Schemes for realizing frequency up- and down-conversions in two-mode cavity QED

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We propose experimental schemes for realizing frequency up- and down-conversion in two-mode cavity QED by considering the atom-cavity interaction in the presence of a strong driving classical field. In contrast to the recent paper based on dispersive atom-cavity interaction [Serra *et al.*, Phys. Rev. A **71**, 045802 (2005)], our scheme is based on resonant interaction of the cavity modes with a single driven three-level atom, so that the quantum dynamics operates at a high speed, which is important in view of decoherence. It is shown that, with the help of a strong driving classical field, frequency up- and down-conversion operations can be realized by initially preparing the atom in a certain state.

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Quantum entanglement is one of the most striking features of quantum mechanics [1–3]. The recent surge of interest and progress in quantum-information theory allows one to take a more positive view of entanglement and regard it as an essential resource for many ingenious applications such as quantum cryptography [4], quantum dense coding [5], and quantum teleportation [6]. These researches motivate an intensive interest in generation and manipulation of quantum entanglement.

Cavity QED, with Rydberg atoms crossing superconducting cavities, offers an almost ideal system for the generation of entangled states and implementation of small-scale quantum-information processing [7]. In the context of cavity QED, numerous theoretical schemes for generating entangled states of many atoms and nonclassical states of cavity fields have been proposed [8], which led to experimental realization of the Einstein-Podolsky-Rosen state [9] of two atoms, the Greenberger-Horne-Zeilinger state [10] of three parties (two atoms plus one cavity mode), and the macroscopic superposition (Schrödinger cat) state [11] and Fock state [12] of a single-mode cavity field. Most of the schemes are based on the interaction of atoms and single-mode cavity fields. An experiment has been reported for preparing two modes of a superconducting cavity in a maximally entangled state by using a sequence of interactions of an atom with two cavity modes [13]. This experiment opens up the possibility for quantum-state engineering and quantum-information processing using multiple modes in a superconducting cavity. In Ref. [14], Solano *et al.* proposed a scheme to generate a two-mode entangled coherent state in a cavity. In Ref. [15], a scheme was proposed for creating quantum entanglement between multiatom Dicke states and two cavity modes.

In this Brief Report, we address the issue of how to engineer two classes of effective interactions between two cavity modes of a cavity. One class is the frequency up-conversion of the form [16]

$$H_{up} = ga^{\dagger}b + g^{*}ab^{\dagger} \tag{1}$$

and the other class is the frequency down-conversion of the form [16]

$$H_{down} = ga^{\dagger}b^{\dagger} + g^{*}ab \tag{2}$$

where $a^{\dagger}(a)$ and $b^{\dagger}(b)$ are creation (annihilation) operators of the cavity modes a and b. g denotes the effective coupling constant between two cavity modes. The interaction (1) is associated with a beam splitter in quantum optics, which generates an active rotation of two cavity modes a and b. The interaction (2) is associated with a parametric amplifier, which can be directly used to generate a two-mode squeezed state. In a recent paper [17], Serra *et al.* proposed an experimental scheme for realizing the effective interactions (1) and (2) by employing the dispersive interaction of the cavity modes with a single driven three-level atom. In contrast to the scheme [17], our scheme is based on resonant interaction of the cavity modes with a single driven three-level atom, so that the quantum dynamics operates at a high speed, which is important in view of decoherence.

We first consider how to engineer interaction (1). Here, we consider the physical model proposed in Ref. [17], which consists of a Λ-configuration three-level atom interacting with two cavity modes and driven additionally by one external classical field. The three-level atomic states are labeled by $|1\rangle$, $|2\rangle$, and $|3\rangle$, with the energies ω_1, ω_2, and ω_3. As shown in Fig. 1, the transition $|1\rangle \Leftrightarrow |3\rangle$ is coupled to cavity mode a with frequency ω_a, and the transition $|2\rangle \Leftrightarrow |3\rangle$ is coupled to cavity mode b with frequency ω_b. A classical field is used to drive the dipole-forbidden atomic transition $|1\rangle \Leftrightarrow |2\rangle$ with the frequency ω_L. In Ref. [17], to obtain the effective interaction (1), the authors consider the dispersive atom-cavity interaction. Here, we consider the different case that the cavity modes and classical fields are resonant with the corresponding atomic transitions. Under the rotating-wave approximation, the Hamiltonian of the system in the interaction picture is

