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Enhancing the observability of the Efimov effect in ultracold atomic gas mixtures

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We discuss the prospects for observing the characteristic features of the Efimov effect in a two-component ultracold atomic gas near an interspecies Feshbach resonance. In the ultracold regime, the Efimov effect is expected to be manifested in the three-body collision rates through the appearance of series of minima or maxima as a function of the two-body s-wave scattering length a. Here, we propose the observation of this Efimov physics through measurements of the inelastic three-body rate constants near a Feshbach resonance. Our analysis suggests that boson-fermion mixtures, where the bosons are much heavier than the fermions, are the most favorable system to observe such features.

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The Efimov effect was originally predicted in nuclear physics in the early 1970s [1]. The Efimov effect is the emergence of a large number of three-body bound states even when the two-body subsystems have none, and it requires that at least two of the interparticle interactions be resonant. Besides this remarkable effect on the three-body bound state spectrum, it has been shown [2–4] that Efimov physics has far-reaching influence on ultracold three-body collisions which, in turn, can be a primary atomic and molecular loss mechanism in ultracold quantum gases. To date, however, it exists only as a theoretical prediction, since it has not been conclusively demonstrated experimentally.

The ability to experimentally control the interatomic interaction by tuning an external magnetic field near a Feshbach resonance in ultracold atomic gases [5–7] creates new possibilities for observing Efimov physics. Near such a resonance, the two-body s-wave scattering length a can vary from $-\infty$ to $+\infty$, revealing Efimov physics in the three-body system whenever $|a|$ is much larger than the characteristic size of the interatomic interactions r_0. In this regime, ultracold three-body observables behave universally in the sense that their dependence on a can be predicted while the details of the interatomic interactions enter only through a small number of short-range parameters. The clearest manifestation of Efimov physics is through the appearance of a series of minima or maxima in the three-body observables that are equally spaced on a $\ln|a|$ scale. The multiplicative factor that separates these features is e^{π/s_0}, where s_0 depends on the mass ratio between the collision partners and the number of resonant interactions. Such series have been predicted for three-body collision processes such as three-body recombination, collision-induced dissociation, and vibrational relaxation [2–4].

While direct experimental evidence of the Efimov *effect* is not available, some recent measurements of three-body collision rates can be interpreted as indirect evidence for Efimov *physics*. Specifically, the predicted a^4 scaling law for recombination of identical bosons [8] has been observed [9] as has the $a^{-2.55}$ suppression of the relaxation of weakly bound molecules in two-spin fermionic gases [10,11], resulting in the observation of long-lived molecules [12–14]. Despite these successes and the experimental control now available, the most convincing signature of Efimov physics, namely equally spaced features on a $\ln|a|$ scale, has not yet been observed. Furthermore, such observations are unlikely to be made in the near future for the equal mass systems most studied theoretically since the features are expected to occur each time a increases by $e^{\pi/s_0}\approx 22.7$. To experimentally verify that their spacing is equal, a minimum of three such features must be observed. Since theory can only predict their spacing and not their positions, a must experimentally cover a range of about 22.7^4, which is not easily achievable.

In this paper, we argue that measuring a sequence of three features arising from Efimov physics can be made much more tractable by simply changing the constituents of the ultracold gas. In particular, boson-fermion mixtures [15,16], where the bosonic atoms are much heavier than the fermionic atoms, exhibit the most favorable conditions for observing Efimov physics. Our proposal is to measure the inelastic loss rates through measurements of the time evolution of both atom and molecule numbers near a Feshbach resonance, as was done, for instance, in Refs. [9,11,16]. In general, the use of mixtures of heavy and light atoms leads to more favorable conditions for observing Efimov physics [1]. In this limit, the space between the minima or maxima is smaller and, therefore, our goal of three Efimov features can be achieved with a more experimentally manageable range of a. This statement applies equally well to fermion-fermion and boson-boson mixtures, but other conditions complicate using these systems. We also present numerical calculations that indicate how other issues, especially the presence of several two-body bound states, might limit the observability of Efimov physics in ultracold mixtures of commonly used alkali-metal atoms.

