Nuclear charge radius for ^{3}He

Donald C. Morton
Herzberg Institute of Astrophysics, National Research Council of Canada, 5071 West Saanich Road, Victoria, British Columbia, Canada V9E 2E7

Qixue Wu and G. W. F. Drake
Department of Physics, University of Windsor, Windsor, Ontario, Canada N9B 3P4
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An rms nuclear charge radius r_c=1.9642(11) fm for ^{3}He is derived from measurements of the $2\,^3S_1$-$2\,^3P_0$ isotope shift combined with the best available data on the fine structure of ^{4}He, the hyperfine structure of ^{3}He, and an assumed r_c=1.673(1) fm for ^{4}He. The result removes a small discrepancy between some older spectroscopic determinations of r_c for ^{3}He from this transition and a more recent measurement.

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The rms charge radius r_c is an important parameter for nuclear structure [1]. Since the difference in the square of the radius contributes to the isotope shift of spectral lines, this difference can be obtained from measurements of the shift if all other contributions can be calculated accurately. This approach has provided spectroscopic values of r_c for the isotopes ^{8}Li and ^{9}Li [2] and the halo nuclei ^{6}He [3] and ^{11}Li [4]. Drake, Nörtershäuser, and Yan [5] published the relevant calculations for many levels of the isotopes of helium and lithium and estimated r_c for ^{3}He and ^{6}Li. Morton, Wu, and Drake [6] recently completed a detailed investigation of the energy levels of ^{4}He and ^{3}He and listed improved values for the parameters used by Zhao, Lawall, and Pipken [7], Shiner, Dixson, and Vedanthan [8], and Marin *et al.* [9] to derive the shifts adopted in Ref. [5]. This Brief Report rederives the experimental shifts for $2\,^3S_1$-$2\,^3P_0$ and $2\,^3S_1$-$3\,^3P_0$ and recalculates r_c for ^{3}He using the latest available data and eliminating one approximation in [5]. Our new results remove a small discrepancy between the older spectroscopic determination of r_c from the $2\,^3S_1$-$2\,^3P_0$ transition [7] and the more recent measurement of [8].

According to Morgan and Cohen [10] and Drake *et al.* [5], the energy difference in esu between ^{3}He and ^{4}He for state j can be represented by

$$(E_3-E_4)_j=\left[\left(\frac{\mu}{M}\right)_3-\left(\frac{\mu}{M}\right)_4\right](E_{\rm NR}^{(1)}+\alpha^2E_{\rm rel}^{(1)}+\alpha^3E_{\rm QED}^{(1)})_j+\left[\left(\frac{\mu}{M}\right)_3^2-\left(\frac{\mu}{M}\right)_4^2\right](E_{\rm NR}^{(2)}+\cdots)_j+\frac{2\pi Ze^2}{3}\left[r_{c3}^2\sum_i\delta_{3j}(r_i)-r_{c4}^2\sum_i\delta_{4j}(r_i)\right],\qquad(1)$$

where μ is the reduced electron mass, M the mass appropriate for each nucleus, α is the fine-structure constant, e is the electronic charge in esu, and Z is the atomic number, while $E_{\rm NR}$ is the nonrelativistic energy, $E_{\rm rel}$ the leading relativistic correction, $E_{\rm QED}$ the leading QED correction, and $\Sigma\delta(r_i)$ the expectation value of the electron density at the nucleus obtained by summing over the two helium electrons. The elimination of other terms, including the mass-independent QED correction, makes the calculation of the isotope shift more accurate than the absolute levels of either isotope. Drake *et al.* [5] tabulated the sum of the first two terms on the right as $\delta E_i(^3\text{He-}^4\text{He})$ in MHz. For the third term they neglected the isotopic dependence of the $\Sigma\delta(r_i)$ and quoted a single value C_j for each level. We have calculated the sum in atomic units separately for each isotope and listed the results in Table I along with $\delta E_i(^3\text{He-}^4\text{He})$ from [5].

