Correlated photons and collective excitations of a cyclic atomic ensemble

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We systematically study the interaction between two quantized optical fields and a cyclic atomic ensemble driven by a classic optical field. This so-called atomic cyclic ensemble consists of three-level atoms with Δ-type transitions due to the symmetry breaking, which can also be implemented in the superconducting quantum circuit by Yu-xi Liu *et al.* [Phys. Rev. Lett. **95**, 087001 (2005)]. We explore the dynamic mechanisms to creating the quantum entanglements among photon states, and between photons and atomic collective excitations by the coherent manipulation of the atom-photon system. It is shown that the quantum information can be completely transferred from one quantized optical mode to another, and the quantum information carried by the two quantized optical fields can be stored in the collective modes of this atomic ensemble by adiabatically controlling the classic field Rabi frequencies.

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I. INTRODUCTION

The electric-dipole selection rule does not allow one- and two-photon processes to coexist for given initial and final states in quantum systems with a center of inversion symmetry, where all states have well-defined parities [1]. Because one-photon transitions, resulted from the electric-dipole interaction between two nondegenerate states, require that these states have opposite parities; but two-photon process needs those states to have the same parity [1]. However, the one- and two-photon processes can coexist in systems with lack of inversion symmetry, e.g., in the semiconductor systems [2–5]. Then, the magnitude and direction of the photocurrent in these systems [2–5] can be controlled by using two different optical paths.

Reference [6] showed that electric-dipole transition between any two states is allowed for chiral molecules and their mirror images due to lack of inverse symmetry. It means that the one- and two-photon processes can also coexist in these systems. Then, the same initial and final states can be connected by two different paths, which result in an interference effect for final state. The different relative phase differences for pulses of the two optical pathways will result in different interference fringes. This implies that final state can be controlled by choosing applied pulse phases. Using an example in Ref. [6], the coherent population transfer (CPT) was studied in a three-level system with cyclic transitions, induced by three classical fields. Different from the usual Λ-type atoms [7], the CPT in these systems is controlled not only by the amplitudes of the electric-dipole transition elements, but also by the phases of applied pulses.

Recently, the microwave control of the quantum states has been investigated for superconducting quantum circuits [8], called "artificial atoms," which possess discrete energy levels. The optical selection rule of microwave-assisted transitions is carefully analyzed for this artificial atom. It was shown [8] that the electric-dipole like transition can appear for any two different states when the symmetry of the potential of the artificial atom is broken by changing microwave bias. Then, so-called Δ-type or cyclic transitions can be formed for the lowest three levels. The populations of these states can be selectively transferred by adiabatically controlling both the amplitudes and phases of the applied microwave pulses.

The previous investigations, e.g., in Refs. [2–6,8], only focus on a single three-level system with Δ-type or cyclic transitions, induced by the three classical fields. In contrast to the above examples [2–6,8], the electric-dipole interaction cannot induce cyclic transitions among three energy levels for a usual atom, due to its symmetry and well-defined parities of its eigenstates. To have cyclic or Δ-type transitions, a coherent radio-frequency field is required to apply such that it can induce a magnetic dipole (or an electric quadrupole) transition [10–12] between two levels, e.g., two lower (or higher) levels which are forbidden to the electric-dipole transition in the Λ (or V)-type atoms [7].

In this paper, we will investigate the collective effects of photonic emissions and excitations of a cyclic three-level system (such as atomic ensemble with symmetry broken, or the chiral molecular gases [6], or manual "atomic" array with symmetry broken [8,9]) where the quantum transitions is induced by two quantized fields and a classical one. We will focus on the photonic properties of emissions resulting from such cyclic transitions, such as the two-mode photon entanglement, the quantum state exchange and swapping between the two-mode optical field and the two collective modes of atomic excitations.

In more details, by utilizing the collective operator approach and the hidden dynamic symmetry as recently discovered [14] for the three-level Λ-type atomic ensemble coupled to a classical control field and a quantum probe field, both

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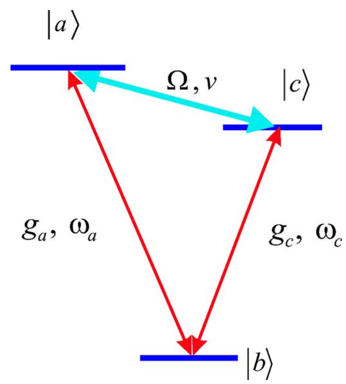

FIG. 1. (Color online) Three-level cyclic atoms are resonantly coupled to two quantized fields and a classical field via electric-dipole interaction.

the adiabatic and dynamic properties for the system of the photons and atomic ensemble are studied systematically. Different from the case of three-level Λ (or V, or Ξ)-type atomic ensemble, due to the interference between one- and two-photon processes in the system of Δ-type atomic ensemble, we find that the electromagnetically induced transparency (EIT) phenomenon [15], appeared in Λ-type system, does not exist here. Instead of dark-state polariton operators [14], a general set of polariton operators is introduced to describe the collective motions of the whole system when the excitation to high energy levels is low. Some unique results are obtained. For example, the entanglement between two quantum optical fields is tunable via classical field, applied to the Δ-type atomic ensemble.

Our paper is organized as follows. In Sec. II, the model is described and the polariton operators are introduced in the limit of the low excitation. In Sec. III, the entanglement between, e.g., the atomic ensemble and quantized fields, or two different optical modes, is discussed. In Sec. IV, we analyze the information transfer from the quantized fields to the atomic ensemble by adiabatic passage, and study the storage of photon information via atomic ensemble.

II. ATOMIC ENSEMBLE WITH CYCLIC TRANSITIONS AND POLARITON EXCITATIONS

We consider an ensemble with N identical three-level "atoms" (such as atomic ensemble or the chiral molecular gases [6] or manual "atomic" array with symmetry broken [8]) interacting with electromagnetic fields. Each atom has cyclic or Δ-type transitions shown in Fig. 1, where a lower level $|b\rangle$ is coupled to two higher levels $|a\rangle$ and $|c\rangle$ by quantized fields through the electric-dipole interaction; and two higher states are coupled by a classical field with a frequency ν through the electric-dipole interaction. The Hamiltonian of the interacting system is given as ($\hbar=1$)

$$H_{ori} = \omega_a a^\dagger a + \omega_b b^\dagger b + \omega_{ab}\sum_{j=1}^{N}\sigma_{\mathrm{aa}}^{(j)} + \omega_{cb}\sum_{j=1}^{N}\sigma_{\mathrm{cc}}^{(j)} + g_a\sum_j e^{i\mathbf{K}_a\cdot\mathbf{r}_j}\sigma_{\mathrm{ab}}^{(j)}a + g_b\sum_j e^{i\mathbf{K}_b\cdot\mathbf{r}_j}\sigma_{\mathrm{cb}}^{(j)}b + \Omega' e^{-i\omega_\nu t}\sum_j e^{i\mathbf{K}_\upsilon\cdot\mathbf{r}_j}\sigma_{\mathrm{ac}}^{(j)} + \mathrm{H.c.} \quad (1)$$

