Light-induced effective magnetic fields for ultracold atoms in planar geometries

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We propose a scheme to create an effective magnetic field for ultracold atoms in a planar geometry. The setup allows the experimental study of classical and quantum Hall effects in close analogy to solid-state systems including the possibility of finite currents. The present scheme is an extention of the proposal in Phys. Rev. Lett. **93**, 033602 (2004), where the effective magnetic field is now induced for three-level Λ-type atoms by two counterpropagating laser beams with shifted spatial profiles. Under conditions of electromagnetically induced transparency the atom-light interaction has a space-dependent dark state, and the adiabatic center-of-mass motion of atoms in this state experiences effective vector and scalar potentials. The associated magnetic field is oriented perpendicular to the propagation direction of the laser beams. The field strength achievable is one flux quantum over an area given by the transverse beam separation and the laser wavelength. For a sufficiently dilute gas the field is strong enough to reach the lowest Landau level regime.

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One of the most fascinating subjects at the interface between ultracold atoms and solid-state systems is the possibility to experimentally study strong-correlation phenomena with the precision and the large degree of variability provided by atomic physics. For example, interacting Bose-Einstein condensates (BEC) or degenerate Fermi gases in rotating two-dimensional traps are studied in several laboratories with the goal to observe quantum-Hall-like effects [1–3]. The trap rotation provides an effective magnetic field for the electrically neutral atoms. However, in order to reach the fractional quantum-Hall regime it is necessary to rotate the trap close to the critical frequency. Furthermore, the atom density needs to be low enough such that the number of magnetic flux quanta approaches the number of atoms, which is an experimental challenge. Besides experimental difficulties this approach has some conceptual drawbacks: It is limited to rotational symmetric setups and does not allow to study transport phenomena, i.e., the effect of magnetic fields to a finite particle current.

In [4–6] we have suggested an alternative method based on light-induced gauge potentials for atoms with a space-dependent dark state. A dark state is created if three-level Λ-type atoms interact with two laser fields under conditions of electromagnetically induced transparency (EIT) [7–11]. If the dark state is space dependent, a vector gauge potential arises for the adiabatic center-of-mass motion [12]. As shown in [4,5] the vector potential is associated with a nonvanishing magnetic field, if at least one of the two light beams has a vortex, i.e., an orbital angular momentum (OAM). Yet the use of vortex light beams has similar drawbacks as the trap rotation regarding the spatial symmetry and transport phenomena.

We here propose a variation of this scheme which is free of the above-mentioned limitations. The scheme, shown in Fig. 1, once again involves two laser beams interacting with three-level atoms in the EIT configuration. Yet we are no longer dealing with light beams posessing an OAM with respect to their propagation axis. As we will show later on a nonvanishing magnetic field requires only a *relative* OAM between the two light beams. This can be achieved by two counter-propagating and overlapping laser beams with shifted spatial profiles. In this case an effective magnetic field appears perpendicular to the propagation direction and to the gradient of the relative intensity of the light beams. This configuration allows a planar geometry and a nonvanishing flow of atoms, e.g., an atomic BEC moving along an atomic waveguide [13].

Let us consider an ensemble of cold three-level atoms with lower levels $|1\rangle$ and $|2\rangle$ and electronically excited state $|3\rangle$. The atoms interact with two resonant laser beams in the EIT configuration, see Fig. 1. The first beam (to be referred to as the control beam) has a frequency ω_c, a wave-vector $\mathbf{k}_c$, and induces the atomic transitions $|2\rangle \to |3\rangle$ with Rabi frequency $\Omega_c \equiv \mu_{32}E_c/2$, where E_c is the electric field strength and μ_{32} is the transition dipole moment. The second (probe) beam with frequency ω_p, wave-vector $\mathbf{k}_p$ causes the transition $|1\rangle \to |3\rangle$ with a Rabi frequency $\Omega_p \equiv \mu_{31}E_p/2$. The two laser beams keep the atoms in their dark state [7–11]:

$$|D\rangle = |1\rangle\cos\theta - |2\rangle\sin\theta\exp(iS) \sim |1\rangle - \zeta|2\rangle, \tag{1}$$

where $\zeta = \Omega_p/\Omega_c = |\zeta|e^{iS} \equiv \tan\theta\, e^{iS}$ is the ratio between the Rabi frequencies of the probe and control fields, S is their relative phase, and θ is the mixing angle between the states $|1\rangle$ and $|2\rangle$ in the atomic dark state $|D\rangle$.