In two-component ultracold atomic gases—of, say, X atoms and Y atoms—only two three-body systems involve interspecies interactions: XXY and XYY. Ideally, both species would be prepared in hyperfine states that minimize two-body losses so that three-body collisions become dominant. In this case, extracting the inelastic three-body rates would then be cleaner, requiring monitoring the atomic and molecular densities over time. The rate equations governing this evolution, however, depend on contributions from both XXY and XYY systems. For instance, the rate equation for the atomic density n_X depends on the three-body recombination rate constant for $X+X+Y$ and $X+Y+Y$ collisions and the vibrational relaxation rate for $XY+X$ collisions. Similarly, the rate equation for n_Y depends on the $X+X+Y$, $X+Y+Y$, and $XY+Y$ collision rates, while the rate equation for n_{XY}, where XY are weakly bound molecules, depends on the $XY+X$ and $XY+Y$ collisions, as well as relaxation due to molecule-molecule collisions, $XY+XY$. Near a Feshbach resonance, however, the importance of each collision process for the atomic and molecular densities evolution is dictated by its energy and scattering length dependence [2,3] and, evidently, on the product of densities of each specie involved in the collision process. In describing the rate equations, we have assumed that the intraspecies interactions are not resonant, that two-atom processes are not important, that collision-induced dissociation is not energetically allowed, and that all inelastic processes result in loss.

In general, to observe Efimov physics via three-body recombination (K_3), the number of molecules trapped should somehow be minimized so that recombination correlates most directly to atomic loss. Otherwise, the atomic densities n_X and n_Y would also be affected by $XY+X$ and $XY+Y$ collisions, complicating the interpretation. If a substantial number of weakly bound molecules are trapped, then vibrational relaxation ($V_{\rm rel}$) is the better observable for Efimov physics. In this case, relaxation due to molecule-molecule collisions must also be considered [10], but no Efimov features have yet been predicted for such collisions.

We have previously obtained [2,3] the energy and scattering length dependence for the ultracold three-body rate constants and found that processes exhibiting minima due to Efimov physics are modulated by the factor

$$M_{s_0}(a) = \sin^2[s_0 \ln(a/r_0) + \Phi] \tag{1}$$

and that processes for which Efimov physics emerges as peaks share the factor

$$P_{s_0}(a) = \frac{\sinh(2\eta)}{\sin^2[s_0 \ln(|a|/r_0) + \Phi] + \sinh^2(\eta)}. \tag{2}$$

In the above equations, Φ is an unknown phase and η parametrizes the probability of making a transition to a deeply bound state—both arise from short range physics associated with details of the interatomic interactions and are expected to differ for each system. It is from these expressions that the spacing factor e^{π/s_0} is derived.

Since one of the main hindrances to observing Efimov physics in a "traditional" equal mass boson system is the need to experimentally vary a by a factor of roughly $(e^{\pi/s_0})^4 \approx 22.7^4$ ($s_0 \approx 1.0064$) in order to see three successive minima or maxima, one way to simplify the experiment is to reduce the spacing between the features. It is clear that this can only be accomplished by changing s_0. As it turns out, it was demonstrated in Ref. [1] that, in fact, s_0 depends on the masses of the constituent atoms. Figure 1 shows this dependence for both boson-boson-fermion (BBF) and boson-fermion-fermion (BFF) systems as a function of the mass ratio $\delta = m_F/m_B$. These results were obtained analytically in Ref. [3] following Ref. [1] after proper symmetrization. The influence of the identical particle symmetry is seen in the difference between the curves labeled s_0 (BBF) and s_0' (BFF). Since we can realistically expect atomic masses that differ by at most a factor of 20–30, s_0 can be increased to roughly two—a dramatic improvement since the spacing is determined by an exponential.

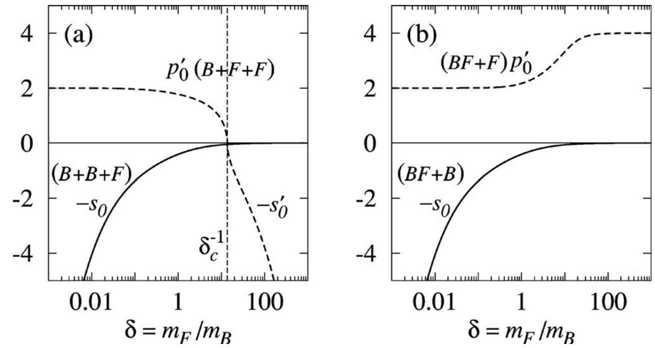

FIG. 1. Coefficients s_0 and p_0 as a function of the mass ratio δ for (a) three-body recombination and (b) vibrational relaxation for boson-fermion mixtures.