Here, $\sigma_{\mathrm{mn}}^{(j)}=|m\rangle_{jj}\langle n|$, with $m,n=a,b,c$ but $n\neq m$, are the flip operators between the levels $|m\rangle_j$ and $|n\rangle_j$ of the jth atom. $\sigma_{\mathrm{mm}}^{(j)}=|m\rangle_{jj}\langle m|$ $(m=a,c)$ represent the population operators. a $(a^\dagger)$ and b $(b^\dagger)$ are the annihilation (creation) operators of the two quantized light fields, with the angular frequencies (wave vectors) ω_a $(\mathbf{K}_a)$ and ω_b $(\mathbf{K}_b)$, respectively. The parameters g_a and g_b denote the coupling constants associated with two quantized fields, coupling to the atomic transitions $|a\rangle\rightarrow|b\rangle$ and $|c\rangle\rightarrow|b\rangle$, respectively. Here, we assume that coupling constants g_a and g_b of all atoms to the two quantized fields are identical. ω_{ab} and ω_{cb} are the angular frequencies of the atomic transitions $|a\rangle\rightarrow|b\rangle$ and $|c\rangle\rightarrow|b\rangle$, respectively. Ω' is the Rabi frequency related to the atomic transition $|a\rangle\rightarrow|c\rangle$, driven by the classic field with the angular frequency ω_ν and the wave vector $\mathbf{K}_\nu$.

For the sake of simplicity, we assume that three light fields are *resonantly* coupled to the relevant atomic transitions, that is, $\omega_{ab}=\omega_a$, $\omega_{cb}=\omega_b$, and $\omega_{ac}=\omega_\nu$. Thus, in the interaction picture, the Hamiltonian (1) can be simplified to

$$H = g_a a\sum_j e^{i\mathbf{K}_a\cdot\mathbf{r}_j}\sigma_{\mathrm{ab}}^{(j)} + g_b b\sum_j e^{i\mathbf{K}_b\cdot\mathbf{r}_j}\sigma_{\mathrm{cb}}^{(j)} + \Omega'\sum_{j=1}^{N} e^{i\mathbf{K}_\nu\cdot\mathbf{r}_j}\sigma_{\mathrm{ac}}^{(j)} + \mathrm{H.c.} \quad (2)$$

Considering that any quantum state is allowed to have a global constant difference of phase factor, one can redefine new atomic states with the phase factors [13] as

$$|a'\rangle_j = e^{i\mathbf{K}_a\cdot\mathbf{r}_j}|a\rangle_j, \quad |b'\rangle_j = |b\rangle_j, \quad |c'\rangle_j = e^{i\mathbf{K}_b\cdot\mathbf{r}_j}|c\rangle_j. \quad (3)$$

We further assume that the *momenta* $\mathbf{K}_a$, $\mathbf{K}_b$, and $\mathbf{K}_\nu$ for the three light fields satisfy the *conversation* condition

$$\mathbf{K}_b + \mathbf{K}_\nu - \mathbf{K}_a = 0. \quad (4)$$

Then, after the factors $\exp(i\mathbf{K}_a\cdot\mathbf{r}_j)$, $\exp(i\mathbf{K}_b\cdot\mathbf{r}_j)$, and $\exp(i\mathbf{K}_\upsilon\cdot\mathbf{r}_j)$ are absorbed into atomic states, the Hamiltonian (2) can be rewritten as

$$H = g_a a\sum_j \sigma_{\mathrm{ab}}^{(j)} + g_b b\sum_j \sigma_{\mathrm{cb}}^{(j)} + \Omega'\sum_j \sigma_{\mathrm{ac}}^{(j)} + \mathrm{H.c.} \quad (5)$$

Here, we still use the notations $|m\rangle$ and $\sigma_{\mathrm{mn}}^{(j)}$ to denote $|m'\rangle$ and $\sigma_{\mathrm{m'n'}}^{(j)}$ $(m',n'=a',b',c')$.

Obviously, the above Hamiltonian describes the homogeneous couplings of the three light fields to atoms in the ensemble. This homogeneity means that there exist various collective excitations that can be characterized by the following operators [14]

$$T^- = \sum_{j=1}^{N}\sigma_{\mathrm{ca}}^{(j)}, \quad T^z = \sum_{j=1}^{N}(\sigma_{\mathrm{aa}}^{(j)} - \sigma_{\mathrm{cc}}^{(j)}),$$

$$A = \frac{1}{\sqrt{N}}\sum_{j=1}^{N}\sigma_{\mathrm{ba}}^{(j)}, \quad C = \frac{1}{\sqrt{N}}\sum_{j=1}^{N}\sigma_{\mathrm{bc}}^{(j)}. \quad (6)$$

Therefore, by using the above collective operators, the Hamiltonian (5) can be expressed as

$$H = g_a\sqrt{N}aA^\dagger + g_b\sqrt{N}bC^\dagger + \Omega' T^+ + \text{H.c.} \tag{7}$$

Equation (7) implies that there exists a dynamic symmetry in the considered system. This symmetry is characterized by Lie algebra generators A, C, T^- and their complex conjugates $A^\dagger$, $C^\dagger$, T^+ (also T^z), that satisfy the following commutation relations:

$$[A,C^\dagger] = [A,C] = 0,$$

$$[A,A^\dagger] = 1, \quad [C,C^\dagger] = 1,$$

$$[T^-,C^\dagger] = 0, \quad [T^+,C^\dagger] = A^\dagger,$$

$$[T^-,A^\dagger] = C^\dagger, \quad [T^+,A^\dagger] = 0,$$

$$[T^+,T^-] = T^z \tag{8}$$

in the large N and low excitation limit [14,16,17]. Where the low excitation means that the most atoms are in the ground state, only a few of them are excited into the higher states. In this case, the average numbers $\langle A^\dagger A\rangle$ and $\langle C^\dagger C\rangle$ of the atoms in the two excited states satisfy the condition $\langle A^\dagger A\rangle/N \ll 1$ and $\langle C^\dagger C\rangle/N \ll 1$. It means that two independent bosonic modes (A and C) of the atomic collective excitation exist in the ensemble.

Since the complex coupling constants can be rewritten as $g_a = g_a^0\exp[i\varphi_a]$, $g_b = g_b^0\exp[i\varphi_c]$, and $\Omega' = \Omega\exp[i\varphi_v]$, where g_a^0 and g_b^0 are positive real numbers, however Ω is a real number. Then the phases φ_a and φ_b can be absorbed into the operators A and C as follows: $A^\dagger\exp[i\varphi_a] \to A^\dagger$, $C^\dagger\exp[i\varphi_c] \to C^\dagger$. In this case, the operator T^+ should be changed as: $T^+ \to T^+\exp[i(\varphi_c - \varphi_a)]$. Without loss of generality, we now consider a simple case with $g_a^0\sqrt{N} = g_b^0\sqrt{N} \equiv g_N$. Under these conditions, the Hamiltonian (7) can be represented as

$$H = g_N aA^\dagger + g_N bC^\dagger + \Omega e^{i\varphi}T^+ + \text{H.c.}, \tag{9}$$

where $\varphi = \varphi_v + \varphi_c - \varphi_a$. The transform from Eq. (7) to Eq. (9) means that only the total phase of the three Rabi frequencies (g_a, g_b, and Ω') is involved in the dynamical evolution.