$$H = H_{cav} + H_{cla}, \tag{3}$$

$$H_{cav} = \Omega_a(a^{\dagger}\sigma_{1,3} + a\sigma_{3,1}) + \Omega_b(b^{\dagger}\sigma_{2,3} + b\sigma_{3,2}), \tag{4}$$

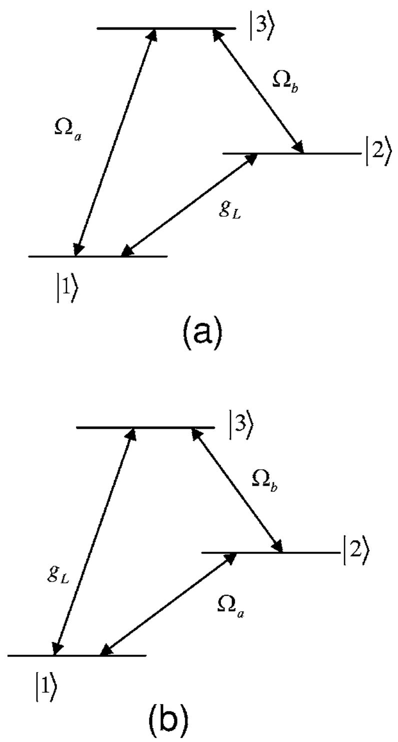

FIG. 1. (a) Atomic level structure of three-level Λ-configuration atoms to obtain effective interaction (1). The atoms have two ground states $|1\rangle$ and $|2\rangle$ and one excited state $|3\rangle$. (b) Atomic level structure of three-level ladder-configuration atoms to obtain effective interaction (2).

$$H_{cla} = g_L(\sigma_{1,2}e^{i\varphi_L} + \sigma_{2,1}e^{-i\varphi_L}), \tag{5}$$

where $\sigma_{i,j}=|i\rangle\langle j|$, Ω_a and Ω_b are the interaction strengths of the atom with the modes a and b, and g_L and φ_L are the amplitude and phase of the classical driving field. In this Brief Report, we consider the case of the strong classical field, i.e., $g_L \gg \Omega_a$, Ω_b.

In general the Hamiltonian $H_{cav}+H_{cla}$ of the system is difficult to treat in an exact way because of the presence of the classical driving term H_{cla}. In order to gain physical insight into the dynamics of such physical system, some approximations are necessary. To demonstrate how the system's dynamics is modified by the strong classical field, we introduce the atomic dressed basis

$$|+\rangle = \frac{1}{\sqrt{2}}(e^{i\varphi_L/2}|1\rangle + e^{-i\varphi_L/2}|2\rangle),$$

$$|-\rangle = \frac{1}{\sqrt{2}}(e^{i\varphi_L/2}|1\rangle - e^{-i\varphi_L/2}|2\rangle). \tag{6}$$

Using this dressed basis, the interaction terms H_{cav} and H_{cla} of the Hamiltonian (3) can be rewritten as follows:

$$H_{cla} = g_L(|+\rangle\langle+| - |-\rangle\langle-|), \tag{7}$$

$$H_{cav} = \frac{\Omega_a}{\sqrt{2}}e^{-i\varphi_L/2}a^\dagger(|+\rangle + |-\rangle)\langle 3| + \frac{\Omega_b}{\sqrt{2}}e^{i\varphi_L/2}b^\dagger(|+\rangle - |-\rangle)\langle 3| + \frac{\Omega_a}{\sqrt{2}}e^{i\varphi_L/2}a|3\rangle(\langle+ \langle-|) + \frac{\Omega_b}{\sqrt{2}}e^{-i\varphi_L/2}b|3\rangle(\langle+| - \langle-|). \tag{8}$$

