All-optical digital logic: Full addition or subtraction on a three-state system

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Stimulated Raman adiabatic passage (STIRAP) is a well-studied pump-probe control scheme for manipulating the population of quantum states of atoms or molecules. By encoding the digits to be operated on as "on" or "off" laser input signals we show how STIRAP can be used to implement a finite-state logic machine. The physical conditions required for an effective STIRAP operation are related to the physical conditions expected for a logic machine. In particular, a condition is derived on the mean number of photons that represent an on pulse. A finite-state machine computes Boolean expressions that depend both on the input and on the present state of the machine. With two input signals we show how to implement a full adder where the carry-in digit is stored in the state of the machine. Furthermore, we show that it is possible to store the carry-out digit as the next state and thereby return the machine to a state ready for the next full addition. Such a machine operates as a cyclical full adder. We further show how this full adder can equally well be operated as a full subtractor. To the best of our knowledge this is the first example of a nanosized system that implements a full subtraction.

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I. INTRODUCTION

A finite-state logic machine [1,2] is a model of computation where the machine has internal states and the output depends not only on the input but also on the initial state of the machine. In response to an input the machine can change its state to a new, present, value and it produces an output. A finite-state machine inherently operates at a higher level than the more familiar combinational logic circuit because the combinational circuit generates an output that depends only on the input. The finite-state machine uses its internal states as a memory so that the present output depends on both the most recent input and its internal state.

In the physics, chemistry, and solid-state engineering communities there is a considerable effort to implement switches and transistors on the nanoscale. The aim is to design combinational circuits that are potentially an order of magnitude or more smaller in their physical dimensions and energetic requirements. If a sufficiently dense switching network can be conceived then one can argue that all of the higher-level logic can be implemented by software. A complementary effort is to realize quantum computations [3] where appeal to parallelism and entanglement [4] can provide qualitative enhancements in the performance of the algorithm, e.g., Refs. [4–8]. We have been following [9–11] what can be described as an intermediate approach where we take advantage of the discrete quantum level structure of nanoscale systems, be they lithographic quantum dots, atoms and molecules, or nanoparticles [12–19] but we do not make use of the coherence of the system, and the reading of the output depends only on the occupancy of the state.

For a nanosystem with resolvable quantum states, the response to a pump or probe depends on the state of the system. The external perturbation can change the state of the system. A finite-state logic machine is therefore a model of computations that is naturally suited to the way quantum systems behave. The particular advantages of stimulated Raman adiabatic passage (STIRAP), as will be demonstrated below, are first that the change of state can be affected by the external perturbation to a very high degree, literally approaching 100%, and, moreover, such residual noise that accumulates with many repeated operations can be wiped out by resetting the machine. The second advantage is that the same perturbation has a distinctly different effect on the system depending on the initial state. To demonstrate these advantages we note that experiments (examples include Ne [20], SO_2 [21], and NO [22]) have shown how well the observed results are described by the quantum-mechanical simulation [23–25] and so we can computationally generate different examples of the possible dynamics.

II. COHERENT POPULATION TRANSFER IN A THREE-LEVEL SYSTEM

The three-level system used in the simulation is shown schematically in Fig. 1. Two photons of frequencies ω_P and ω_S are used and these are, respectively, nearly resonant with the $1\rightarrow 2$ and $2\rightarrow 3$ transitions. Levels 1 and 3 are long living so that the output is the spontaneous emission from level 2. The kinetic or "intuitive" pumping scheme applies first photons of frequency ω_P so as to pump to the excited level 2. From this level the system can fluoresce or the population can be transferred from level 2 to level 3 by applying photons with the frequency ω_S. The point is that there is also a "counter intuitive" route that works remarkably well and it has the advantage that it can be arranged so that there is practically no population in state 2. In this route the photon of frequency ω_S is applied first with photons of frequency ω_P trailing it. In this paper we use only one kind of optical input:

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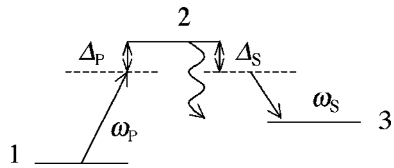