The Hamiltonian (9) can be diagonalized by using polariton operators D_i ($i = 1,2,3,4$) as

$$H = \sum_{i=1}^{4} \varepsilon_i D_i^\dagger D_i, \tag{10}$$

here the polariton operators

$$D_{1,2} = \frac{\sin\theta}{\sqrt{2}}(a \pm be^{i\varphi}) + \frac{\cos\theta}{\sqrt{2}}(Ce^{i\varphi} \pm A), \tag{11}$$

$$D_{3,4} = \frac{\cos\theta}{\sqrt{2}}(a \pm be^{i\varphi}) - \frac{\sin\theta}{\sqrt{2}}(Ce^{i\varphi} \pm A), \tag{12}$$

describe the normal bosonic modes with frequencies

$$\varepsilon_1 \equiv -\varepsilon_2 = \frac{\Omega + \sqrt{\Omega^2 + 4g_N^2}}{2}, \tag{13}$$

$$\varepsilon_3 \equiv -\varepsilon_4 = \frac{\Omega - \sqrt{\Omega^2 + 4g_N^2}}{2}. \tag{14}$$

In Eqs. (11) and (12), the first indexes of the left hand side correspond to the above symbols of the right hand side, and

$$\theta = \arctan\frac{2g_N}{\Omega + \sqrt{\Omega^2 + 4g_N^2}}. \tag{15}$$

It is obvious $\theta \in [0,\pi/2]$ for the positive real numbers g_a^0, g_b^0, and real number Ω. From Eq. (10), the eigenstates of the system can be given as

$$|\Psi_{lmnk}\rangle = |l,m,n,k\rangle_{D_1D_2D_3D_4} \equiv \frac{1}{\sqrt{l!m!n!k!}}D_1^{\dagger l}D_2^{\dagger m}D_3^{\dagger n}D_4^{\dagger k}|\mathbf{0}\rangle, \tag{16}$$

with the ground state $|\mathbf{0}\rangle \equiv |0,0\rangle_{ab} \otimes |\mathbf{b}\rangle$. Here, $|0,0\rangle_{ab}$ is the vacuum state of the two quantized optical fields, $|\mathbf{b}\rangle = \otimes\Pi_j|b\rangle_j$ is the ground state for all atoms with the definition $C|\mathbf{b}\rangle = A|\mathbf{b}\rangle = 0$. The eigenvalue of the state $|\Psi_{lmnk}\rangle$ is

$$\varepsilon_{lmnk} = (l-m)\varepsilon_1 + (n-k)\varepsilon_3. \tag{17}$$

It should be pointed out that the polaritons D_i obtained in present cyclic ensemble are different from the dark state polaritons in the Λ-type ensemble [14]. The latter are the dark state polaritons and commute with the interaction Hamiltonian, but the former ones do not commute with the interaction Hamiltonian, and also are not dark state polaritons.

III. GENERATION OF QUANTUM ENTANGLEMENTS AND THE COHERENT OUTPUT

Now, we study how to generate the entangled states by using solutions of the polaritons and their eigenstates. We first calculate the dynamical evolution driven by the Hamiltonian (10) with the constants Ω, g_N, and $\varphi \equiv 0$. In this case, the polariton operators in Eqs. (11) and (12) are simplified to

$$D_{1,2} = \frac{\sin\theta}{\sqrt{2}}(a \pm b) + \frac{\cos\theta}{\sqrt{2}}(C \pm A), \tag{18}$$

$$D_{3,4} = \frac{\cos\theta}{\sqrt{2}}(a \pm b) - \frac{\sin\theta}{\sqrt{2}}(C \pm A). \tag{19}$$

The Heisenberg equations

$$\partial_t D_j = -i[D_j,H] = -i\varepsilon_j D_j \quad (j = 1,2,3,4)$$

describe the time evolution of the normal modes of the polaritons

$$D_j(t) = e^{-i\phi_j}D_j(0) \quad (j = 1,2,3,4), \tag{20}$$

where $\phi_j \equiv \phi_j(t) = \varepsilon_j t$ is a time-dependant phase. Since the physical modes can be expressed by the normal modes as

$$a = \frac{1}{\sqrt{2}}[(D_1 + D_2)\sin\theta + (D_3 + D_4)\cos\theta], \tag{21}$$

$$b = \frac{1}{\sqrt{2}}[(D_1 - D_2)\sin\theta + (D_3 - D_4)\cos\theta], \tag{22}$$

$$A=\frac{1}{\sqrt{2}}[(D_1-D_2)\cos\theta-(D_3-D_4)\sin\theta], \tag{23}$$

$$C=\frac{1}{\sqrt{2}}[(D_1+D_2)\cos\theta-(D_3+D_4)\sin\theta], \tag{24}$$

the time-dependent operators $a(t)$, $b(t)$, $A(t)$, and $C(t)$ [also $a^\dagger(-t)$, $b^\dagger(-t)$, $A^\dagger(-t)$, and $C^\dagger(-t)$] can be obtained by a straightforward replacement $D_j\rightarrow D_j(0)\exp[-i\phi_j(t)]$ ($D_j^\dagger\rightarrow D_j^\dagger(0)\exp[-i\phi_j(t)]$). The explicit expressions for these operators are given in the Appendix.

In what follows in this section, we will investigate the dynamical evolution of the above cyclic system and show that the entanglement and the information exchange between two optical modes can occur in the present cyclic system for an initial direct-product Fock states of two optical modes. We will also show that the atomic coherent excitation and coherent output of photons can occur when the system is initially in a direct-product coherent states of two optical modes.

A. Generation of entanglement between two optical modes

If the system is initially in the two-mode photon number state

$$|\psi(0)\rangle=\frac{1}{\sqrt{m!n!}}a^{\dagger m}b^{\dagger n}|\mathbf{0}\rangle,$$

where $|\mathbf{0}\rangle\equiv|0,0\rangle_{ab}\otimes|\mathbf{b}\rangle\equiv|0,0,0,0\rangle_{abAC}$ is the ground state of the system. Then, according to Eqs. (21) and (22), at time t, the wave function can be expressed as

$$\begin{aligned}|\psi(t)\rangle&=\frac{1}{\sqrt{m!n!}}[a^\dagger(-t)]^m[b^\dagger(-t)]^n|\mathbf{0}\rangle\\&=\frac{1}{\sqrt{m!n!}}[F_a^a(t)a^\dagger(0)+F_b^a(t)b^\dagger(0)+F_A^a(t)A^\dagger(0)\\&\quad+F_C^a(t)C^\dagger(0)]^m[F_a^b(t)a^\dagger(0)+F_b^b(t)b^\dagger(0)\\&\quad+F_A^b(t)A^\dagger(0)+F_C^b(t)C^\dagger(0)]^n|\mathbf{0}\rangle,\end{aligned} \tag{25}$$

with the time-dependent coefficients $F_\beta^\alpha(t)$ $(\alpha\beta=a,b,A,C)$ given in the Appendix.

