Atom localization via interference of dark resonances

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A scheme of atom localization based on the interference of resonance of double-dark states is proposed, in which the atom interacts with a classical standing-wave field. It is found that the localization property is significantly improved due to the interaction of double-dark resonances. It is realized that the atom is localized just at the nodes of the standing-wave field with higher precision. Moreover, an improvement by a factor of 2 in the detecting probability of a single atom within the subwavelength domain can be achieved by adjusting the probe-field detuning. This scheme shows more advantages than other schemes of atom localization.

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The subwavelength localization of an atom has attracted extensive attention [1–12]. Because the strength of the interaction in a standing-wave field is position dependent, the dynamics of atomic systems are different at different positions. Thus the measurement of position-dependent quantities can provide information on the atomic position and lead to atom localization. Some schemes of atom localization, based on the fact that the strength of the atom-field interaction depends on the position of the atom in the field, have been proposed using the phase shift of either the standing wave or the atomic dipole [1–3], the entanglement between the atom's position and its internal state [4], or other methods [5,6]. Subsequently Zubairy and co-workers [7–9] improved these schemes and proposed two simple localization schemes using either measurement of the Autler-Townes spontaneous spectrum in a three-level system [7,8] or the resonant fluorescence from a standing-wave field in a two-level system [9]. Recently, Ghafoor *et al.* and Liu *et al.* realized the phase control of the atom localization and reduced the uncertainty in a particular position measurement of the single atom by a factor of 2 in a four-level system [10] and in a loop three-level system [11], respectively. Paspalakis and Knight [12] proposed a related method for localizing a three-level Λ-type atom in a standing-wave field. Their scheme is based on measurement of the upper-state population by standard spectroscopic methods [13,14] and simplifies the demands on the initial-state preparation of an atom from the schemes of Zubairy and co-workers [7–9].

On the other hand, the phenomenon of "dark resonances" or coherent population trapping is a well-known concept in optics and laser spectroscopy [15]. Recently Lukin *et al.* [16] have shown that in a generic four-level system coherent perturbation leads to a splitting of dark states and pointed out that the "double-dark resonance" structure as a whole is the definite signature of a new type of quantum interference effect. In this article, on the basis of the atom localization's scheme given in Ref. [12], we want to use this quantum interference effect and to explore a more efficient scheme of atom localization. It is found that, due to the interference of double-dark resonances, the property of atom localization can be significantly improved.

The atomic system under consideration is shown in Fig. 1. Here $|a\rangle$ is an upper excited state and $|b\rangle$, $|c\rangle$, and $|d\rangle$ are three lower metastable states. The transition $|a\rangle \leftrightarrow |c\rangle$ is taken to be nearly resonant with a driving field with Rabi frequency $\Omega=\Omega_0 \sin(kx)$, where $k=2\pi/\lambda$ and λ is the wavelength. Here we consider that the driving field is a classical standing-wave field aligned along the x direction. In addition, a weak probe field with Rabi frequency ε couples the states $|a\rangle$ and $|b\rangle$. Therefore, they form a conventional and simple Λ configuration. An additional coherent perturbation field with Rabi frequency Ω_c couples the state $|c\rangle$ to another state $|d\rangle$, and this leads to the occurrence of double-dark resonances [16]. There are many different mechanisms to realize this coherent perturbation coupling: for example, using a microwave field driving a magnetic dipole transition, using optical fields inducing multiple two-photon transitions, by a static field, or by a nonadiabatic coupling mechanism in time-dependent laser fields [17].

In the interaction picture with the rotating-wave approximation, the Hamiltonian of the system can be written as

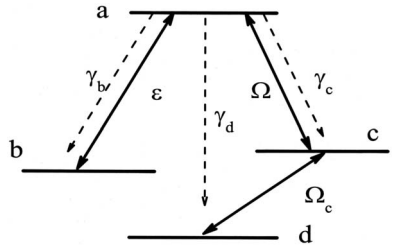

FIG. 1. Four-state atomic system displaying double-dark resonances. γ_b, γ_c, and γ_d represent the spontaneous decay rates from the upper excited level $|a\rangle$ to the three metastable states $|b\rangle$, $|c\rangle$, and $|d\rangle$. One driving field Ω, a weak probe field ε, and an additional coherent perturbation field Ω_c couple their corresponding transitions, respectively.