FIG. 1. The level scheme in a STIRAP experiment, and the pump and Stokes transitions. The population in level 2 is detected by its fluorescence. Noise is introduced if the population built in the final level into which level 2 radiatively decays. We have shown [10] that it is possible to reset the machine. The pump frequency is defined as $\omega_P = \omega_2 - \omega_1 - \Delta_p$ and the Stokes frequency as $\omega_S = \omega_2 - \omega_3 - \Delta_S$ [see Eq. below]. In the figure and in the simulations shown in Fig. 2, $\Delta_S = \Delta_P$.

a Stokes pulse at the frequency ω_S followed in time by a pump pulse at the frequency ω_P. We call this input a SP pulse or "the pump." If the system is in level 1 we expect that the SP pulse takes it to level 3 without any fluorescence from level 2. But if the system is in level 3 then the same SP pulse takes it to level 1 via the kinetic route and so level 2 will be populated and this can be detected by its fluorescence. The purpose of the simulation is to show that these expectations are fully borne out and that by either route it is possible to achieve an essentially 100% population transfer. The main source of noise is the spontaneous emission from level 2 that can terminate on both levels 1 and 3. If the radiative decay is to another level we lose that molecule from our ensemble. (This loss can be taken into account by giving a width to level 2.) To achieve near-perfect population transfer the pump and Stokes pulses need to be rather intense with the result that in the kinetic route there is not much emission from level 2 (see Fig. 2). The few photons that we need for detecting the output are possible without degrading the signal. After several cycles the noise accumulates but we can reset the machine, as discussed below and in Ref. [10].

For the three-level system as shown in Fig. 1, with two pairs of levels coupled by nearly resonant transient laser pulses, the Hamiltonian in the rotating-wave approximation is, in atomic units [23,26,27],

$$\mathbf{H} = \frac{1}{2}\begin{pmatrix} 2\omega_1 & \Omega_P(t)\exp(i\omega_P t) & 0 \\ \Omega_P(t)\exp(-i\omega_P t) & 2\omega_2 & \Omega_S(t)\exp(-i\omega_S t) \\ 0 & \Omega_S(t)\exp(i\omega_S t) & 2\omega_3 \end{pmatrix}. \tag{1}$$

The P and S pulses are as defined above. We reiterate that in this paper we consider only one possible type of input: a Stokes pulse followed closely in time by a pump pulse that we call a SP pulse. The amplitude of the laser pulse, $E(t)$, times the transition dipole μ, the Rabi frequency [23,28], is denoted as $\Omega(t)$, $\Omega(t) = \mu E(t)/\hbar$. The central frequency of the pump and Stokes lasers is about resonant with the $1 \rightarrow 2$ and the $2 \rightarrow 3$ transitions, i.e., $\omega_P = \omega_2 - \omega_1 - \Delta_P$ and $\omega_S = \omega_2 - \omega_3 - \Delta_S$ where the detuning $\Delta_P = \Delta_S = \Delta$ is small. Therefore the two lasers are off resonance for the transitions for which they are not intended. The rotating-wave approximation means that the Hamiltonian couples between levels using only that component of the oscillating electrical field which is in resonance or nearly so for the two levels. The Hamiltonian is that used in earlier studies of STIRAP [24] and other aspects of the problem are thoroughly covered in Refs. [23,29,30].

The wave function for the system is a linear combination of its three possible components, with time-dependent coefficients

$$\psi(t) = \sum_{i=1}^{3} \tilde{c}_i(t)|i\rangle, \quad \tilde{c}_i(t) = c_i(t)\exp(-i\omega_i t). \tag{2}$$

$\tilde{c}_i(t)$ are the coefficients in the interaction picture. The Hamiltonian in the interaction picture has the form

$$\tilde{\mathbf{H}} = \frac{1}{2}\begin{pmatrix} 0 & \Omega_P(t)\exp(-i\Delta_P t) & 0 \\ \Omega_P(t)\exp(i\Delta_P t) & 0 & \Omega_S(t)\exp(i\Delta_S t) \\ 0 & \Omega_S(t)\exp(-i\Delta_S t) & 0 \end{pmatrix}. \tag{3}$$

As usual, the coefficients $\tilde{\mathbf{c}}$ in Eq. (2) satisfy the matrix Schrödinger time-dependent equation $i d\tilde{\mathbf{c}}/dt = \tilde{\mathbf{H}}\tilde{\mathbf{c}}$, which we solve numerically without invoking an adiabatic approximation [31]. The Hamiltonian is Hermitian so that the total probability $\mathbf{c}^T\mathbf{c} = \tilde{\mathbf{c}}^T\tilde{\mathbf{c}}$ is conserved in time.