Figure 1 also shows the mass dependence of the parameter p_0, determined in Ref. [3] for three-body systems with two indistinguishable fermions. This parameter is also influenced by Efimov physics and is the key parameter determining the suppression of vibrational relaxation in $FX+F$ collisions. Even though p_0 is influenced by Efimov physics, rate constants that depend on it do not show minima or maxima as do rate constants that depend on s_0. We note, though, that even the BFF systems can show Efimov features. In Fig. 1(a), the BFF curve is labeled with p_0' at small δ, but transitions to s_0' at δ larger than δ_c^{-1} (defined this way for consistency with Ref. [3]). Therefore, δ_c^{-1} is the critical mass ratio above which BFF systems display the Efimov effect (since they then have an attractive potential). Recombination in BFF systems will thus only show minima or maxima for δ larger than 13.607, spaced by $e^{\pi/s_0'}$.

Table I summarizes the scattering length and energy dependence of all of the three-body rate constants relevant to mixtures of bosons and fermions. The formulas in Table I, however, do not include a proportionality constant that gives the correct units for each rate constant. The cases in bold are expected to exhibit Efimov physics either through minima

TABLE I. Energy and scattering length dependence of the three-body rate constants in boson-fermion (B-F) mixtures. Boldface indicates the processes expected to exhibit the Efimov effect, characterized by a series of minima (M_{s_0}) or maxima (P_{s_0}) given, respectively, by Eqs. (1) and (2).

K_3	Energy	$a>0$	$a<0$
$B+B+F$	**const**	$\boldsymbol{M_{s_0}a^4}$	$\boldsymbol{P_{s_0}\lvert a\rvert^4}$
$B+F+F$ ($\delta<\delta_c^{-1}$)	E	a^6	$\lvert a\rvert^{6-p_0'}$
$B+F+F$ ($\delta>\delta_c^{-1}$)	$\boldsymbol{E}$	$\boldsymbol{M_{s_0}a^6}$	$\boldsymbol{P_{s_0}\lvert a\rvert^6}$
$V_{\rm rel}$	Energy	$a<0$	$a>0$
$BF+B$	**const**	$\boldsymbol{P_{s_0}a}$	**const**
$BF+F$	const	$a^{1-2p_0'}$	const

[M_{s_0} from Eq. (1)] or maxima [P_{s_0} from Eq. (2)]. A word of caution is in order at this point. The scaling behavior listed in the table holds only so long as the system is in the threshold regime, i.e., when the collision energy is the smallest energy [18]. Since most experiments with ultracold atoms are done at essentially fixed collision energy (temperature), the requirement of being in the threshold regime defines the maximum value of the scattering length for which threshold behavior can be expected. Explicitly, $a_{\max}=\hbar/(2\mu_{XY}E)^{1/2}$, where μ_{XY} is the two-body reduced mass. For $a>a_{\max}$, the threshold scaling laws break down and finite energy effects quickly wash out the features related to Efimov physics. As a consequence, even at ultracold temperatures only a finite number of minima or maxima can be expected to be observed.

In general, the observability of Efimov physics depends not only on the rate constants, but also on the various atomic and molecular densities. This is certainly the case for boson-boson and fermion-fermion mixtures. For recombination in boson-fermion mixtures, however, the exact values of the atomic densities are not crucial since $B+F+F$ collisions are suppressed by a factor of the energy at ultracold temperatures. $B+B+F$ collisions will thus dominate, making boson-fermion mixtures the top candidate for observing Efimov physics. Furthermore, choosing $m_B \gg m_F$ makes s_0 large and puts BFF recombination in the regime without minima or maxima. Of course, as mentioned before, if the molecular density is low, the atomic loss is due mainly to recombination in $B+B+F$ collisions. Taking all of these points into consideration, the rate equations are approximately

$$\dot{n}_B \approx \dot{n}_F \approx -K_3^{B+B+F} n_B^2 n_F, \quad (3)$$

and minima and peaks (Table I) should be observed for $a>0$ and $a<0$, respectively. If the molecular density is not negligible, however, making $BF+B$ and $BF+F$ collisions possible, Efimov physics should still be cleanly observable in the fermion density for $a>0$, since $BF+F$ collisions are suppressed (see Table I).