In practice laboratory measurements give the energy of the isotope shift of a transition j to k, which Drake *et al.* [5] called $\delta\nu_{jk}$. Thus

$$\begin{aligned}\delta\nu_{jk}&=(E_3-E_4)_j-(E_3-E_4)_k\\&=\delta E_j(^3\text{He}-{}^4\text{He})-\delta E_k(^3\text{He}-{}^4\text{He})\\&\quad+\frac{4\pi e^2}{3}\left[r_{c3}^2\left(\sum_i\delta_{3j}(r_i)-\sum_i\delta_{3k}(r_i)\right)\right.\\&\quad\left.-r_{c4}^2\left(\sum_i\delta_{4j}(r_i)-\sum_i\delta_{4k}(r_i)\right)\right],\end{aligned}\qquad(2)$$

from which r_c can be calculated because all the other variables are known.

For this revision we adopted the $2\,^3P$ fine-structure separations of 29 616.9518(6) for J=2 to 1 and

TABLE I. Calculated parameters for ^{3}He and ^{4}He for use with Eq. (2).

State j	δE (^{3}He-^{4}He) (MHz)	$(4\pi e^2/3h)\Sigma\delta_{3j}$ (MHz)	$(4\pi e^2/3h)\Sigma\delta_{4j}$ (MHz)
$2\,^3S_1$	53 897.130 1(6)	25.976 172 3(2)	25.979 667 1(2)
$2\,^3P_2$	20 229.628 3(4)	24.766 088 8(3)	24.769 486 0(3)
$2\,^3P_1$	20 230.619 1(4)	24.766 088 8(3)	24.769 486 0(3)
$2\,^3P_0$	20 230.346 1(4)	24.766 088 8(3)	24.769 486 0(3)
$3\,^3P_2$	11 713.898 0(2)	24.968 167 7(4)	24.971 541 7(4)
$3\,^3P_1$	11 714.165 6(2)	24.968 167 7(4)	24.971 541 7(4)
$3\,^3P_0$	11 713.911 2(2)	24.968 167 7(4)	24.971 541 7(4)

TABLE II. Original measurements, revised isotope shifts, and the resulting charge radius.

Transition	Measurement (MHz)	Isotope shift $\delta\nu_{jk}$ (MHz)	$r_c(^3\mathrm{He})$ (fm)
^{3}He($2\,^3S_1 3/2$-$2\,^3P_0 1/2$)-^{4}He($2\,^3S_1$-$2\,^3P_1$)	1480.573(30)[a]	33 668.062(30)	1.963(6)
^{3}He($2\,^3S_1 3/2$-$2\,^3P_0 1/2$)-^{4}He($2\,^3S_1$-$2\,^3P_2$)	810.608(30)[a]	33 668.057(30)	1.962(6)
^{3}He($2\,^3S_1 3/2$-$2\,^3P_0 1/2$)-^{4}He($2\,^3S_1$-$2\,^3P_2$)	810.599(3)[b]	33 668.066(3)	1.9643(11)
^{3}He($2\,^3S_1 1/2$-$3\,^3P_0 1/2$)-^{4}He($2\,^3S_1$-$3\,^3P_0$)	45 394.413(137)[c]	42 184.368(166)	1.985(41)
Electron-nucleus scattering			1.959(30)[d]
Nuclear theory			1.96(1)[e]

[a]Zhao, Lawall, and Pipken [7].
[b]Shiner, Dixson, and Vedantham [8].
[c]Marin *et al.* [9].
[d]Amroun *et al.* [15].
[e]Pieper and Wiringa [16] with Eq. (29) form [5].