In this paper we are interested in using only the amplitude of the states but not the phase as the carrier of information. The results are therefore displayed as the probabilities of the states, $|c_i(t)|^2$, vs time. Two typical results from the simulations are shown as level population vs time in Fig. 2. For either case an input is a set of two laser pulses that are overlapping in time. The time sequence of the two SP pulses applied is shown in Fig. 2(a) in reduced time units t/σ where σ is the width of the S and of the P pulse ($\sigma_P = \sigma_S = \sigma$). In Fig. 2(b) the system is initially in level 1 so the first optical input sends it to level 3 with hardly any population (none, if fully adiabatic) in the intermediate level 2. After the first optical input the system is essentially 100% in level 3 so that the second optical input transfers all the population to level 1 with a brief intermediate sojourn in level 2. In Fig. 2(c) the

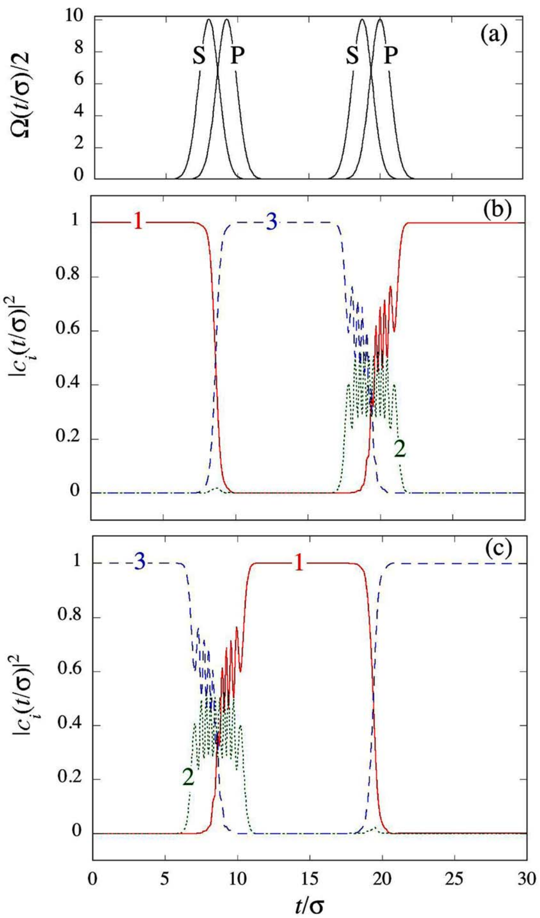

FIG. 2. (Color online) Probabilities of the three levels (level 1, full line; level 2, dotted line; level 3, dashed line) for a sequence of two SP pulses vs time (in reduced units t/σ where σ is the width of the S and of the P pulses that are taken to be equal). (a) The sequence of the two SP pulses. (b) The initial state is level 1, so that the first SP pulse corresponds to the STIRAP route while the second SP pulse corresponds to the kinetic route. (c) The initial state is level 3 so that the first population transfer is via the kinetic route while the second one follows the STIRAP one. Parameters of the simulation given in reduced time units (t/σ) are $\Omega_P(t/\sigma) = \Omega_S(t/\sigma) = 20.05 \exp\{-[(t/\sigma)-\tau_i]^2/2\}$, with $\tau_{s1}=8$, $\tau_{p1}=9.25$, $\tau_{s2}=18.75$, and $\tau_{p2}=20$. The detuning $\Delta=\Delta_S=\Delta_P=4(\sigma/t)$. The area of the pulse, $A(t)=\int\Omega(t/\sigma)d(t/\sigma)$ is 6.38π. We quote these details since achieving an essentially complete population transfer by the kinetic route, as shown in the figure, is sensitive to the intensity of the pulse and also to the detuning.

system is initially in level 3 and so the output (=fluorescence from level 2) follows the first but not the second optical input.