In order to simplify the dynamics of system, we switch to the interaction picture respect to the H_{cla}, the Hamiltonian $H_{cav}+H_{cla}$ of the system becomes

$$\begin{aligned} H' = {} & \frac{\Omega_a}{\sqrt{2}}e^{-i\varphi_L/2}a^\dagger(e^{igt}|+\rangle + e^{-igt}|-\rangle)\langle 3| \\ & + \frac{\Omega_b}{\sqrt{2}}e^{i\varphi_L/2}b^\dagger(e^{igt}|+\rangle - e^{-igt}|-\rangle)\langle 3| \\ & + \frac{\Omega_a}{\sqrt{2}}e^{i\varphi_L/2}a|3\rangle(e^{-igt}\langle+| + e^{igt}\langle-|) \\ & + \frac{\Omega_b}{\sqrt{2}}e^{-i\varphi_L/2}b|3\rangle(e^{-igt}\langle+| - e^{igt}\langle-|). \end{aligned} \tag{9}$$

In the strong laser regime $g_L \gg \Omega_a, \Omega_b$, it is convenient to consider the interaction (9) in terms of a coarse-grained Hamiltonian which neglects the effect of rapidly oscillating terms. Using the time-averaging method of Ref. [18], one can arrive at the effective Hamiltonian

$$\begin{aligned} H'' = {} & \frac{1}{2g_L}[(|+\rangle\langle+| - |-\rangle\langle-|)(\Omega_a^2 a^\dagger a + \Omega_b^2 b^\dagger b) \\ & + \Omega_a\Omega_b(|+\rangle\langle+| + |-\rangle\langle-|)(a^\dagger b e^{-i\varphi_L} + ab^\dagger e^{-i\varphi_L}) \\ & - 2\Omega_a\Omega_b|3\rangle\langle 3|(a^\dagger b e^{-i\varphi_L} + ab^\dagger e^{-i\varphi_L})]. \end{aligned} \tag{10}$$

Thus, if we prepare the initial state of the atom in level $|3\rangle$, the dynamics generated by Eq. (10) acting on this state factors out and leaves the atomic state unchanged. This allows us to reduce the dynamics to that of the cavity fields only and we obtain the effective interaction

$$H_{eff} = \Omega_{eff}a^\dagger b + \Omega_{eff}^* ab^\dagger \tag{11}$$

where $\Omega_{eff}=-(\Omega_a\Omega_b/g_L)e^{-i\varphi_L}$ is the effective coupling constant. Equation (11) is the expected frequency up-conversion process, which generates an active rotation of the two cavity modes.

In the following, we consider how to engineer interaction (2) by considering the physical mode of the ladder-configuration three-level atom interacting with two cavity modes and driven additionally by one external classical field [17]. As shown in Fig. 1(b), the transition $|1\rangle \Leftrightarrow |2\rangle$ is coupled to cavity mode a with frequency ω_a, and the transition $|2\rangle \Leftrightarrow |3\rangle$ is coupled to cavity mode b with frequency ω_b. A classical field is used to drive the dipole-forbidden atomic transition $|1\rangle \Leftrightarrow |3\rangle$ with the frequency ω_L and amplitude g_L. We assume that the cavity modes and classical fields are resonant with the corresponding atomic transitions. Under the rotating-wave approximation, the Hamiltonian of the system in the interaction picture is

$$H = H_{cla} + H_{cav}, \tag{12}$$

$$H_{cla} = g_L(\sigma_{1,3}e^{i\varphi_L} + \sigma_{3,1}e^{-i\varphi_L}), \tag{13}$$

$$H_{cav} = \Omega_a(a^\dagger\sigma_{1,2} + a\sigma_{2,1}) + \Omega_b(b^\dagger\sigma_{2,3} + b\sigma_{3,2}). \tag{14}$$

In order to gain physical insight into the dynamics of this physical system, we introduce the atomic dressed basis

$$|+\rangle = \frac{1}{\sqrt{2}}(e^{i\varphi_L/2}|1\rangle + e^{-i\varphi_L/2}|3\rangle),$$

$$|-\rangle = \frac{1}{\sqrt{2}}(e^{i\varphi_L/2}|1\rangle - e^{-i\varphi_L/2}|3\rangle). \tag{15}$$