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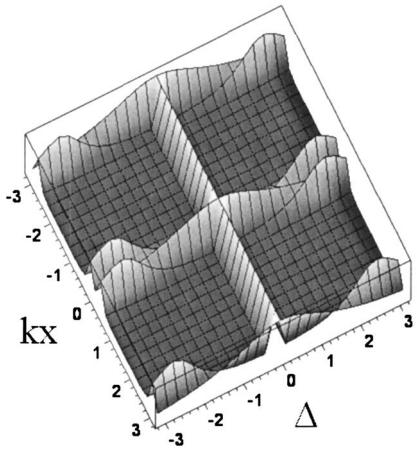

FIG. 2. The conditional position probability distribution $W(x)$ (in arbitrary units) as a function of kx and Δ. Other parameters are $\Delta_0=0.0$, $\Delta_c=0.0$, $\Omega_0=10\gamma$, and $\Omega_c=0.2\gamma$.

$$H=-\hbar\begin{pmatrix}\Delta_0 & \Omega & 0 & \varepsilon\\ \Omega & 0 & \Omega_c & 0\\ 0 & \Omega_c & -\Delta_c & 0\\ \varepsilon & 0 & 0 & \Delta_0-\Delta\end{pmatrix}. \tag{1}$$

We can easily get the density matrix equations of the system as follows:

$$(d/dt)\rho_{aa}=-(\gamma_b+\gamma_c+\gamma_d)\rho_{aa}+i\varepsilon(\rho_{ba}-\rho_{ab})+i\Omega(\rho_{ca}-\rho_{ac}),$$

$$(d/dt)\rho_{ab}=-\Gamma_{ab}\rho_{ab}+i\varepsilon(\rho_{bb}-\rho_{aa})+i\Omega\rho_{cb},$$

$$(d/dt)\rho_{ac}=-\Gamma_{ac}\rho_{ac}+i\varepsilon\rho_{bc}-i\Omega_c\rho_{ad}+i\Omega(\rho_{cc}-\rho_{aa}),$$

$$(d/dt)\rho_{ad}=-\Gamma_{ad}\rho_{ad}+i\varepsilon\rho_{bd}+i\Omega\rho_{cd}-i\Omega_c\rho_{ac},$$

$$(d/dt)\rho_{cd}=-\Gamma_{cd}\rho_{cd}+i\Omega_c(\rho_{dd}-\rho_{cc})+i\Omega\rho_{ad},$$

$$(d/dt)\rho_{cc}=\gamma_c\rho_{aa}+i\Omega_c(\rho_{dc}-\rho_{cd})+i\Omega(\rho_{ac}-\rho_{ca}),$$

$$(d/dt)\rho_{db}=-\Gamma_{db}\rho_{db}-i\varepsilon\rho_{da}+i\Omega_c\rho_{cb},$$

$$(d/dt)\rho_{dd}=\gamma_d\rho_{aa}+i\Omega_c(\rho_{cd}-\rho_{dc}),$$

$$(d/dt)\rho_{bb}=\gamma_b\rho_{aa}+i\varepsilon(\rho_{ab}-\rho_{ba}),$$

$$(d/dt)\rho_{bc}=-\Gamma_{bc}\rho_{bc}+i\varepsilon\rho_{ac}-i\Omega\rho_{ba}-i\Omega_c\rho_{bd}, \tag{2}$$

with $\rho_{ij}=\rho_{ji}^{*}$ and the closure relation $\Sigma_i\rho_{ii}=1$ $(i,j=\{a,b,c,d\})$. Here γ_b, γ_c, and γ_d represent the spontaneous decay rates from the state $|a\rangle$ to states $|b\rangle$, $|c\rangle$, and $|d\rangle$, respectively. Here $\Gamma_{ab}=\gamma_{ab}-i\Delta$, $\Gamma_{ac}=\gamma_{ac}-i\Delta_0$, $\Gamma_{ad}=\gamma_{ad}-i(\Delta_0+\Delta_c)$, $\Gamma_{cd}=\gamma_{cd}-i\Delta_c$, $\Gamma_{db}=\gamma_{db}-i(\Delta-\Delta_c-\Delta_0)$, $\Gamma_{bc}=\gamma_{bc}-i(\Delta_0-\Delta)$, and $\Gamma_{ij}=\Gamma_{ji}^{*}$ and γ_{ij} $(i\neq j)$ are the relaxation rates of the respective coherences. In the nonradiative limit, $\gamma_{ab}=\gamma_{ac}=\gamma_{ad}=(\gamma_b+\gamma_c+\gamma_d)/2$ and $\gamma_{cd}=\gamma_{db}=\gamma_{bc}=0$ and $\Delta=\nu_p-\omega_{ab}$ $(\Delta_0=\nu-\omega_{ac}$, $\Delta_c=\nu_c-\omega_{cd})$ is the detuning of the probe (driving, coherent perturbation) field with frequency $\nu_p(\nu,\nu_c)$ and $\omega_{ij}(i\neq j)$ is the atomic transition frequency between levels $|i\rangle$ and $|j\rangle$.