The physics shown in Fig. 2 is all that we need to implement a full binary addition or full binary subtraction. The "full" is an essential point. The technical meaning of full is that we take into consideration the carry digit from the previous addition or the borrow digit from the previous subtraction. This digit that we bring in from the previous stage can be 0 or 1. We encode it by the initial state of the machine; say level 1 if the digit is 0 and level 3 if it is 1. The two digits to be added (or subtracted; see below) are encoded by the two optical SP inputs being on, for 1, or off, for 0. Complete disclosure requires that we clearly state that for a perfectly efficient device after the two optical inputs we should have the machine in a state that correctly codes for the carry (or borrow) digit so that the machine is ready for the next addition (or subtraction). Unfortunately we are unable to quite do so. We need two more operations before the machine is ready for the next cycle. Specifically, we first read the state of the machine in order to get the sum out. This can be done by applying a SP pulse, as explained in [10] and also illustrated by Fig. 2. If the machine is in logical state 0 (physical level 1), level 2 is not populated when a SP pulse is applied, while if it is in logical state 1 (level 3), level 2 is populated and fluoresces when a SP pulse is applied. Next, whatever is the sum out, we reset the machine to state 0 by applying a SP pulse if needed. (See [10] for resetting a STIRAP machine.) If needed, we can also drain level 2 by stimulated emission. If level 2 fluoresced during one of the first two time intervals, it means the carry out is 1 and we input a SP pulse so that the state of the machine is 1 [and physically in level 3; Fig. 2(c)]. These two operations prepare the machine so that its logical state is the carry out of the previous addition and therefore it is the carry in of the next addition. Depending on the previous input, the preparation of the machine may be automatic. In other words, for certain values of the sum out and the carry out, the encoding of the carry may coincide with the reading of the sum out.

III. COUPLING CRITERIA

Before we turn to the design of a full adder or a full subtractor we discuss the physics of an optical input. This is necessary because we operate with short pulses and it therefore can conceivably be the case that the energy of the pulse is so low that the mean number of photons is below unity. It is then not clear that we can say that such a laser field represents an "on" as opposed to an "off" input. It is therefore very reassuring for us to show that the very condition required for adiabatic following [24,32,33] is equivalent to a large number of atoms (or molecules) undergoing the transition. In the quantum-mechanical simulations we do not assume that the system behaves adiabatically. We want the adiabatic behavior to follow from the intensity and shape of the pulses that we use rather than to be an assumption. But in the end we need the system to evolve effectively adiabatically and so we operate under conditions where adiabatic following is expected. We now show that the very same conditions have the classical meaning that many molecules made the transition and therefore that under such conditions we can reliably distinguish on and off.

The STIRAP literature [24,32,33] reports that for pulsed lasers the condition for adiabatic following is most conveniently written as

$$\Omega_{\rm eff}^2\sigma > 100/\sigma \qquad (4)$$

where, as before, σ is the width in time of the laser pulse and the left-hand side is the pulse energy.

For the pulse shape we use the same form as in the numerical simulations:

$$\Omega(t) = \mu E(t)/\hbar \equiv \frac{\mu E_0}{\hbar} \exp\left(-\frac{t^2}{2\sigma^2}\right). \quad (5)$$

Thereby, an effective number of photons can be defined by the semiclassical correspondence [34]

$$n = (1/2\pi\hbar\omega)E^2(t) = \frac{\hbar}{\mu^2 2\pi\omega}\Omega^2(t). \quad (6)$$