Equation (25) shows that the entanglement between the optical modes and atomic collective modes can be generated when the coefficients in Eq. (25) satisfy certain conditions. However, in the following, we will only focus on how to generate quantum entanglement between two optical modes. When the coefficients of the atomic operators A and C of Eq. (25) vanish in some instants or in the certain limit, i.e.,

$$F_A^a(t)=F_C^a(t)=F_A^b(t)=F_C^b(t)=0, \tag{26}$$

the state $|\psi(t)\rangle$ in Eq. (25) only contains the variables of photons, namely

$$\begin{aligned}|\psi(t)\rangle&=\frac{1}{\sqrt{m!n!}}[F_a^a(t)a^\dagger(0)+F_b^a(t)b^\dagger(0)]^m[F_a^b(t)a^\dagger(0)\\&\quad+F_b^b(t)b^\dagger(0)]^n|\mathbf{0}\rangle.\end{aligned} \tag{27}$$

There are three cases in which the above photon state (27) can be generated during the dynamical evolution satisfying Eq. (26).

Case I: When the classical field is strongly coupled to the atomic ensemble such that the coupling constants Ω and g_N satisfy the condition $\Omega/g_N\rightarrow+\infty$, then $\theta\approx0$ and the polariton operators in Eqs. (11) and (12) can be simplified to

$$D_{1,2}=\frac{1}{\sqrt{2}}(C\pm A),\quad D_{3,4}=\frac{1}{\sqrt{2}}(a\pm b). \tag{28}$$

In such condition, the time-dependent state in Eq. (25) becomes into

$$\begin{aligned}|\psi(t)\rangle&=\frac{1}{\sqrt{m!n!}}[a^\dagger(0)\cos\phi_3-ib^\dagger(0)\sin\phi_3]^m[-ia^\dagger(0)\sin\phi_3\\&\quad+b^\dagger(0)\cos\phi_3]^n|\mathbf{0}\rangle.\end{aligned} \tag{29}$$

Equation (29) shows that the entanglement of optical modes a and b is obtained if the condition $\phi_3(t)\neq l\pi/2$ with the integer l is satisfied. When $\phi_3(t)=\pi/2 \pmod{\pi}$, the state $|\psi(t)\rangle=a^{\dagger n}b^{\dagger m}|\mathbf{0}\rangle/\sqrt{m!n!}$ with a negligibly global factor. This process means that the information between the modes a and b is exchanged. When $\phi_2(t)=0 \pmod{\pi}$, the state $|\psi(t)\rangle=|\psi(0)\rangle=a^{\dagger m}b^{\dagger n}|\mathbf{0}\rangle/\sqrt{m!n!}$, which returns to the initial state.

Case II: When the coupling of the classical field to the atomic ensemble is much stronger than that of the quantized fields. That is, the Rabi frequencies satisfy the condition $\Omega/g_N\rightarrow-\infty$, then $\theta=\pi/2$. In this case, the polariton operators can be simplified to

$$D_{1,2}=\frac{1}{\sqrt{2}}(a\pm b),\quad D_{3,4}=-\frac{1}{\sqrt{2}}(C\pm A). \tag{30}$$

The state in Eq. (25) becomes into

$$\begin{aligned}|\psi(t)\rangle&=\frac{1}{\sqrt{m!n!}}[a^\dagger(0)\cos\phi_1-ib^\dagger(0)\sin\phi_1]^m[-ia^\dagger(0)\sin\phi_1\\&\quad+b^\dagger(0)\cos\phi_1]^n|\mathbf{0}\rangle.\end{aligned} \tag{31}$$

Similar to the case I, the entanglement between optical modes a and b can also be obtained. When $\phi_1(t)=0 \pmod{\pi}$, the state $|\psi(t)\rangle=|\psi(0)\rangle$; when $\phi_1(t)=\pi/2 \pmod{\pi}$, the state $|\psi(t)\rangle=a^{\dagger n}b^{\dagger m}|\mathbf{0}\rangle/\sqrt{m!n!}$. Same as the case I, the above processes mean that the state can return to the initial one, or the quantum information between modes a and b can be exchanged in some instants.

Case III: Under the condition of $\phi_1(t)=\phi_3(t)+2\pi l$ with the integer l, we can also obtain the similar results as the above. Comparing with the cases I and II that are in the special limit of the ratio $|\Omega/g_N|$, here we consider a general case of Ω/g_N. At the instants $t_s=st_0$ $(s=0,1,2,\ldots)$ with $t_0=2\pi/\sqrt{\Omega^2+4g_N^2}$, the time-dependant phases satisfy

$$\phi_1(t_s)=\phi_3(t_s) \pmod{2\pi}, \tag{32}$$

then Eq. (25) is

$$|\psi(t_s)\rangle = \frac{1}{\sqrt{m!n!}}[a^\dagger(0)\cos\phi_s - ib^\dagger(0)\sin\phi_s]^m[-ia^\dagger(0)\sin\phi_s + b^\dagger(0)\cos\phi_s]^n|\mathbf{0}\rangle, \tag{33}$$

which is a two-mode photonic entangled state when $\phi_s \neq l\pi/2$ with the integer l, where $\phi_s \equiv \phi(t_s) = \phi_1(t_s) = \phi_3(t_s)$ (mod 2π).

Moreover, when the the special value of Ω/g_N is taken in case III, the modes a and b can be disentangled in some certain instants. For examples, if $\Omega^2/g_N^2 = 4p^2/(q^2-p^2)$ [p, q are integers], then at time $\tau_s^{(q)} = qt_s = 2\pi qs/\sqrt{\Omega^2+4g_N^2}$ ($s=0,1,2,\ldots$), one has $|\cos\phi_{1,3}(\tau_s^{(q)})|=1$, and

$$|\psi(\tau_s^{(q)})\rangle = \frac{1}{\sqrt{m!n!}}a^{\dagger m}(0)b^{\dagger n}(0)|\mathbf{0}\rangle, \tag{34}$$

which is just the same state as the initial one. If $\Omega^2/g_N^2 = p^2/(q^2-p^2)$ (p, q are integers), then at time $\tau_s^{(q)} = qt_s = 2\pi qs/\sqrt{\Omega^2+4g_N^2}$ ($s=0,1,2,\ldots$), one has $|\sin\phi_{1,3}(\tau_s^{(q)})|=1$, and then we have

$$|\psi(\tau_s^{(q)})\rangle = \frac{1}{\sqrt{m!n!}}a^{\dagger n}(0)b^{\dagger m}(0)|\mathbf{0}\rangle, \tag{35}$$

which means the information carried by two optical modes has been exchanged between modes a and b.