If substantial numbers of molecules are present in the gas, the density dependence in the rate equations favors vibrational relaxation as the best process to measure for effects of Efimov physics. In particular, since relaxation for $BF+F$ collisions is suppressed as $a^{1-2p_0'}$ [$2 \leqslant p_0' \leqslant 4$ from Fig. 1(b)], measuring peaks in the $BF+B$ relaxation constant will reveal Efimov physics. In addition, molecule-molecule collisions are also suppressed since BF molecules are composite fermions, eliminating s-wave collisions by symmetry. The time evolution for both atomic and molecular species is given by

$$\dot{n}_B \approx \dot{n}_{BF} \approx -V_{\rm rel}^{BF+B} n_{BF} n_B$$

$$\dot{n}_F \approx -V_{\rm rel}^{BF+F} n_{BF} n_F. \quad (4)$$

In this case, Efimov physics is manifested only for $a>0$ (Table I) due to $BF+B$ collisions.

Figure 2 shows numerical results for $a>0$ recombination using a model system with masses corresponding to ^{133}Cs + ^{133}Cs + ^{6}Li collisions, disregarding the same species interactions. We have calculated the rates for scattering lengths up to 5×10^4 a.u. at a collision energy of 1 nK by using a two-body model potential adjusted to support two s-wave and one p-wave bound states, and assuming $r_0=15$ a.u. In Fig. 2, we show recombination to weakly bound s-wave ^{133}Cs^{6}Li molecules and to deeply bound p-wave ^{133}Cs^{6}Li molecules—recombination to deeply bound s-wave was about two orders of magnitude smaller than recombination to deeply bound p-wave molecules. In this range of a, we found four minima with spacing given by $e^{\pi/s_0}=4.88$ ($s_0=1.982766$). For large a, the numerical results confirm the analytical formula from Table I. Not unexpectedly, the results for small a differ from the analytical formula as details of the interatomic interactions become important. As a result, the first minimum in Fig. 2 does not follow the e^{π/s_0} spacing. The expected spacing is recovered, though, between the second and higher minima.

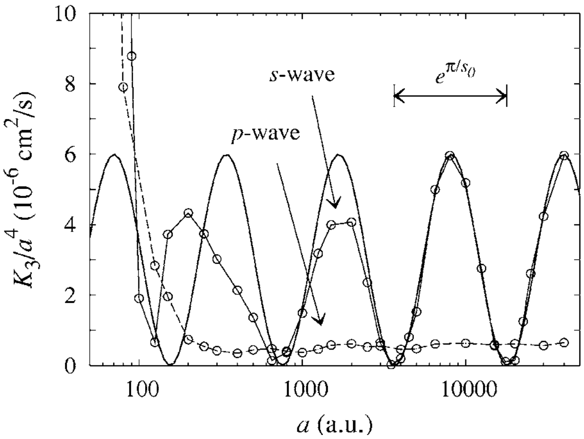

FIG. 2. Recombination for model ^{133}Cs+^{133}Cs+^{6}Li collisions into weakly bound s-wave ^{133}Cs^{6}Li molecules (solid line with circles) and in to deeply bound p-wave ^{133}Cs^{6}Li molecules (dashed line with circles). The solid line represents the analytical expression [Eq. (1)].

Figure 2 also shows that even in the presence of recombination to deeply bound states, the contrast ratio of the oscillations should be measurable. Further, since recombination to highest vibrationally excited channels usually dominates, we expect this result to hold even for more realistic potentials with many two-body bound states. Nevertheless, recombination to deeply bound states could set limits on the observation of Efimov physics by washing out the minima or reducing the contrast. Preliminary numerical calculations suggest, however, that for a more realistic two-body potential with a hard core, recombination to deeply bound states is less important than indicated in Fig. 2.