To describe the time evolution by classical rate equations [35,36] one defines the kinetic rate of transitions as $R \equiv \Omega^2/\Gamma$ (in the notation of [35] where Γ is the width of the transition; $R=c\Sigma n$ where Σ is the cross section). When the laser intensity is high, as it is under STIRAP conditions and other perturbations by the environment are, by comparison, negligible the width is dominated by the power broadening (i.e., by the rate of pumping) and so $R \cong \Omega$. We define A as the mean (over the duration of the pulse) number of transitions per input. A is therefore given by integrating $\Omega(t)$ over the duration of the pulse and using Eq. (4) this number of transitions is large compared to unity:

$$A \equiv \int \Omega(t)dt \gg 1. \quad (7)$$

A, the number of transitions, is, in a kinetic description, the number of molecules that collided with a photon during the pulse duration. In a two-state picture [24] where the population at resonance scales as $\sin^2(A/2)$ one can obtain a tighter result because the condition of approaching 100% transfer is

$$A = (2k+1)\pi \quad (8)$$

where k is an integer. The number of transitions is the number of photons absorbed by the system during the pulse so semiclassically Eq. (8) can be interpreted as a restriction on the number of photons. This makes intuitive sense because efficient transfer requires that the number of transitions is odd while the condition for 100% return to the initial state, $A=2k\pi$, corresponding to an even number of transitions. Also note that for two lasers $A^2=A_P^2+A_S^2$. See [37] for the multistate case.

IV. IMPLEMENTATION OF A FULL ADDER

In a full addition, unlike in a full subtraction as discussed below, the order into which the two input digits x and y and the carry in are combined does not matter. In order that the first steps for a full addition and a full subtraction are identical, we operate the full adder using two optical inputs as follows. In the first step we add the carry in and the input digit y, $y=0,1$. The initial time is t and we take the unit of computer time to be somewhat longer than the duration of the optical input. Then the first step is summarized by the Boolean equations

$$\text{state}(t+1) = \text{carry in XOR } y, \quad (9)$$

$$\text{carry 1} = \text{carry in AND } y. \quad (10)$$

Carry 1 is an intermediate result needed to compute the carry out by Eq. (13) below of the next stage. Carry 1 is logically represented as the output of the machine at time $t+1$ and its value is 1 if we detect fluorescence from level 2. The only time this happens is when the optical input is on and when it induces a kinetic route, namely, we have an output if the optical input is on and the initial state is level 3 [Fig. 2(c)]. Rather than by logic equations we can show the operations in tabular form, Table I, a format that is known as a truth table. At the next time interval, we input x, the second binary digit to be added, and its value is also encoded as a SP pulse. The truth table is given in Table II.

TABLE I. Truth table for the half addition of y and the carry in.

State(t) ≡carry in	$y(t)$ ≡SP pulse	State(t+1) (XOR) ≡midway sum	Output(t+1) (AND) ≡carry 1
0 (level 1)	0	0 (level 1)	0
0 (level 1)	1	1 (level 3)	0
1 (level 3)	0	1 (level 3)	0
1 (level 3)	1	0 (level 1)	1

State $(t+2)$ is the XOR sum of the three inputs (x, y, and the carry in):

$$\text{state}(t+2) = \text{state}(t+1)\text{XOR } x = [\text{state}(t)\text{XOR } y]\text{XOR } x$$
$$= \text{carry in XOR } y \text{ XOR } x \quad (11)$$

and using an overbar to denote negation:

$$\text{carry 2} = \text{state}(t+1)\text{AND } x = (\text{carry in XOR } y)\text{AND } x$$
$$= (\overline{\text{carry in}} \text{ AND } y + \text{carry in AND } \bar{y})\text{AND } x$$
$$= x \text{ AND } y \text{ AND } \overline{\text{carry in}} + x \text{ AND } \bar{y} \text{ AND carry in}. \quad (12)$$

The carry out is given by reading fluorescence from level 2, either at time $t+1$ or at time $t+2$,

$$\text{carry out} = \text{carry 1} + \text{carry 2} = (\text{carry in AND } y)$$
$$+ x \text{ AND } y \text{ AND } \overline{\text{carry in}} + x \text{ AND } \bar{y} \text{ AND carry in}$$
$$= \text{carry in AND } y \text{ AND}(x+\bar{x})$$
$$+ x \text{ AND } y \text{ AND } \overline{\text{carry in}} + x \text{ AND } \bar{y} \text{ AND carry in}$$
$$= x \text{ AND } y \text{ AND carry in} + \bar{x} \text{ AND } y \text{ AND carry in}$$
$$+ x \text{ AND } y \text{ AND } \overline{\text{carry in}} + x \text{ AND } \bar{y} \text{ AND carry in} \quad (13)$$

TABLE II. Truth table for the half addition of the midway sum and the x input.