For the entangled states generated by the above three cases, we can further calculate their entanglement degree in order to make them more clearly. In fact, for each pure state, the entanglement can be defined as the entropy of either of the two subsystems [24–26]. For example, the expression of entanglement for Eq. (29) is given as

$$E(|\psi(t)\rangle) = -\operatorname{Tr}[\rho_a \log_2 \rho_a(t)], \tag{36}$$

where

$$\rho_a(t) = \operatorname{Tr}_b \rho(t) = \operatorname{Tr}_b[|\psi(t)\rangle\langle\psi(t)|] \tag{37}$$

is the reduced density matrix of mode a. For simplicity, we consider the case of $m=n=1$ in Eq. (29). In this case, we have

$$\begin{aligned}\rho_a(t) &= \operatorname{Tr}_b[|\psi(t)\rangle\langle\psi(t)|] \\ &= 2\sin^2\phi_3\cos^2\phi_3(|0\rangle_{aa}\langle 0| + |2\rangle_{aa}\langle 2|) \\ &\quad + (\cos^2\phi_3 - \sin^2\phi_3)^2|1\rangle_{aa}\langle 1|,\end{aligned} \tag{38}$$

and the entanglement is given as

$$\begin{aligned}E[|\psi(t)\rangle] &= -\operatorname{Tr}[\rho_a \log_2 \rho_a(t)] \\ &= (\cos^2\phi_3 - \sin^2\phi_3)^2 \log_2(\cos^2\phi_3 - \sin^2\phi_3)^2 \\ &\quad + 4\sin^2\phi_3\cos^2\phi_3 \log_2(2\sin^2\phi_3\cos^2\phi_3),\end{aligned} \tag{39}$$

here $\phi_3(t) \equiv \varepsilon_3 t = (\Omega - \sqrt{\Omega^2+4g_N^2})t/2$.

Figure 2 shows the entanglement in Eq. (39) during the time evolution. In fact, at time $t=(k\pm 1/4)\pi/\varepsilon_3$ ($k=0,1,2,\ldots$), $|\psi(t)\rangle = (|20\rangle_{ab}+|02\rangle_{ab})/\sqrt{2}$, and $E(|\psi(t)\rangle)=1$, as shown in Fig. 2 with those A points. It means that $|\psi(t)\rangle$ is a maximally entangled state for two optical modes in the two dimensional space, constructed by the states $|2,0\rangle_{ab}$ and $|0,2\rangle_{ab}$. At the time $t=\{k\pi\pm\arcsin[\sqrt{(3+\sqrt{3})/6}]\}/\varepsilon_3$ or $t=\{k\pi\pm\arcsin[\sqrt{(3-\sqrt{3})/6}]\}/\varepsilon_3$ ($k=0,1,2,\ldots$), it has $|\psi(t)\rangle = (|20\rangle_{ab}+|02\rangle_{ab}\pm i|11\rangle_{ab})/\sqrt{3}$ and $E(|\psi(t)\rangle)=\log_2 3$, which shows that $|\psi(t)\rangle$ is a maximally entangled state in the three dimensional space, constructed by the states $|20\rangle_{ab}$, $|02\rangle_{ab}$, and $|11\rangle_{ab}$ of two optical modes (see the points B in Fig. 2). These so-called maximally entangled states are very useful in the field of quantum information, e.g., to implement the quantum teleportation.

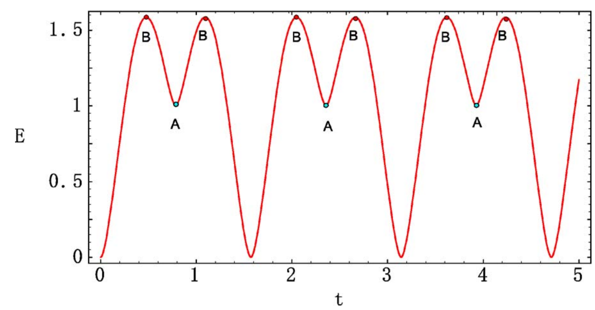

FIG. 2. (Color online) The entanglement in the state Eq. (39) vs the time t [in units of $1/\varepsilon_3$]. Points A show the maximally entangled states for 2×2 dimension. Points B show the maximally entangled states for 3×3 dimension.

In this subsection, we have studied the entanglement and the information exchange between two optical modes in the present cyclic system for an initial direct-product Fock states of optical modes. Such an entanglement or information exchange phenomenon cannot be occurred in a noncyclic three-level system, e.g., V-type three-level system [16]. Physically, the classical field, which is used to couple two higher levels, assists to implement the entanglement (or information exchange) between these optical modes. This can be seen from Eq. (9), in a noncyclic three-level V-type system obtained from the Δ-type system with $\Omega=0$, two optical modes only interact independently with two collective excitation modes respectively. So it cannot realize the entanglement between two optical modes.

B. Coherent output of collective excitations and photons

Here we study the dynamical evolution of the cyclic system when the system is initially in the several kinds of coherent state as follows.

(i) If the system is initially in a direct product state of coherent states for four modes a, b, A and C

$$|\psi(0)\rangle = D_a(\alpha)D_b(\beta)D_A(\zeta)D_C(\eta)|\mathbf{0}\rangle \equiv |\alpha,\beta,\zeta,\eta\rangle_{abAC},$$

where $D_Q(\gamma)=\exp[\gamma Q^\dagger - \text{H.c.}]$ ($Q=a,b,A,C$) is the displacement operator. Then the state evolves into

$$|\psi(t)\rangle = D_a(\alpha')D_b(\beta')D_A(\zeta')D_C(\eta')|\mathbf{0}\rangle \tag{40}$$

with

$$\alpha'(t) = \alpha F_a^a(t) + \beta F_a^b(t) + \zeta F_a^A(t) + \eta F_a^C(t),$$

$$\beta'(t) = \alpha F_b^a(t) + \beta F_b^b(t) + \zeta F_b^A(t) + \eta F_b^C(t),$$

$$\zeta'(t) = \alpha F_A^a(t) + \beta F_A^b(t) + \zeta F_A^A(t) + \eta F_A^C(t),$$

$$\eta'(t) = \alpha F_C^a(t) + \beta F_C^b(t) + \zeta F_C^A(t) + \eta F_C^C(t).$$

Here, the relation

$$\begin{aligned}U(t)D_Q(\gamma)U^\dagger(t) &= \exp[\gamma Q^\dagger(-t) - \text{H.c.}] \\ &= \exp\{\gamma[F_a^Q(t)a^\dagger(0) + F_b^Q(t)b^\dagger(0) \\ &\quad + F_A^Q(t)A^\dagger(0) + F_C^Q(t)C^\dagger(0)] - \text{H.c.}\}\end{aligned}$$

has been used in Eq. (40). Equation (40) shows that any initial direct-product coherent state is still a direct-product coherent state during the time evolution. However, the intensity of each mode varies with the time evolution.

(ii) If the atoms are initially in the ground states, but one of the optical modes, e.g., mode a, is initially in a coherent state

$$|\psi(0)\rangle = D_a(\alpha)|\mathbf{0}\rangle = |\alpha,0,0,0\rangle_{abAC}. \tag{41}$$

Then, at time t, the state is

$$|\psi(t)\rangle = |\alpha'(t),\beta'(t),\zeta'(t),\eta'(t)\rangle_{abAC},$$

where

$$\alpha'(t) = \alpha F_a^a(t), \quad \beta'(t) = \alpha F_b^a(t),$$

$$\zeta'(t) = \alpha F_A^a(t), \quad \eta'(t) = \alpha F_C^a(t).$$

This means that *a new coherent optical field* of mode b and two new coherent atomic collective excitations are generated.