The dark state depends on the atomic position through the $\mathbf{r}$ dependence of the Rabi frequencies $\Omega_p(\mathbf{r})$ and $\Omega_c(\mathbf{r})$, so an effective vector potential (generally known as the Berry connection [14,15]) appears in the adiabatic equation of motion for the atomic center of mass. The effective vector and trapping potentials governing the translational motion of the dark-state atoms read [5,6]

$$\mathbf{A}_{\mathrm{eff}} = -\hbar\frac{|\zeta|^2}{1+|\zeta^2|}\nabla S = -\hbar\sin^2\theta\nabla S \tag{2}$$

and

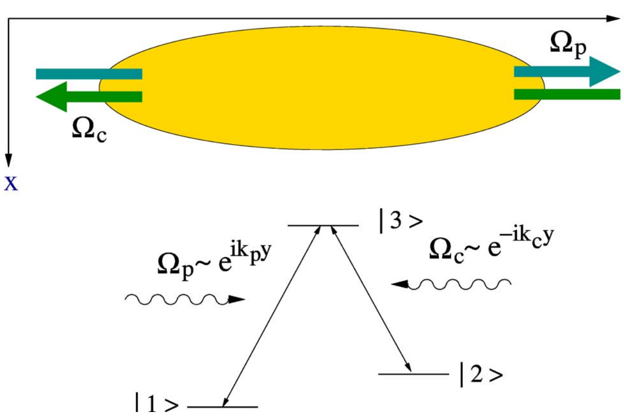

FIG. 1. (Color online) (top) Schematic representation of setup for light-induced effective magnetic fields: Two counterpropagating and overlapping laser beams interact with a cloud of cold atoms. (bottom) The level scheme for the Λ-type atoms interacting with the resonant probe and control beams characterized by Rabi frequencies Ω_p and Ω_c.

$$V_{\text{eff}}(\mathbf{r}) = V_{\text{ext}}(\mathbf{r}) + \frac{\hbar^2}{2m}\frac{|\zeta|^2(\boldsymbol{\nabla}S)^2 + (\boldsymbol{\nabla}|\zeta|)^2}{(1+|\zeta|^2)^2}, \tag{3}$$

where

$$V_{\text{ext}}(\mathbf{r}) = \frac{V_1(\mathbf{r}) + |\zeta|^2(V_2(\mathbf{r}) + \hbar\omega_{21})}{1+|\zeta|^2} \tag{4}$$

is the external potential for the dark-state atoms, $V_j(\mathbf{r})$ is the trapping potential for an atom in the internal state j, and $\omega_{21}=\omega_2-\omega_1+\omega_c-\omega_p$ is the frequency of the two-photon detuning.

One easily recognizes that the vector gauge potential $\mathbf{A}_{\text{eff}}$ yields a nonvanishing magnetic field only if the gradients of the relative intensity and the relative phase are both nonzero and not parallel to each other:

$$\mathbf{B}_{\text{eff}} \equiv \boldsymbol{\nabla} \times \mathbf{A}_{\text{eff}} = -\hbar\, \boldsymbol{\nabla}\,(\sin^2\theta) \times \boldsymbol{\nabla}S. \tag{5}$$

This equation has a very intuitive interpretation: $\boldsymbol{\nabla}(\sin^2\theta)$ is a vector that connects the "center of mass" of the two light beams; $\boldsymbol{\nabla}S$ is proportional to the vector of their relative momentum. Thus a nonvanishing $\mathbf{B}_{\text{eff}}$ requires a *relative orbital angular momentum* of the two light beams. As discussed in [4–6] this is the case, e.g., for light beams with a vortex.

Here we consider, however, a different scenario. We suggest to use two counter-propagating light beams of finite diameter with an axis offset: $\Omega_p=\Omega_p^{(0)}e^{ik_p y}$ and $\Omega_c=\Omega_c^{(0)}e^{-ik_c y}$, where $\Omega_p^{(0)}$ and $\Omega_c^{(0)}$ are real amplitudes with shifted transverse profiles. The beams possess a relative orbital angular momentum similarly to two point particles with constant momenta passing each other at some finite distance. In such a situation the phase of the ratio $\zeta=\Omega_p/\Omega_c$ is given by

$$S = ky, \quad k = k_p + k_c, \tag{6}$$

so that $\boldsymbol{\nabla}S=k\hat{\mathbf{e}}_y$ where $\hat{\mathbf{e}}_y$ is a unit Cartesian vector.

The spatial dependence of the intensity ratio $|\zeta|^2=|\Omega_p/\Omega_c|^2$ is determined by the spatial profiles of both $|\Omega_p|^2$ and $|\Omega_p|^2$. Since the control and probe beams counter propagate along the y axis, their intensities depend weakly on y. Furthermore we shall disregard the z dependence of the intensity ratio $|\zeta|^2$. This is legitimate, for instance, if the atomic motion is confined to the xy plane due to a steep trapping potential in the z direction. Hence one finds

$$\mathbf{B}_{\text{eff}} = \hat{\mathbf{e}}_z\hbar k\frac{\partial}{\partial x}\sin^2\theta. \tag{7}$$

The field strength B_{eff} depends generally on the x coordinate and has a weak y dependence as long as the paraxial approximation holds.