State(t+1) ≡midway sum	$x(t+1)$ ≡SP pulse	State(t+2) (XOR) ≡sum	Output(t+2) (AND) ≡carry 2
0 (level 1)	0	0 (level 1)	0
0 (level 1)	1	1 (level 3)	0
1 (level 3)	0	1 (level 3)	0
1 (level 3)	1	0 (level 1)	1

TABLE III. Truth table for the full addition of x and y.

x	y	Carry in	Sum out	Carry out
0	0	0	0	0
1	0	0	1	0
0	1	0	1	0
1	1	0	0	1
0	0	1	1	0
1	0	1	0	1
0	1	1	0	1
1	1	1	1	1

where we used the so-called idempotent law [1] $(x+\bar{x})=1$ to express carry out as a sum of AND terms. The last line is the direct way to check that we have a carry digit of 1 if at least two or if all three of the three inputs x, y, and carry in are unity. Equation (13) can be simplified into a sum of three terms that each has only two variables. That can be done by hand or using Karnaugh maps [1,2] to finally give

$$\text{carry out} = x \text{ AND } y + x \text{ AND carry in} + y \text{ AND carry in}. \quad (14)$$

Similarly we get for the sum out (which equals the state at time $t+2$)

$$\text{sum out} = x \text{ AND } \bar{y} \text{ AND } \overline{\text{carry in}} + \bar{x} \text{ AND } y \text{ AND } \overline{\text{carry in}} + x \text{ AND } y \text{ AND carry in} + \bar{x} \text{ AND } \bar{y} \text{ AND carry in}. \quad (15)$$

Equation (13), (11), and (15) can be verified using the truth table of the full adder (Table III).

V. IMPLEMENTATION OF A FULL SUBTRACTOR

Turning next to subtraction, in addition to x and y, the minuend and subtrahend, there is a third Boolean variable, the borrow from the previous computation, also called "borrow in." The truth table for the two outputs, the difference out and the borrow out, is given in Table IV.

Comparing Tables III and IV we verify that in binary arithmetic the sum out and the difference out have the same value. Therefore the Boolean equation for the difference out is

TABLE IV. Truth table for a full subtraction $x-y$.

x	y	Borrow in	Difference out	Borrow out
0	0	0	0	0
1	0	0	1	0
0	1	0	1	1
1	1	0	0	0
0	0	1	1	1
1	0	1	0	0
0	1	1	0	1
1	1	1	1	1

TABLE V. Truth table for the half addition of y and borrow in.

y ≡input SP pulse	Borrow in ≡state(t)	Mid difference ≡state(t+1)	Borrow 1 ≡output from level 2
0	0	0	0
0	1	1	0
1	0	1	0 (STIRAP)
1	1	0	1 (kinetic route)

$$\text{difference out} = x \text{ AND } \bar{y} \text{ AND } \overline{\text{borrow in}} + \bar{x} \text{ AND } y \text{ AND } \overline{\text{borrow in}} + x \text{ AND } y \text{ AND borrow in} + \bar{x} \text{ AND } \bar{y} \text{ AND borrow in}. \quad (16)$$

On the other hand, the borrow out is almost but not quite the carry out of the full addition. Rather, the borrow out requires that in comparison to Eq. (14) the minuend x is negated,

$$\text{borrow out} = \bar{x} \text{ AND } y + \bar{x} \text{ AND carry in} + y \text{ AND carry in}. \quad (17)$$

What Eqs. (16) and (17) imply is that the first stage of the full subtractor is completely analogous to that of the full adder (see Table I). Table V is therefore a half adder for y and the borrow in. As for addition here too we store the borrow in in the state of the machine while the subtrahend y is coded as an input pulse. Keeping the same convention as in Table I, namely, level 1≡borrow in=0 and level 3 ≡borrow in=1, we have Table V

The output is 1 when the system is in level 3 so that the input pulse drives it to level 1 via level 2. The two logic equations that describe Table V are

$$\text{mid difference} = y \text{ XOR state}(t), \quad (18)$$

$$\text{borrow } 1 = y \text{ AND } b_{\text{in}}. \quad (19)$$

In the second stage (see Table VI) we implement a second half adder to get the difference out:

$$\text{difference out} = \text{state}(t+2) = y \text{ XOR state}(t) \text{XOR } x. \quad (20)$$

