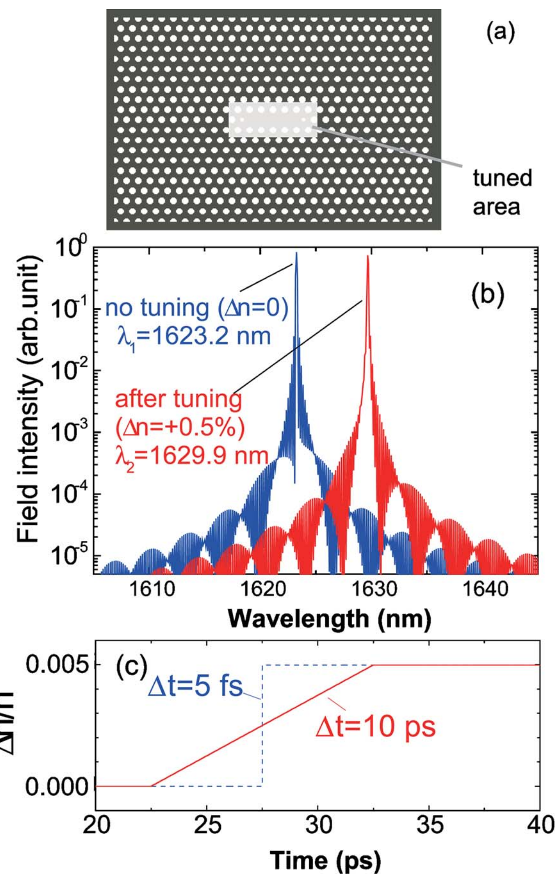

FIG. 1. (Color online) Wavelength conversion in a four-point-defect PC cavity. (a) Schematic of the cavity in a 2D hexagonal air-hole PC slab. The lattice constant $a=420$ nm, the hole radius $r=0.275a$, the end-hole radius $r_e=0.125a$, and $n_{eff}=2.78$. The index of the gray region is tuned. (b) Wavelength spectra with and without the index tuning calculated by 2D FDTD. (c) Temporal variation of the refractive index.

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ways. For example, we have recently demonstrated index tuning in a similar Si PC cavity system as a result of the optical nonlinear effect [8–10].

Figure 1(b) shows calculated wavelength spectra of the electromagnetic field in the cavity with and without index tuning. The spectra were obtained by Fourier conversion of the temporal field data for $t > t_2$. We used two different tuning times $\Delta t (=t_2 - t_1)$ of 5 fs and 10 ps and obtained the same spectra. We later consider the effect of the tuning rate. As clearly seen in the two spectra, the wavelength of the light in the cavity is converted from λ_1 to λ_2 after the dynamic tuning. Note that there is no noticeable peak at the original wavelength λ_1 after the tuning, which means that $\sim$100% wavelength conversion occurs in this process. The relative wavelength shift $\Delta\lambda/\lambda$ almost coincides with the relative index shift $\Delta n/n$. To be more accurate, we numerically confirmed that the final wavelength λ_2 is exactly the same as the new resonance wavelength of the cavity with a refractive index of n_2. In other words, the wavelength of the light in the cavity follows the resonance wavelength of the cavity with a changed n [we refer to this as $\lambda_{res}(n)$]. Consequently, we found the following relation for this dynamic process: $\Delta\lambda/\lambda = \Delta\lambda_{res}(n)/\lambda \sim \Delta n/n$. The slight deviation between $\Delta n/n$ and $\Delta\lambda_{res}(n)/\lambda$ is due to the fact that we do not change the refractive index of air in which there is slight light field. In addition, this wavelength shift does not depend on the rate of the dynamic change, which directly proves that this conversion process is fundamentally different from the various pulse spectrum distortion effects induced by the Kerr effect or sideband generation by high-speed optical modulators.

Next, we investigate the energy conservation relation in this conversion process. In conventional wavelength conversion processes, the field energy U is conserved at the photon level as $\hbar\omega_0 = \hbar\omega_1 + \hbar\omega_2$, where ω_0 is the angular frequency of the initial light, ω_1 and ω_2 are those of the converted light in the case for the $\chi^{(2)}$ process (or ω_2 could be that of the phonon in the case for acousto-optic effects). In our case, however, the energy conservation is fundamentally different since no multiphoton process or interaction with other elementary excitation is involved. In fact, U does not have to be conserved because the variation of n must affect U (whose dielectric part is expressed as $n^2|E|^2$). Figure 2(a) shows the calculated U in the cavity as a function of time during the conversion process. All the conditions are the same as those in Fig. 1. We integrated the field energy density inside the cavity. Clearly, U is not conserved during the conversion process even if we exclude the energy loss due to the finite Q. Note that the net change in U is the same whatever the tuning rate. Figure 2(b) summarizes the estimated ΔU and $\Delta\lambda$ for various Δn values. This plot clearly shows that when n increases, λ is converted to longer wavelength and simultaneously U *decreases*. Although the fact that $U \sim n^2|E|^2$ might imply that the increase of n leads to the *increase* in U in the case of conversion to longer wavelength, the result in Fig. 2 is opposite. In addition, the increase in λ is exactly the same as the reduction of U, that is, $\Delta\lambda/\lambda = -\Delta U/U$. To observe this more directly, we plot $\Delta(U\lambda)$ in the same figure, which clearly shows that $(U\lambda)$ is conserved during the whole process. We confirmed that in all