(iii) If the atoms are initially in the ground states, and the two optical modes are initially in their coherent states:

$$|\psi(0)\rangle = D_a(\alpha)D_b(\beta)|\mathbf{0}\rangle = |\alpha,\beta,0,0\rangle_{abAC}, \tag{42}$$

then, the evolved state will be direct-product coherent state of four modes

$$|\psi(t)\rangle = D_a(\alpha')D_b(\beta')D_A(\zeta')D_C(\eta')|\mathbf{0}\rangle \tag{43}$$

with

$$\alpha'(t) = \alpha F_a^a(t) + \beta F_a^b(t),$$

$$\beta'(t) = \alpha F_b^a(t) + \beta F_b^b(t),$$

$$\zeta'(t) = \alpha F_A^a(t) + \beta F_A^b(t),$$

$$\eta'(t) = \alpha F_C^a(t) + \beta F_C^b(t).$$

This means the initial optical modes can lead to the coherent output of new modes of atomic collective excitation.

(iv) If the two optical modes are initially in the vacuum state, but the two atomic collective excitation modes are initially in the coherent states

$$|\psi(0)\rangle = D_A(\zeta)D_C(\eta)|\mathbf{0}\rangle. \tag{44}$$

Then, the state will evolve into a direct-product state of the two optical modes and the atomic excitation modes

$$|\psi(t)\rangle = D_a(\alpha')D_b(\beta')D_A(\zeta')D_C(\eta')|\mathbf{0}\rangle \tag{45}$$

with

$$\alpha'(t) = \zeta F_a^A(t) + \eta F_a^C(t),$$

$$\beta'(t) = \zeta F_b^A(t) + \eta F_b^C(t),$$

$$\zeta'(t) = \zeta F_A^A(t) + \eta F_A^C(t),$$

$$\eta'(t) = \zeta F_C^A(t) + \eta F_C^C(t).$$

Equations (43) and (45) show that the coherent optical modes or coherent atomic excitation modes will result in the generation of the coherent atomic excitation modes or coherent optical modes in the cyclic atomic ensemble system.

(v) We now consider that one of the optical modes, e.g., the mode a, is initially in odd or even coherent states $\mathcal{N}(|\alpha\rangle_a \pm |-\alpha\rangle_a)$ with the normalization constant $\mathcal{N}=(2\pm 2e^{-2|\alpha|^2})^{-1/2}$. But another optical mode b is in the vacuum state, and also the two atomic modes are in their ground states. That is, the system is initially in the state

$$|\psi(0)\rangle = \mathcal{N}(|\alpha\rangle_a \pm |-\alpha\rangle_a) \otimes |0,0,0\rangle_{bAC}. \tag{46}$$

At instant τ when $\phi_1(\tau)=\phi_3(\tau)=\phi(\tau)$ (mod 2π), the state will evolve to a so-called *entangled coherent state* of these two optical modes [18–20]

$$\begin{aligned}|\psi(\tau)\rangle = |\mathbf{0}\rangle_{AC} \otimes \mathcal{N}[&|\alpha\cos\phi, -i\alpha\sin\phi\rangle_{ab} \\ &\pm |-\alpha\cos\phi, i\alpha\sin\phi\rangle_{ab}].\end{aligned} \tag{47}$$

When $\phi(\tau)$ in Eq. (47) satisfies $\phi(\tau)=\pi/4$ (mod π), the state will be in a maximally entangled state [27]. These states have recently been proposed as an important tool in the theories and experiments relating to the quantum information processing [21–23].

Especially, if $\phi(\tau)=0$ (mod π) at certain time τ, the instantaneous state in Eq. (47) returns to the initial state in Eq. (46). If $\phi(\tau)=\pi/2$ (mod π) at certain time τ, the instantaneous state in Eq. (47) becomes into

$$|\psi(\tau)\rangle = \mathcal{N}(|\alpha\rangle_b \pm |-\alpha\rangle_b) \otimes |0,0,0\rangle_{aAC}. \tag{48}$$

It means that a new coherent state for the mode b is generated. It also shows that the quantum information is transferred from mode a to mode b.

So far we have showed the atomic coherent excitation (or coherent output of photons) when the optical fields (or atomic collective excitations) are initially in the coherent states. Moreover, if one of the optical modes is initially in an odd (or even) coherent state but another one is in the vacuum state, the system will evolve to an entangled state for two optical modes with coherent states each. When a special condition is satisfied, the information can be transferred from the first optical mode to the second one, which has been described in Eqs. (46) and (48). These interesting results is due to the classical optical field, which induces the atomic transition between two higher states. However these interesting phenomena cannot be found in the noncyclic three-level

V-type atomic ensemble, where only the classical optical field is removed as a comparison with the cyclic system.

IV. THE STATE STORAGE OF PHOTONS BASED ON ADIABATIC MANIPULATION

In Sec. III, we have studied the dynamic properties of the atomic ensemble with the cyclic transitions. Here, we consider the adiabatic evolution of the cyclic atomic ensemble, controlled by the time-dependent classical field. In this case, the Rabi frequency Ω should become into the time-dependent one, i.e., $\Omega(t)$. Using the diagonalized Hamiltonian (9) and following the method of collective excitations shown in Ref. [14], we will discuss how to transfer the information from the two quantized light fields to the atomic ensemble by the adiabatic passage.

In the following, the value of φ is fixed, i.e., $\varphi=0$, but Ω will be changed within the range $(-\infty,+\infty)$ according to the constant g_N. In general, the polariton operators D_i $(i=1,2,3,4)$ consist of two photonic modes and two atomic collective excitation modes. For simplicity, we consider two simple cases for D_i. One is $\Omega/g_N\rightarrow+\infty$. In this limit, $\theta\rightarrow 0$, and the polariton operators are given in Eqs. (28) with the relative values $\varepsilon_1\equiv-\varepsilon_2\rightarrow\Omega$, $\varepsilon_3\equiv-\varepsilon_4\rightarrow 0$. Another is $\Omega/g_N\rightarrow-\infty$, then we have $\theta\rightarrow\pi/2$, and the polariton operators become into Eq. (30) with the relative values $\varepsilon_1\equiv-\varepsilon_2\rightarrow 0$, $\varepsilon_3\equiv-\varepsilon_4\rightarrow\Omega$.