TABLE VI. Truth table for the half addition of the x input and the mid difference.

x input ≡input SP pulse	Mid difference ≡state(t+1)	Difference out ≡state(t+2)	Output
0	0	0	0
0	1	1	0
1	0	1	0 (STIRAP)
1	1	0	1 (kinetic route)

TABLE VII. Encoding of the input x into the state of the machine.

x (SP pulse)	state(t)	state($t+1$) $\equiv x$
0	0 (level 1)	0
1	0 (level 1)	1 (level 3, by STIRAP)

In this second stage, when we read the signal that is the fluorescence from level 2, if any, we give the complementary logic meaning to the input signal. This leads to the following logic equation:

$$\text{borrow } 2 = \bar{x} \text{ AND mid diff} = \bar{x} \text{ AND}(y\bar{b}_{\text{in}} + \bar{y}b_{\text{in}}) = \bar{x} \text{ AND } y \text{ AND } \bar{b}_{\text{in}} + \bar{x} \text{ AND } \bar{y} \text{ AND } b_{\text{in}}. \quad (21)$$

Finally we get borrow out as the sum (logical OR) of the two intermediate borrows:

$$\begin{aligned}\text{borrow out} &= \text{borrow } 1 + \text{borrow } 2 = y \text{ AND } b_{\text{in}} + \bar{x} \text{ AND mid diff} = y \text{ AND } b_{\text{in}} \\ &\quad + \bar{x} \text{ AND } y \text{ AND } \bar{b}_{\text{in}} + \bar{x} \text{ AND } \bar{y} \text{ AND } b_{\text{in}} \\ &= y \text{ AND } b_{\text{in}}(x+\bar{x}) + \bar{x} \text{ AND } y \text{ AND } \bar{b}_{\text{in}} \\ &\quad + \bar{x} \text{ AND } \bar{y} \text{ AND } b_{\text{in}} = x \text{ AND } y \text{ AND } b_{\text{in}} \\ &\quad + \bar{x} \text{ AND } y \text{ AND } b_{\text{in}} + \bar{x} \text{ AND } y \text{ AND } \bar{b}_{\text{in}} \\ &\quad + \bar{x} \text{ AND } \bar{y} \text{ AND } b_{\text{in}}. \quad (22)\end{aligned}$$

It is now seen that Eq. (20) for the difference out is the same as Eq. (15) for the sum out while Eq. (22) for the borrow out corresponds to Eq. (13) for the carry out and the two are the same if x is replaced by $\bar{x}$.

Finally, we note that we can also implement a half subtractor where the borrow out is obtained as the output of an INHIBIT function, which is the equivalent of a controlled-NOT logic operation in quantum computing [3]. Like a half adder, a half subtractor has only two inputs x and y and two outputs the difference and the borrow. In order to calculate x-y, we this time encode the x input (minuend) into the state of the machine. This can be done by applying a SP pulse to the machine prepared in level 1, that is, with the logical state 0 (see Table VII).

To obtain the INHIBIT logic function for the borrow, we then encode the y input (subtrahend) into a PS pulse, meaning that in this case, for the input, the pump pulse precedes the Stokes pulse. This leads to the truth table shown in Table VIII.

We show in the Appendix how one can build a full subtractor based on two half subtractors. The scheme in the Appendix has two disadvantages as compared to the implementation of the full subtractor by two half adders discussed above. The first one is that strictly speaking it is not a finite-state machine in the sense that the borrow in is not encoded into the state of the machine. The second one is that proceeding this way, full subtraction and full addition require a different experimental setup.