Using this dressed basis, and following the steps leading from Eq. (7) to Eq. (10), we can obtain the effective interaction

$$\begin{aligned} H' = \frac{1}{2g_L}[&(|+\rangle\langle+| - |-\rangle\langle-|)(\Omega_a^2 a^\dagger a + \Omega_b^2 bb^\dagger) \\ &+ \Omega_a\Omega_b(|+\rangle\langle+| + |-\rangle\langle-|)(a^\dagger b e^{-i\varphi_L} + ab^\dagger e^{-i\varphi_L}) \\ &- 2\Omega_a\Omega_b|2\rangle\langle2|(a^\dagger b^\dagger e^{-i\varphi_L} + abe^{i\varphi_L})]. \end{aligned} \tag{16}$$

Thus, if we prepare the initial state of the atom in the level $|2\rangle$, the dynamics generated by Eq. (16) acting on this state factors out and leaves the atomic state unchanged. This allows us to reduce the dynamics to that of the cavity fields only and we obtain the effective interaction

$$H_{eff} = \Omega_{eff}a^\dagger b^\dagger + \Omega_{eff}^* ab \tag{17}$$

where $\Omega_{eff} = -(\Omega_a\Omega_b/g_L)e^{-i\varphi_L}$ is the effective coupling constant. Equation (17) is the expected frequency down-conversion process. The associated time evolution operator is

$$U_{eff}(t) = \exp[-i\Omega_{eff}t(a^\dagger b^\dagger + \Omega_{eff}^* ab)] \tag{18}$$

which is a two-mode squeezed operator that can produce two-mode squeezing on any initial field state. For example, if two cavity modes are initially prepared in vacuum states, a two-mode squeezed vacuum state will be generated. As a by-product of the present scheme, if we consider mode a identical to mode b in Eqs. (12) and (13), we can generate a degenerate parametric down-conversion corresponding to the interaction

$$H_{eff} = \Omega_{eff}a^{\dagger 2} + \Omega_{eff}^* a^2 \tag{19}$$

which can be used to generate a squeezed state of cavity fields.

We give a brief discussion of the experimental feasibility of the proposed scheme within microwave cavity QED. The scheme presented here requires (1) resonant interaction between atom and cavity modes, (2) negligible cavity loss during the whole preparation process, (3) no atomic spontaneous decay during the atom-cavity interaction, and (4) detection of atoms in given states. In microwave cavity QED, we can consider Rb atoms with higher Rydberg atomic states as Λ-type or ladder-type atoms, which have lifetimes of the order of 0.01 s [7,17]. In that case, we can choose an appropriate single-mode or bimodal microwave cavity to couple atomic transitions [17]. A practical superconducting cavity has the typical value of Q about 10^9 if the cavity temperature is low enough, i.e., the cavity lifetimes for a high-Q superconducting cavity can be as long as 0.01 s, which is three orders of magnitude longer than typical atom-cavity interaction times [7]. The detection process of an atom in the desired state can be implemented by passing the atom through the classical microwave field zone and field ionization counters. The interaction time between atom and cavity can be controlled by using a velocity selector and applying Stark field adjustment in order to make the atom resonant with the field for the right amount of time. In comparison with Ref. [17], we consider the same atom-cavity interactions, i.e., $\Omega_a = \Omega_b = 7\times10^5\ s^{-1}$ [17]. In order to get a good approximation in Eqs. (10) and (16), the amplitude g_L should be much bigger than Ω_a and Ω_b. With the choice $g_L = 10\Omega_a$, the effective interaction is about $7\times10^4\ s^{-1}$, which is bigger than that obtained in Ref. [17], i.e., the present scheme provides a more efficient scheme for engineering effective interaction of two cavity modes.

In summary, we have proposed schemes to engineer frequency up- and down-conversion in two-mode cavity QED. Our schemes are based on the resonant atom-cavity interactions so that the quantum dynamics operates at a high speed, which is important in view of decoherence.

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