Assuming $\varepsilon\ll\Omega,\Omega_c$ and $\rho_{bb}^{(0)}\approx 1$ ($\rho_{bb}^{(0)}$ is the initial population), then in the long-time limit, the conditional position probability distribution [7]—i.e., the probability of finding the atom in the internal excited state $|a\rangle$ and at position x in the standing-wave field [12]—is given by

$$F(x,t\to\infty)=|N|^2|f(x)|^2\varepsilon^2A^2/[(B-\Delta A)^2+\gamma_{ab}^2A^2]. \tag{3}$$

Here $A=\Omega_c^2-(\Delta_0-\Delta)(\Delta_0+\Delta_c-\Delta)$, $B=\Omega^2(\Delta_0+\Delta_c-\Delta)$, N is a normalization factor, and $f(x)$ is the center-of-mass wave function of the atom. As $f(x)$ is assumed to be nearly constant over many wavelengths of the standing-wave field, the conditional position probability distribution is determined by the filter function [7–12]

$$W(x)=\varepsilon^2A^2/[(B-\Delta A)^2+\gamma_{ab}^2A^2]. \tag{4}$$

When $\Omega_c=0$ and $\Delta_c=0$—i.e., the coherent perturbation field is absent—from Eq. (4) we obtain

$$W(x)=\varepsilon^2(\Delta_0-\Delta)^2/\{[(\Delta_0-\Delta)\Delta-\Omega^2]^2+\gamma_{ab}^2(\Delta_0-\Delta)^2\}. \tag{5}$$

The filter function of Eq. (5) has the same form as that derived in Ref. [12], and this is easy to understand based on the fact that the system becomes a conventional Λ-type atomic system when the coherent perturbation field is absent.

It is worthwhile to point out that for the case of $\Delta_0+\Delta_c-\Delta=0$, i.e., satisfying the three-photon resonance,

$$W(x)=\varepsilon^2/\gamma_{ab}^2. \tag{6}$$

The filter function is a constant for fixed ε and γ_{ab}, and not dependent on the spatial position x; this means that atom cannot be localized. Therefore, $\Delta_0+\Delta_c-\Delta\neq 0$ is a fundamental condition for realizing atom localization in our scheme.

For the convenience of discussion and without lost of generality, we have $\gamma_b=\gamma_c=\gamma_d=\gamma=1.0$. Equation (4) shows that the conditional position probability distribution depends not only on three controllable detunings Δ, Δ_0, and Δ_c, but also on Rabi frequencies of the driving and coherent perturbation fields, Ω and Ω_c.

First, we investigate the effect of the probe detuning on the atom localization. Considering the driving and coherent perturbation fields on resonance with the corresponding transitions—i.e., $\Delta_0=0$ and $\Delta_c=0$—and setting $\Omega_0=10\gamma$ and $\Omega_c=0.2\gamma$, we present a three-dimensional (3D) demonstration of the conditional position probability distribution $W(x)$ as a function of kx and the probe-field detuning Δ, shown in Fig. 2. Because of the periodicity of the standing-wave field, $\Omega=\Omega_0\sin(kx)$, here we only study the variance of $W(x)$ in the subwavelength domain, $-\pi\leqslant kx\leqslant\pi$. Figure 2 shows that the probe-field detuning has a significant effect on the atom localization. When Δ is zero, the conditional probability distribution is space invariant; therefore, there does not exist any atom localization. This has been clarified for the case of three-photon resonance in the above. When the probe field has a little deviation from resonance, $\Delta\neq 0$, some distribution peaks corresponding to different positions occur. Therefore, atom localization is realized. Moreover, it is noteworthy