TABLE VIII. Truth table for a half subtractor $x-y$.

x [state(t)]	y (PS pulse)	Diff [state($t+1$)]	Borrow
0 (level 1)	0	0 (level 1)	0
0 (level 1)	1	1 (level 3)	1 (kinetic route)
1 (level 3)	0	1 (level 3)	0
1 (level 3)	1	0 (level 1)	0 (STIRAP)

VI. CONCLUDING REMARKS

Quantum-mechanical simulations for realistic parameters show that the population-transfer control method known as STIRAP can be used to operate a finite-state logic circuit. Specifically one can select such operating conditions that even without imposing adiabatic passage the population transfer in the so-called, counterintuitive direction fully approaches 100%. In the opposite direction, called intuitive or kinetic, the same laser input gives rise to a population transfer that does proceed via very small occupancy of the intermediate level. This is exactly what is needed. For the same laser pulse sequence the system responds differently depending on what is its initial state. This difference can be detected and is one part of the output. The other output is the final state of the system. We show how this rather simple three-level structure (two stable physical states that are used as the states of the machine and an intermediate state that can fluoresce) is sufficient to perform a full addition and also a full subtraction. The initial state of the machine stores the digit that is the carry in or the borrow in as computed by the previous cycle. The machine is left with its final state storing the digit needed in for the next cycle.

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APPENDIX: A FULL SUBTRACTOR MADE OF TWO HALF SUBTRACTOR

To operate a full subtractor on the basis of combining two half subtractors, we first use the half subtractor shown in Table VIII, where the minuend x digit is encoded into the state of the machine and the borrow in provided as a PS pulse. The corresponding truth table is shown in Table IX.

TABLE IX. First half subtractor that performs $x-$borrow in.

x [state(t)]	borrow in (PS pulse)	Mid [state($t+1$)]	Borrow 1
0 (level 1)	0	0	0
0 (level 1)	1	1 (level 3)	1 (kinetic route)
1 (level 3)	0	1 (level 3)	0
1 (level 3)	1	0 (level 1)	0 (STIRAP)

TABLE X. Truth table for the half subtraction mid−y.

Mid [state(t+1)]	$y(t+1)$ (PS pulse)	Difference out	Borrow 2
0 (level 1)	0	0	0
0 (level 1)	1	1 (level 3)	1 (kinetic)
1 (level 3)	0	1 (level 3)	0
1 (level 3)	1	0 (level 1)	0 (STIRAP)

The logic equations for the intermediate difference and borrow are

$$\text{mid} = x \text{ XOR } b_{\text{in}} = \bar{x} \text{ AND } b_{\text{in}} + x \text{ AND } \bar{b}_{\text{in}} \tag{A1}$$

and

$$\text{borrow 1} = \bar{x} \text{ AND } b_{\text{in}}. \tag{A2}$$

The subtraction mid−y with the subtrahend y encoded as a PS pulse is implemented in the second half subtractor. The truth table is given in Table X.

The final equation for the difference out is the same as Eq. (16):

$$\text{difference out} = \text{mid XOR } y = x \text{ XOR } y \text{ XOR borrow in}, \tag{A3}$$

and for the borrow out, we obtain the same equations as Eq. (22) by combining the borrow 1 and the borrow 2:

$$\text{borrow 2} = \overline{\text{mid}} \text{ AND } y = (x \text{ AND } b_{\text{in}} + \text{not } \bar{x} \text{ AND not } \bar{b}_{\text{in}}) \times \text{AND } y = x \text{ AND } y \text{ AND } b_{\text{in}} + \bar{x} \text{ AND } y \text{ AND } \bar{b}_{\text{in}} \tag{A4}$$

so that

$$\begin{aligned} \text{borrow out} = \text{borrow 1} + \text{borrow 2} &= \bar{x} \text{ AND } b_{\text{in}} + x \text{ AND } y \text{ AND } b_{\text{in}} + \bar{x} \text{ AND } y \text{ AND } \bar{b}_{\text{in}} \\ &= \bar{x} \text{ AND } (y+\bar{y}) \text{ AND } b_{\text{in}} + x \text{ AND } y \text{ AND } b_{\text{in}} + \bar{x} \text{ AND } y \text{ AND } \bar{b}_{\text{in}}, \\ \text{borrow out} &= \bar{x} \text{ AND } y \text{ AND } b_{\text{in}} + \bar{x} \text{ AND } \bar{y} \text{ AND } b_{\text{in}} + x \text{ AND } y \text{ AND } b_{\text{in}} + \bar{x} \text{ AND } y \text{ AND } \bar{b}_{\text{in}}. \end{aligned} \tag{A5}$$

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