Table II summarizes the prospects for observing two and three Efimov features in boson-fermion mixtures of commonly used alkali atoms. Although the most convincing experimental demonstration of the Efimov effect requires the observation of three features, the observation of two features with the predicted spacing would be very suggestive. Table II shows the expected spacing e^{π/s_0} of Efimov features given the masses of the atoms, the minimum magnitude $a_{\min}$ to which a must be tuned, and the maximum energy $E_{\max}$ for which the system remains in the threshold regime for a up to $a_{\min}$. These quantities are determined by

TABLE II. Values of $a_{\rm min}$ and $E_{\rm max}$ required for observing two and three Efimov features. The table also gives the spacing e^{π/s_0} between the features in $B+B+F$ and $BF+B$ collisions in boson-fermion mixtures. $a_{\rm min}$ is given in atomic units.

		Two features		Three features	
B-F	e^{π/s_0}	$\lvert a_{\rm min}\rvert$	$E_{\rm max}$(nK)	$\lvert a_{\rm min}\rvert$	$E_{\rm max}$(nK)
^{133}Cs-^{6}Li	4.877	3×10^3	1500	2×10^4	60.0
^{87}Rb-^{6}Li	6.856	8×10^3	230	6×10^4	5.00
^{23}Na-^{6}Li	36.28	9×10^5	$\ll0.1$	3×10^7	$\ll0.1$
^{7}Li-^{6}Li	$>10^2$	$\gg10^8$	$\ll0.1$	$\gg10^8$	$\ll0.1$
^{133}Cs-^{40}K	47.02	2×10^6	$\ll0.1$	9×10^7	$\ll0.1$
^{87}Rb-^{40}K	$>10^2$	$\gg10^8$	$\ll0.1$	$\gg10^8$	$\ll0.1$
^{23}Na-^{40}K	$>10^2$	$\gg10^8$	$\ll0.1$	$\gg10^8$	$\ll0.1$
^{7}Li-^{40}K	$>10^2$	$\gg10^8$	$\ll0.1$	$\gg10^8$	$\ll0.1$

$$a_{\rm min}=r_0\frac{(e^{\pi/s_0})^N}{f(\delta)} \quad \text{and} \quad E_{\rm max}=\frac{\hbar^2}{2\mu_{BF}a_{\rm min}^2}, \tag{5}$$

where $f(\delta)=[\sqrt{\delta(\delta+2)}/(\delta+1)]^{1/2}$ is a mass ratio dependent term determined in Ref. [3]. Since the first Efimov feature is not expected to follow the predictions (as shown in Fig. 2), our estimates here use $N=3$ and 4 for observing two and three features, respectively. In Table II, we have assumed $r_0=15$ a.u. for all mixtures; Table II thus gives only a rough estimate for $a_{\rm min}$ and $E_{\rm max}$. Table II includes boson-fermion mixtures used in recent experiments: ^{23}Na–^{6}Li [15] and ^{87}Rb–^{40}K [16], as well as ^{7}Li–^{6}Li[17]. Not surprisingly, the systems with the largest mass ratios, ^{133}Cs–^{6}Li and ^{87}Rb–^{6}Li, provide the most favorable prospects for observing Efimov physics since both mixtures offer manageable temperatures and a reasonable range of a.

Before closing, we briefly consider observing Efimov physics in other systems. As shown in Refs. [2,3] for boson-boson mixtures, Efimov physics is observable in K_3 and $V_{\rm rel}$. Unlike boson-fermion mixtures, however, none of the collision rates are suppressed in the ultracold regime. Thus, the competition might wash out the features related to Efimov physics. The situation is similar in fermion-fermion mixtures. In this case, though, Efimov physics is visible only in K_3 for mixtures of heavy and light fermions and is suppressed by a factor of E at ultracold temperatures. In both cases, the control of atomic and molecular densities would be crucial for observing Efimov physics.

In this paper, we have discussed the prospects for experimentally observing Efimov physics in two-component ultracold quantum gases by measuring the inelastic loss rates near a Feshbach resonance. We have shown that boson-fermion mixtures with heavy bosons and light fermions are probably most likely to exhibit the signatures of Efimov physics. Both the range of a and the maximum allowed temperature are experimentally accessible, allowing a sufficient number of Efimov features to be observed to establish their equal spacing in $\ln(a)$. These results suggest that a first experimental demonstration of the Efimov effect is, in fact, currently possible.

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