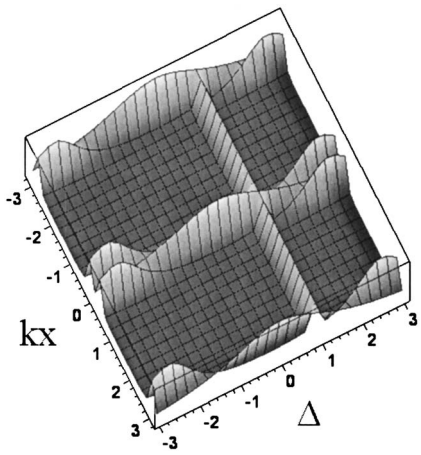

FIG. 3. The conditional position probability distribution $W(x)$ (in arbitrary units) as a function of kx and Δ but with $\Delta_c=1.0\gamma$ and other parameters are the same as in Fig. 2.

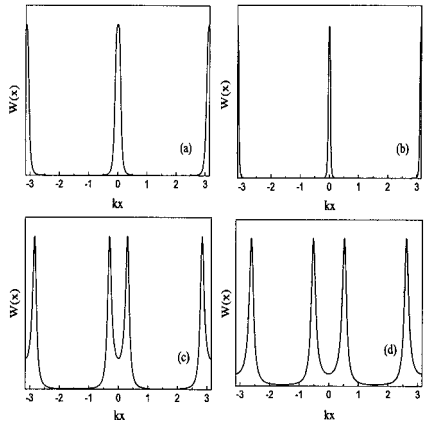

FIG. 4. The conditional position probability distribution $W(x)$ (in arbitrary units) as a function of kx for four different probe-field detunings, (a) $\Delta=0.05\gamma$, (b) $\Delta=0.15\gamma$, (c) $\Delta=3.0\gamma$, and (d) $\Delta=5.0\gamma$. All other parameters are the same as in Fig. 2.

that these position probability peaks just lie at the nodes of the standing-wave field; that is, the atom is localized just at the nodes of the standing-wave field. This can significantly improve the measurement precision for this atom. At the same time, on average, only two sharp peaks occur in the subwavelength domain and the numbers of the distribution peaks decrease obviously compared with that in a conventional scheme of atom localization [7–9,12]. In the conventional scheme, the periodicity of the standing-wave field yields four equally probable different positions of the atom in the subwavelength domain of the optical field when a spontaneously emitted photon is detected, and for a single required frequency measurement, the probability of finding the atom at a particular position is 1/4. While in our present scheme the existence of the two equally probable sharp peaks means that the detecting probability of this atom in the subwavelength domain increases to 1/2. With a further increment of Δ, there gradually occur four sharp peaks of the atom localization in the subwavelength domain. This means that the atomic detecting probability is again reduced to 1/4. We can say that adjusting the probe-field detuning can realize the quantum control of atom localization and reduce the uncertainty in measuring a particular position of the single atom by a factor of 2. Recently, such an improvement of the detecting probability has also realized in a four-level atomic system [10] and a loop three-level system [11], respectively. The method used in [10,11] is to realize atom localization via phase control, which is based on the fact that the property of a loop system is sensitive to the relative phase of the coupled fields, while in our present scheme the improvement is based on the interference of double-dark resonances.

Why does this phenomenon occur? It is not difficult to get an explanation from our four-level atomic system model: Without the additional coherent perturbation field, this model is just a conventional Λ-type configuration which has only one dark state related to electromagnetically induced transparency (EIT) [18], while when the additional perturbation field is added, there exist two dark states, and their mutual interference can induce a sharp light absorption [16]. In the case around the three-photon resonance, the two dark-state resonances play nearly equal roles and their interference induces the occurrence of three sharp peaks of atom localization in the subwavelength domain. When the probe-field detuning is large, the role of one of the two dark states is weakened while the other is built up. In the limit of $|\Delta|\to\infty$, only one dark state is dominant and this is similar to that in a Λ-type system [12]. Therefore, there exist four sharp peaks of atom localization in the subwavelength domain.