If we are interested in fractional quantum-Hall physics and thus in the possibility to enter the lowest Landau level (LLL) regime we have to estimate the maximum strength of the magnetic field. For this we determine the minimum area needed for a magnetic flux corresponding to a single flux quantum $2\pi\hbar$. From Eq. (7) we recognize that this area is given by the product λx_{eff}, where x_{eff} is the effective separation between the two beam centers. To reach the LLL in a two-dimensional gas the atomic density has thus to be smaller than one atom per λx_{eff}.

The above analysis holds as long as the atoms move sufficiently slow to remain in their dark states. This is the case if the adiabatic condition [5] holds: $\Omega \gg F$, where $F=|\nabla\zeta\cdot\mathbf{v}|/(1+|\zeta|^2)$ reflects the two-photon Doppler detuning. In the present situation we have

$$F^2 = \cos^2\theta\left[\left(v_x\frac{\partial}{\partial x}|\zeta|\right)^2 + (|\zeta|kv_y)^2\right] \ll \Omega^2. \tag{8}$$

where $\Omega=(|\Omega_c|^2+|\Omega_p|^2)^{1/2}$ is the rms Rabi frequency. The adiabatic condition implies that the quantity Ω^{-1} should be much less than the time an atom travels a characteristic length over which the amplitude or the phase of the ratio $\zeta=\Omega_p/\Omega_c$ changes considerably. For atoms moving along the y axis, such a length is $1/k\approx 1/2k_p\sim 10^{-7}$ m. On the other hand, the Rabi frequency can be of the order of 10^7 to 10^8 s^{-1} [16]. Therefore, the adiabatic condition should hold for atomic velocities up to meters per second.

The above estimation does not take into account a finite lifetime of the excited atoms, typically $\tau_3\sim 10^{-7}$ s. If this is included, the atomic dark state acquires a finite lifetime $\tau_D\sim\tau_3\Omega^2/F^2$ due to nonadiabatic coupling [5]: For instance, if the atomic velocities are of the order of a centimeter per second, the atoms should survive in their dark states up to a second.

Much larger atomic velocities are possible, however, as long as the velocity spread $\Delta\mathbf{v}$ is much smaller than the central velocity $\mathbf{v}_0$. For atoms moving along the y axis, one can set a two-photon detuning $\omega_{21}=-(k_p+k_c)v_0$ to compensate the Doppler shift associated with $\mathbf{v}_0$. In that case it is the velocity spread $\Delta\mathbf{v}$ rather than the whole atomic velocity $\mathbf{v}$ that determines the nonadiabatic term F. For instance, in a recent experiment [17] on propagation of a BEC in a waveguide, the central atomic velocity is 5 cm/s, whereas the velocity spread is only 1.4 mm/s. Note that the two-photon detuning will also lead to a transversal slope in the trapping potential represented by the term with ω_{21} in Eq. (4).

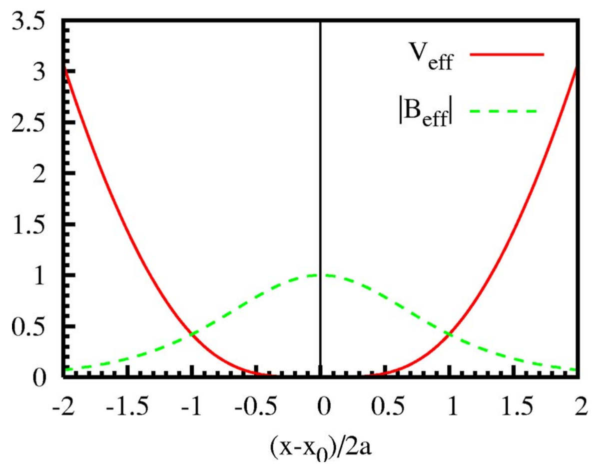

FIG. 2. (Color online) Effective trapping potential V_{eff} and an effective magnetic field B_{eff} produced by counter-propagating Gaussian beams. The external harmonic potential V_{ext} cancels the quadratic term in the overall potential V_{eff}. The effective magnetic field is plotted in the units of $B_{\text{eff}}(0) \equiv \hbar k/4a$, whereas the effective trapping potential is plotted in the units of $\hbar\omega_{\text{rec}}(1+1/4a^2k^2)$, with $\omega_{\text{rec}}=\hbar k^2/2m$.