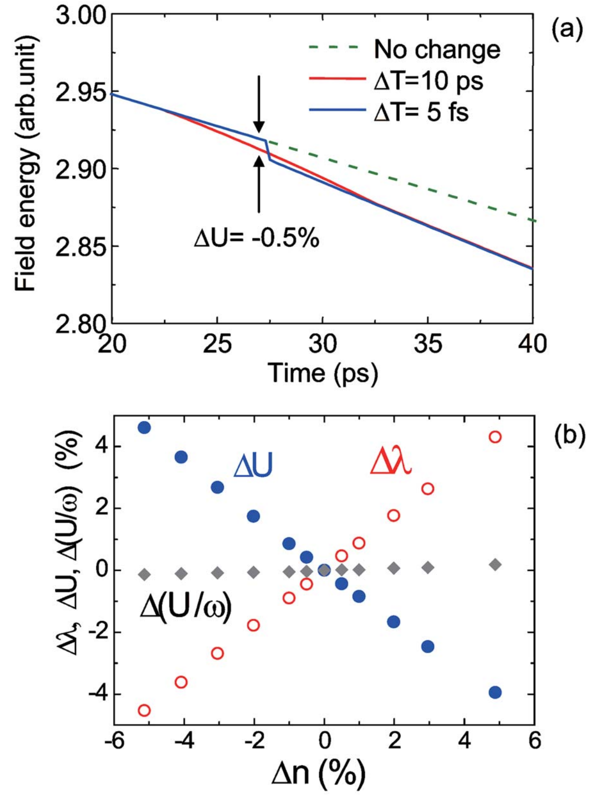

FIG. 2. (Color online) Temporal variation of the integrated electromagnetic field energy in the cavity during the conversion process shown in Fig. 1. (a) Field energy as a function of time for different tuning times. (b) ΔU, $\Delta\lambda$, and $\Delta(U\lambda)$ vs Δn.

cases, $(U\lambda)$ is the conserved quantity during the conversion process. As is well known in classical mechanics, the quantity $J = \oint p dq = U/\omega$ $(\sim U\lambda)$ called the action integral is an adiabatic invariant for classical oscillators when some of the oscillator parameters are dynamically changed in an adiabatic fashion [11]. In our case, $U\lambda$ corresponds to the adiabatic invariant for electromagnetic oscillations in this system. This explains the energy conservation rule for the current conversion process. This conservation rule naturally holds for a photon level as $N\hbar\omega_1\lambda_1 = N\hbar\omega_2\lambda_2$ (N is the number of photons). This is reasonable because the energy quanta and Planck constant were originally defined as adiabatic invariant in quantum mechanics.

It is now clear that this conversion process is fundamentally different from the conventional wavelength conversion described as a multiphoton process. The former is adiabatic where the number of energy quanta is conserved, but the latter is nonadiabatic where the number is not conserved. We can understand this adiabatic wavelength conversion process by analogy with the dynamical tuning of classical oscillators. Let us think of a musical instrument such as a guitar, which is a simple example of a classical oscillator. We pluck a string to generate the note of A. If we then turn the tuning peg before the vibration is dying out, we can raise the note from A to a higher one. In this process, U/ω is conserved. We can change the tone by dynamically changing the resonance frequency of the guitar. Note that the adiabatic process has been studied in various atom-photon interactions in