The analysis on Eq. (15) shows that when Ω varies in the range $(-\infty,+\infty)$, θ will vary in the range $(0,\pi/2)$. In the above two limit cases, the polariton operators $D_{1,2}$ (or $D_{3,4}$) consist of only the optical modes a,b (or only the atomic collective excitation modes A,C). This implies that the information can be transferred from two quantized light fields to the atomic ensemble, and then can also be stored in the atomic ensemble, as given in Ref. [14],

For example, initially, if we set $\Omega(t)/g_N\rightarrow+\infty$, and the system is in a direct-product Fock state of two optical modes

$$|\Psi(0)\rangle=|m,n\rangle_{ab}\otimes|\mathbf{b}\rangle=\frac{a^{\dagger m}b^{\dagger n}}{\sqrt{m!n!}}|\mathbf{0}\rangle. \tag{49}$$

Then, using expressions of the polariton operators in Eq. (28), Eq. (49) can be rewritten as a superposition

$$|\Psi(0)\rangle=\sum_{j,k}f_{mn}^{jk}(D_3^\dagger)^{j+k}(D_4^\dagger)^{m+n-j-k}|\mathbf{0}\rangle, \tag{50}$$

where

$$f_{mn}^{jk}=(-1)^{n-k}\frac{C_m^jC_m^{m-j}C_n^kC_n^{n-k}}{\sqrt{2^{m+n}m!n!}},$$

$$C_m^j=\frac{m!}{j!(m-j)!}.$$

In the process of the adiabatical evolution, the state at time t will be

$$|\Psi(t)\rangle=\sum_{j,k}f_{mn}^{jk}U_{mn}^{jk}(t)[D_3^\dagger(t)]^{j+k}[D_4^\dagger(t)]^{m+n-j-k}|\mathbf{0}\rangle, \tag{51}$$

where $U_{mn}^{jk}(t)$ is the relative dynamic phase

$$U_{mn}^{jk}(t)=(2j+2k-m-n)\exp\left[-i\int_0^t\varepsilon_3(t')dt'\right]. \tag{52}$$

When $\Omega(t)$ is adiabatically changed to $-\infty$ in certain time τ, the state will be

$$\begin{aligned}|\Psi(\tau)\rangle&=\sum_{j,k}f_{mn}^{jk}U_{mn}^{jk}(\tau)[D_3^\dagger(\tau)]^{j+k}[D_4^\dagger(\tau)]^{m+n-j-k}|\mathbf{0}\rangle\\&=\sum_{j,k}f_{mn}^{jk}U_{mn}^{jk}(\tau)\left[-\frac{A^\dagger(0)+C^\dagger(0)}{\sqrt{2}}\right]^{j+k}\\&\quad\times\left[\frac{A^\dagger(0)-C^\dagger(0)}{\sqrt{2}}\right]^{m+n-j-k}|\mathbf{0}\rangle.\end{aligned} \tag{53}$$

Equation (53) shows that the quantum information carried by optical modes (a and b) has been completely transferred to the atomic collective excitation modes A and C. Since atoms are local and robust, the above adiabatic process means that the information of two quantized light fields has been stored in an atomic ensemble.

Especially, for certain evolution path of $\Omega(t)$, if the relative dynamic phase $U_{mn}^{jk}(\tau)\equiv 1$ holds for any integer m,n,j,k, that is

$$\int_0^\tau\varepsilon_3(t)dt=2l\pi \tag{54}$$

with an integer l, $|\Psi(\tau)\rangle$ will have a simple form

$$|\Psi(\tau)\rangle=(-1)^{m+n}|0\rangle_a|0\rangle_b|n\rangle_A|m\rangle_C. \tag{55}$$

That is, when adiabatically changing Ω/g_N: $+\infty\rightarrow-\infty$, under the special case of $U_{mn}^{jk}(\tau)\equiv 1$ for any j, k, m, and n, the state transfer can be realized as follows:

$$\begin{aligned}|\Psi(0)\rangle&=|m\rangle_a|n\rangle_b|0\rangle_A|0\rangle_C\rightarrow|\Psi(\tau)\rangle\\&=(-1)^{m+n}|0\rangle_a|0\rangle_b|n\rangle_A|m\rangle_C.\end{aligned} \tag{56}$$

Such an adiabatic passage means $a\rightarrow -C$ and $b\rightarrow -A$, so the initial state involved only for the optical modes will evolve to the final state involved only for the atomic excitation modes. This also means that the quantum information carried by the optical fields has been transferred to and stored in the atomic ensemble.

An inverse adiabatic passage, which makes $\Omega(\tau)/g_N=-\infty\rightarrow\Omega(T)/g_N=+\infty$, will result in information transfer from the atomic ensemble to the two optical modes, i.e., $C\rightarrow -a$ and $A\rightarrow -b$. And then, the initial atomic state $|n\rangle_A|m\rangle_C$ will evolve to the final state $|\Psi(T)\rangle=|m\rangle_a|n\rangle_b|0\rangle_A|0\rangle_C$ by the inverse adiabatic passage with the relative dynamic phase $U_{mn}^{jk}(T)\equiv 1$.

It is *worth stressing* that if the initial state is $|\Psi(0)\rangle=|0\rangle_a|0\rangle_b|m\rangle_A|n\rangle_C$, after an adiabatically changing Ω/g_N: $+\infty\rightarrow-\infty$, this state will become into the Fock state of the two optical modes a and b (depicted according to the polaritons $D_{1,2}$), where we also assume that the dynamic phase factor is 1 during the adiabatic evolution, that is

$$U_{mn}^{\prime jk}(\tau)\equiv 1\ \text{(for any integer } m,n,j,k), \tag{57}$$

which is equal to $\int_0^\tau\varepsilon_1(t)dt=2l\pi$ with an integer l.

The inverse adiabatical passage $\Omega/g_N:-\infty\rightarrow+\infty$ will result in the information carried by the optical fields to be transferred to that by atomic ensemble.

So far, we have achieved the quantum information exchange between optical fields and atomic ensemble with initially in the Fock states. For general states, e.g.,

$$|\Psi(0)\rangle=\sum_{m,n}u_{mn}|m\rangle_a|n\rangle_b|0\rangle_A|0\rangle_C \tag{58}$$

or $|\Psi(0)\rangle=|\alpha\rangle_a|\beta\rangle_b|0\rangle_A|0\rangle_C$, the information can also be transferred in the similar way as done in Ref. [14].

In Sec. III, we mainly study the generation of entanglement states between two optical modes, and the quantum information transfer from one optical mode to another optical mode by virtue of *the dynamical evolution*. In this section, we discuss the information transfer and storage from the optical fields to the cyclic atomic ensemble through *the adiabatic passage*. Moreover, the quantum information can also be retrieved from the atomic collective excitation modes. It is well known that photons are nonlocal and not easy to be stored, but atoms are local and robust. The above process provides a way to implement retrievable storage of the optical information in an atomic ensemble.

V. CONCLUSION

We have investigated various protocols of quantum information processing based on the photonic properties of the emission and excitation of a Δ-type (or cyclic) "atomic" ensemble, which coupled to two quantum optical fields and one classical field. The classical field controls the coupling between two upper energy levels. By means of collective operator approach, we studied the dynamical evolution and adiabatic manipulation for such a unique system. Our results show that the two-mode photon entanglement and quantum information exchange between two optical modes can be realized when the optical modes are initially in the direct-product Fock states or coherent states.

It is remarked that, even without symmetry broken, a three-level system can also form a cyclic one. The electric-dipole interaction of the classic field coupled to the two higher states in our model can be replaced by the magnetic-dipole transition. However this magnetic-dipole interaction is generally very weak compared with the electric-dipole interaction and disposed as perturbation. The significant phenomenon of cyclic three-level configuration only occurs in the systems with symmetry broken as given in the present work.