Let us assume that both the control and probe beams are characterized by Gaussian profiles with the same amplitude Ω_0 and width σ:

$$|\Omega_j| = \Omega_0 \exp\left(-\frac{(x-x_j)^2}{\sigma^2}\right), \quad j = p,c. \tag{9}$$

In the paraxial approximation, the Gaussian beams have the width $\sigma \equiv \sigma(y) = \sigma_0[1+(\lambda y/\pi\sigma_0^2)]^{1/2}$, where $\sigma_0 \equiv \sigma(0)$ is the beam waist and λ is the wavelength. Since $k_p \approx k_c \approx k/2$, we have $\lambda \approx 4\pi/k$ both for the control and probe beams. We are interested mostly in distances $|y|$ much less than the confocal parameter of the beams $b=2\pi\sigma_0^2/\lambda \approx k\sigma_0^2/2$. For such distances, $|y| \ll b$, the width $\sigma(y)$ is close to the beam waist: $\sigma(y) \approx \sigma_0$.

Suppose the beams are centered at $x_p = x_0 + \Delta x/2$ and $x_c = x_0 - \Delta x/2$, The intensity ratio reads then $|\zeta|^2 \equiv |\Omega_p/\Omega_c|^2 = \exp[(x-x_0)/a]$, where $a \equiv a(y) = \sigma^2/4\Delta x$ is the relative width of the two beams. Thus we have

$$\mathbf{B}_{\text{eff}} = -\hbar k \frac{1}{4a\cosh^2[(x-x_0)/2a]}\mathbf{e}_z, \tag{10}$$

$$V_{\text{eff}}(\mathbf{r}) = V_{\text{ext}}(\mathbf{r}) + \frac{\hbar^2 k^2}{2m}\frac{(1+1/4a^2k^2)}{4\cosh^2[(x-x_0)/2a]}. \tag{11}$$

It is evident that both $\mathbf{B}_{\text{eff}}$ and $V_{\text{eff}}(\mathbf{r})$ are maximum at the central point $x=x_0$ and decrease quadratically for $|x-x_0| \ll a$. Similar to Ref. [5], the term quadratic in the displacement $x-x_0$ can be canceled in the effective trapping potential (11) by taking an external potential V_{ext} with the appropriate quadratic term. The frequency of the external potential fulfilling such a condition is

$$\omega_{\text{ext}} = \frac{\hbar k}{4am}\sqrt{1+1/4a^2k^2}. \tag{12}$$

With this the overall effective trapping potential becomes constant up to terms of the fourth order in $x-x_0$. In the vicinity of the central point ($|x-x_0| \ll a$) the magnetic field strength is: $B_{\text{eff}} \approx \hbar k/4a$. The corresponding magnetic length and cyclotron frequency are: $\ell_B \approx \sqrt{\hbar/B_{\text{eff}}} = 2\sqrt{a/k}$ and $\omega_c = B/m \approx \hbar k/4am$. The magnetic length ℓ_B is much smaller than the relative width of the two beams $\ell_B \ll a$ provided the latter is much larger than the optical wavelength: $ak \gg 1$. In that case many magnetic lengths fit within the interval $|x-x_0| < a$ across the beams. Furthermore the cyclotron frequency equals then approximately to the frequency of the external trap: $\omega_c \approx \omega_{\text{ext}}$, both of them being much less than the recoil frequency.

Figure 2 shows the effective trapping potential and effective magnetic field calculated using Eqs. (10) and (11), with the external harmonic potential V_{ext} of frequency ω_{ext} [Eq. (12)] added to cancel the quadratic term in the overall potential V_{eff}. The magnetic field is seen to be close to its maximum value in the area of a constant potential where $|x-x_0| \ll a$. For larger distances the effective trapping potential forms a barrier, so the atoms can be trapped in the region of a large magnetic field.

In summary, we have shown how to create an effective magnetic field in ultracold gases with a planar geometry using two counter-propagating laser beams acting on three-level atoms in the EIT configuration. If the amplitude ratio of the two beams changes substantially in the transverse direction, an effective magnetic field appears in the plane perpendicular to the propagation direction of the beams. This can be achieved if the beams are shifted relative to each other (see Fig. 1), such that they have a relative OAM.

The suggested method provides a possibility to create an effective magnetic field over an extended area along the propagation direction. This allows for a geometrical setup similar to that used in solid-state systems for classical and quantum Hall measurements. In particular, finite currents perpendicular to the magnetic field are possible and Hall "voltages" can be detected by observing changes in the chemical potential perpendicular to both the current and magnetic field. Finally the suggested method is much more robust than that of Refs. [4,5], as it does not require vortex light beams.

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