31 908.1271(15) MHz for $J=2$ to 0 in ^{4}He from Giusfredi *et al.* [11] compared with 29 616.844(22) and 31 908.040(22) used by [7] or 31 908.135(3) used by [8]. We also adopted the displacement 323.9503(12) MHz of $2\,^3P_0 F=1/2$ above the hypothetical $2\,^3P_0$ measured by [8] and the hyperfine shift of 2246.5873 MHz of $2\,^3S_1 F=3/2$ below the hypothetical $2\,^3S_1$ in ^{3}He from the precise measurement of $2\,^3S_1 F=\frac{3}{2}$ to $\frac{1}{2}$ by Rosner and Pipken [12] and the calculations of [6]. Zhao *et al.* [7] used 323.977(12) and 2246.559 MHz for these. Drake *et al.* [5] did take advantage of the improved splitting of $3\,^3P$ in ^{4}He by Mueller *et al.* [13] to update the isotope shift for $2\,^3S_1$-$3\,^3P_0$ in [9], and we have included a small revision of 1283.069(93) MHz from [6] for the hyperfine shift of $3\,^3P_0 F=1/2$.

The first four entries in Table II list the original measurements by [7–9], the isotope shifts derived with the above numbers, and the resulting $r_c(^3\mathrm{He})$ obtained from Eq. (2) with $r_c(^4\mathrm{He})=1.673(1)$ from Borie and Rinker [14]. For completeness, the table repeats from [5] the scattering measurement by Amroun *et al.* [15] and a theoretical value for $r_c(^3\mathrm{He})$ from Pieper and Wiringa [16].

All six results plotted in Fig. 1 show excellent consistency, supporting a recommended $r_c(^3\mathrm{He})=1.9642(11)$ fm. There is a small decrease and a reduced error from the best value of 1.9659(14) obtained by Drake *et al.* [5] and adopted by [6], and the other two results for $2\,^3S_1$-$3\,^3P_0$ now show much better agreement. We found that the use of separate $\delta(r_i)$ for each isotope affects the fifth significant figure of $r_c(^3\mathrm{He})$ and hence is important for only the most accurate measurement used here.

Unfortunately, the charge radius for ^{4}He remains the weak link in the isotopic method. Its error of 0.001 fm contributes almost one-half of the final error of 0.0011 fm, and it could be worse. As noted in [5], the adopted $r_c(^4\mathrm{He})=1.673(1)$, derived from the Lamb shift in muonic helium, has not been reproduced, though it is consistent with a theoretical 1.670(4) derived from the point proton radius listed in [16] and Eq. (29) of [5].

Accurate measurements of other isotope shifts in helium would be very useful in testing our preferred value, as would an improved determination for $2\,^3S_1$-$3\,^3P_0$.

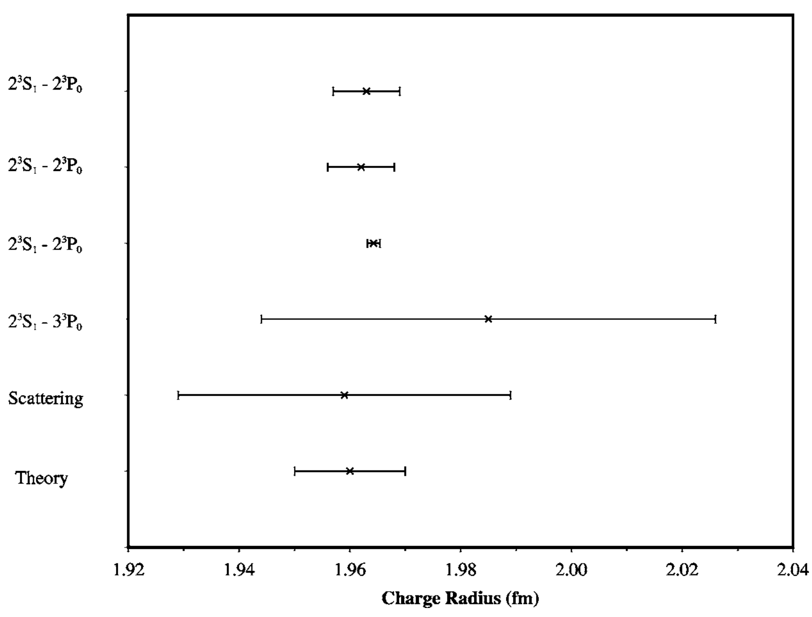

FIG. 1. The nuclear charge radius for ^{3}He plotted in the same order as in Table I.

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