We also need to point out that, it is the classical field that result in various phenomena, found in this paper. Without this classical field, it would be impossible to generate the entangled states of the two optical modes. As a quantum memory, the collective excitations of the Δ-type atomic ensemble can store the quantum information carried by two quantum optical modes through the adiabatical manipulation. The corresponding adiabatical evolution is realized by choosing certain classical Rabi frequency $\Omega(t)$. We expect that our proposal can be confirmed and implemented experimentally in the near future.

In this paper, we just consider an ideal case. Actually, in a realistic system, two kinds of decoherence mechanisms may play a role. The first one comes from the multimode radiation, which will result in the collective decay of atoms from the excited states to the ground state. This kind of radiation properties due to atomic decay in present system will be investigated in a following paper. The second decoherence effect is due to the inhomogeneous coupling of atoms to the light fields. The influence of inhomogeneous coupling has been studied in detail in a two-level-atom ensemble by Sun *et al.* [28]. In the realistic experiments, the atoms can be fixed and the coupling can be taken as the constant by a special design to avoid the decoherence of inhomogeneous coupling. Our assumption for homogeneous coupling is reasonable in a short interaction time.

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APPENDIX: DERIVATION OF POLARITON OPERATORS

Here we rewrite the forms of $D_{1,2,3,4}$ in Eqs. (18) and (19)

$$D_1=\frac{\sin\theta}{\sqrt{2}}(a+b)+\frac{\cos\theta}{\sqrt{2}}(A+C), \tag{A1}$$

$$D_2=\frac{\sin\theta}{\sqrt{2}}(a-b)-\frac{\cos\theta}{\sqrt{2}}(A-C), \tag{A2}$$

$$D_3=\frac{\cos\theta}{\sqrt{2}}(a+b)-\frac{\sin\theta}{\sqrt{2}}(A+C), \tag{A3}$$

$$D_4=\frac{\cos\theta}{\sqrt{2}}(a-b)+\frac{\sin\theta}{\sqrt{2}}(A-C). \tag{A4}$$

According to Eq. (10), it has

$$\partial_t D_j=-i[D_j,H]=-i\varepsilon_j D_j \quad (j=1,2,3,4),$$

and then

$$D_j(t)\equiv e^{-i\phi_j}D_j(0) \quad (j=1,2,3,4), \tag{A5}$$

where $\phi_j(t)\equiv\varepsilon_j t$. Following Eqs. (A1)–(A4), we can obtain the inverse transformation

$$a=\frac{1}{\sqrt{2}}[(D_1+D_2)\sin\theta+(D_3+D_4)\cos\theta], \tag{A6}$$

$$b=\frac{1}{\sqrt{2}}[(D_1-D_2)\sin\theta+(D_3-D_4)\cos\theta], \tag{A7}$$

$$A=\frac{1}{\sqrt{2}}[(D_1-D_2)\cos\theta-(D_3-D_4)\sin\theta], \tag{A8}$$

$$C=\frac{1}{\sqrt{2}}[(D_1+D_2)\cos\theta-(D_2+D_3)\sin\theta]. \tag{A9}$$

By means of Eq. (A5), we have

$$\begin{aligned}a(t)=&\frac{1}{\sqrt{2}}\{[D_1(0)e^{-i\phi_1}+D_2(0)e^{i\phi_1}]\sin\theta\\&+[D_3(0)e^{-i\phi_3}+D_4(0)e^{i\phi_3}]\cos\theta\}\\&\equiv F_a^a(t)a(0)+F_b^a(t)b(0)+F_A^a(t)A(0)+F_C^a(t)C(0),\end{aligned} \tag{A10}$$

$$\begin{aligned}b(t)=&\frac{1}{\sqrt{2}}\{[D_1(0)e^{-i\phi_1}-D_2(0)e^{i\phi_1}]\sin\theta\\&+[D_3(0)e^{-i\phi_3}-D_4(0)e^{i\phi_3}]\cos\theta\}\\&\equiv F_a^b(t)a(0)+F_b^b(t)b(0)+F_A^b(t)A(0)+F_C^b(t)C(0),\end{aligned} \tag{A11}$$

$$\begin{aligned}A(t)=&\frac{1}{\sqrt{2}}\{[D_1(0)e^{-i\phi_1}-D_2(0)e^{i\phi_1}]\cos\theta\\&-[D_3(0)e^{-i\phi_3}-D_4(0)e^{i\phi_3}]\sin\theta\}\\&\equiv F_a^A(t)a(0)+F_b^A(t)b(0)+F_A^A(t)A(0)+F_C^A(t)C(0),\end{aligned} \tag{A12}$$

$$\begin{aligned}C(t)=&\frac{1}{\sqrt{2}}\{[D_1(0)e^{-i\phi_1}+D_2(0)e^{i\phi_1}]\cos\theta-[D_3(0)e^{-i\phi_3}\\&+D_4(0)e^{i\phi_3}]\sin\theta\}\\&\equiv F_a^C(t)a(0)+F_b^C(t)b(0)+F_A^C(t)A(0)+F_C^C(t)C(0),\end{aligned} \tag{A13}$$

where we have used $\phi_1(t)=-\phi_2(t)$ and $\phi_3(t)=-\phi_4(t)$ and the related coefficients are

$$F_a^a(t)=F_b^b(t)=\cos\phi_1\sin^2\theta+\cos\phi_3\cos^2\theta,$$

$$F_b^a(t)=F_a^b(t)=-i(\sin\phi_1\sin^2\theta+\sin\phi_3\cos^2\theta),$$

$$F_A^a(t)=F_a^A(t)=-i\sin\theta\cos\theta(\sin\phi_1-\sin\phi_3),$$

$$F_C^a(t)=F_a^C(t)=\sin\theta\cos\theta(\cos\phi_1-\cos\phi_3),$$

$$F_A^b(t)=F_b^A(t)=(\cos\phi_1-\cos\phi_3)\sin\theta\cos\theta,$$

$$F_C^b(t)=F_b^C(t)=-i\sin\theta\cos\theta(\sin\phi_1-\sin\phi_3),$$

$$F_A^A(t)=F_C^C(t)=\cos\phi_1\cos^2\theta+\cos\phi_3\sin^2\theta,$$

$$F_C^A(t)=F_A^C(t)=-i(\sin\phi_1\cos^2\theta+\sin\phi_3\sin^2\theta).$$

It also has

$$\begin{aligned}a^\dagger(-t)&\equiv F_a^a(t)a^\dagger(0)+F_b^a(t)b^\dagger(0)+F_A^a(t)A^\dagger(0)+F_C^a(t)C^\dagger(0),\\b^\dagger(-t)&\equiv F_a^b(t)a^\dagger(0)+F_b^b(t)b^\dagger(0)+F_A^b(t)A^\dagger(0)+F_C^b(t)C^\dagger(0),\\A^\dagger(-t)&\equiv F_a^A(t)a^\dagger(0)+F_b^A(t)b^\dagger(0)+F_A^A(t)A^\dagger(0)+F_C^A(t)C^\dagger(0),\\C^\dagger(-t)&\equiv F_a^C(t)a^\dagger(0)+F_b^C(t)b^\dagger(0)+F_A^C(t)A^\dagger(0)+F_C^C(t)C^\dagger(0).\end{aligned} \tag{A14}$$

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