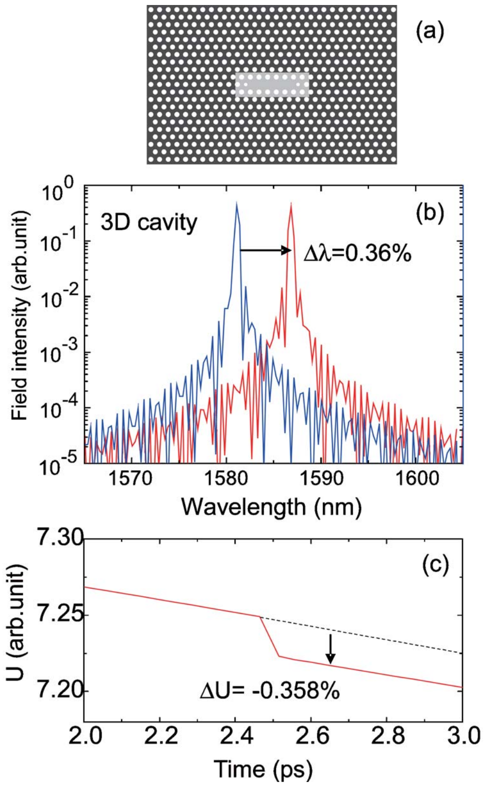

FIG. 3. (Color online) Dynamic tuning in a realistic partial PBG cavity in a finite-thickness PC slab obtained by full 3D calculation. (a) A schematic of the 3D five-point-defect cavity. $a=420$ nm, $n=3.46$, $r=0.275a$, the end-hole radius $r_e=0.125a$, and the thickness of the slab $t=0.5a$. (b) Wavelength conversion for the partial PBG cavity shown in (a) at $\Delta t=5$ fs. (c) Integrated field energy in the cavity as a function of time.

quantum optics (for example, one can change the state of *atoms* by coherent light using adiabatic following) [12], but the present situation is fundamentally different since the state of *light* is adiabatically modified.

Since all the previous reports (Refs. [2–4]) and our results in Figs. 1 and 2 assumed a perfect 2D photonic band gap (PBG), this phenomenon may be explained by the fact that the field energy U stored in the cavity cannot escape at any instance during the transition due to the existence of the PBG, and thus is inevitably converted to the new resonant mode at λ_2, which is the only allowed mode in the final state. However, we will see that this explanation is too simple. We next investigate more realistic three-dimensional (3D) cavities which have only partial PBGs to check whether the existence of perfect PBGs is essential or not. Indeed, this is an important issue because in practice it is very difficult to realize perfect PBGs. In terms of experimental observability, the verification in realistic 3D cavities with partial PBGs is crucial. We calculated a basically similar cavity shown in Fig. 3(a) assuming a finite thickness by the full 3D FDTD method. Due to leakage into the vertical direction, this cavity has a finite Q of 230 000. Figure 3(b) shows calculated spectra for this 3D cavity with and without the dynamic tuning at $\Delta t=5$ fs. Figure 3(c) shows U as a function of time. The result in Figs. 3(b) and 3(c) is similar to that in Figs. 1(b) and 2(a). This clearly shows that a perfect PBG is not necessary for this phenomenon, and that the conversion is possible for experimentally realizable cavities.

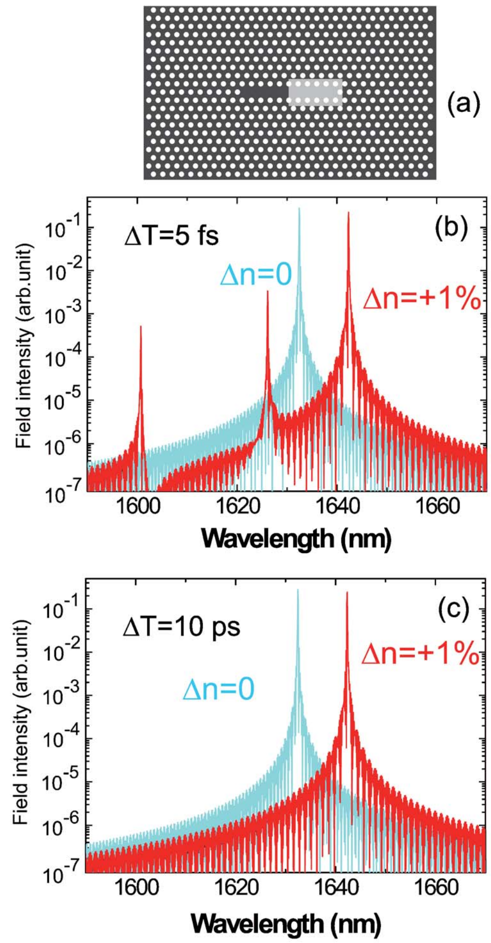

FIG. 4. (Color online) Effect of spatial homogeneity and mode separation on adiabaticity criterion. (a) A schematic of the ten-point-defect cavity. The index of the gray area is tuned. Only the right half of the cavity area is tuned. (b) Wavelength spectra with and without index tuning in the case of spatially inhomogeneous tuning for a ten-point-defect cavity (2D calculation) at $\Delta t=5$ fs. Other geometrical parameters are the same as those in Fig. 1(a). (c) The same as (b) at $\Delta t=10$ ps.