Figure 2 also shows that, when both the coherent perturbation and driving fields are resonant, the spatial variance of atom localization is even symmetric about the zero-probe-field detuning, $\Delta=0$. When one of the two fields is nonresonant, this symmetry will be destroyed. As an example, Fig. 3 illustrates the case that the coherent perturbation field is nonresonant. This can be explained from the viewpoint of the loss of symmetry in double-dark states [16].

In order to see more clearly the effect of probe-field detuning on atom localization, in Fig. 4 we present a 2D demonstration of the conditional position probability distribution $W(x)$ as a function of kx for four different probe-field detunings. Figure 4 shows that probe-field detuning has a significant effect not only on the numbers of atom localization peaks, but also on the degree of atom localization. When Δ is small [Figs. 4(a) and 4(b)], there exist three peaks in the subwavelength domain. The peaks in Fig. 4(a) are more pronounced than those in Fig. 4(b). This indicates that a suitable increment in the probe-field detuning can improve the degree of atom localization, which is different from that in the scheme proposed by Paspalakis and Knight [12] where the smaller the probe detuning is, the more pronounced the localization peak is. When the probe-field detuning is large [Figs. 4(c) and 4(d)], the degree of atom localization becomes worse and the signal-to-noise ratio is larger than those in Figs. 4(a) and 4(b).

Finally, in Fig. 5 we also present a 3D demonstration of the conditional position probability distribution $W(x)$ as a function of kx and one of other parameters Δ_0, Ω_0, Δ_c, and Ω_c, respectively [Figs. 5(a)–5(d)]. These figures show that the degree of atom localization depends crucially on these

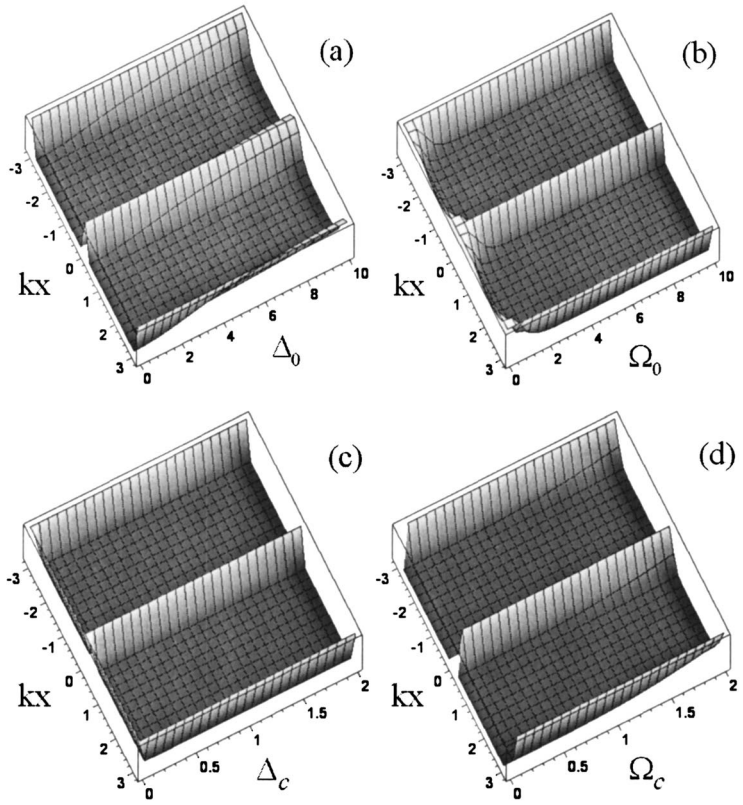

FIG. 5. The conditional position probability distribution $W(x)$ (in arbitrary units) as a function of kx and (a) Δ_0, (b) Ω_0, (c) Δ_c, and (d) Ω_c, respectively. All other parameters are the same as in Fig. 3.

parameters. For a much better degree of atom localization, an optimal combination of all related parameters is needed.

In summary, a scheme of atom localization based on the interference of double-dark resonances was proposed. Adjusting the probe-field detuning not only can make an atom localized at the nodes of the standing-wave field with high precision, but also can increase the detecting probability of an atom at a particular position by a factor of 2. Our scheme is related to that proposed by Paspalakis and Knight [12] but based on the interference of double-dark resonances, which has shown more advantages than other schemes of atom localization.

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