Next, we discuss the realizability of this phenomenon. As the first criterion, adiabatic continuity between the initial and final states should be ensured. This will be violated if the tuning time is too short and some untargeted modes are excited. Thus a small high-Q cavity having a large mode separation is generally preferable, or one can avoid the excitation of other modes by symmetry conservation even for instant tuning. For example, if we change *n homogeneously* within the cavity, adiabatic continuity is maintained even for a very short tuning time because of the momentum conservation. Note that these two conditions are satisfied in the results shown in Figs. 1–3 because the cavity size is small and we assumed a homogeneous index variation. This is why these results do not exhibit the tuning rate dependence even for an extremely fast tuning. To check adiabaticity conditions directly, we investigated a relatively long cavity having smaller mode separation assuming asymmetric index tuning (the tuned area is the right half of the cavity) as in Fig. 4(a). For $\Delta t = 5$ fs, three different modes are excited [Fig. 4(b)], indicating that the process is no longer adiabatic. However if we employ a much slower tuning rate ($\Delta t = 10$ ps) as shown in Fig. 4(c), the adiabatic conversion recovers. Furthermore, if we tune the index homogeneously, adiabatic conversion occurs in both cases ($\Delta t = 5$ fs and 10 ps). In the case of the short cavity shown in Fig. 1(a), we confirmed that the excitation of other modes is negligible (less than 10^{-5}) even with asymmetric index tuning and $\Delta t = 5$ fs. In terms of the cavity size and mode separation, the PC cavity shown in Fig. 1(a) is similar to most of PC microcavities investigated recently [5,13–15]. Thus, adiabaticity should be maintained for most of wavelength-scale PC cavities under most experimental conditions.

To discuss the experimental observability more practically, we need to consider the way we change n. This leads to the second criterion for the observability and also the available $\Delta\lambda$. The second criterion is that the photon lifetime (τ_{ph}) must be longer than the index tuning time. This is the most critical issue and apparently high-Q cavities and slow-light waveguides are suitable. In addition (third criterion), the wavelength shift should be larger than the wavelength width of the cavity resonance which is also determined by τ_{ph}. The typical τ_{ph} in high-Q PC cavities is 0.1–1 ns (corresponding to $\Delta\lambda/\lambda \sim 10^{-5}$–$10^{-6}$). These conditions can be met by various methods. One way is to use the carrier-plasma effect that we recently used for all-optical switching in Si PC cavities [9,10]. For this, we illuminate the cavity from the top with a light pulse whose wavelength is within the absorption band of Si. Alternatively, we can excite the cavity's other low-Q resonant mode via a coupled waveguide so as to induce two-photon absorption inside the cavity [9,10]. For both cases, the generated carriers cause Δn due to the plasma effect. We experimentally observed $\Delta n \sim 10^{-3}$ with 10 mW illumination without degrading Q. Besides, we can make use of Kerr effect or even thermo-optic nonlinearity induced by the rapid thermalizing process of hot carriers. The available $\Delta\lambda/\lambda$ is determined by the largest $\Delta n/n$, which is approximately 10^{-4}, 10^{-3}, and 5×10^{-2} for Kerr, plasma, and thermo-optic effects, respectively. The way is not limited to optical. We can use electro-optic tuning (e.g., quantum-confined Stark effect enables $\Delta n/n \sim 10^{-3}$). All the effects are faster (psec or less) than τ_{ph} and large enough for the criterions for existing cavities. Moreover, we can directly alter λ_{res} (instead of n) by deforming the cavity mechanically or using piezoelectric effect, which may become practical for extremely high-Q cavities.

Finally, we summarize the features of this conversion process in comparison with other conversion processes: (i) it is a linear process, (ii) it does not depend on the initial light intensity, (iii) the shift does not depend on the tuning rate, (iv) it can occur in any materials if the resonance can be dynamically varied, (v) no need for phase matching, and (vi) the conversion efficiency is close to 100% (we define the efficiency as the ratio of the number of photons converted) if the leakage is negligible. These features will enable efficient wavelength conversion of even single photons stored in ultrasmall cavities, which may be important for quantum communication since all the information including entanglement is preserved. One drawback is the fact that $\Delta\lambda$ is limited by the available Δn and is much smaller than conventional wavelength conversion.

The analogy with the tuning of a guitar shows that the phenomenon we investigated here is purely classical, but no one has explicitly discussed the use of this *tuning of light* for wavelength conversion as far as we know. The reason for this is that such classical tuning of electromagnetic oscillation is normally impossible due to the generally short τ_{ph} in small structures. However, recent advances in ultrahigh-Q microresonators [14,15] and ultraslow-light waveguides [16,17] are making such classical tuning of light possible. We believe that the wavelength conversion investigated here will be an important optical process in such ultra-long τ_{ph} media.

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