Calculation of metastable states associated with $dt\mu_{J\nu}$ ion using the variational method

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A series of metastable states below the $t\mu(2s)+d$ threshold have been discovered. The structure of the metastable $dt\mu_{J\nu}$ ion is studied here by the related suitable wave function and using the variational method. Few resonance energies and the related widths for $dt\mu_{J\nu}$ molecular states located below the $2s$ threshold are determined. For precise assessment of the reliability of the given wave function, the resonance formation rate is also calculated for the states which take place in the Vesman region. The obtained results are close to those previously reported and give strong indications that the related wave function is good enough to be useful for further calculations.
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I. INTRODUCTION

Recent investigations have demonstrated that the bound spectral structure of muonic molecular ions is richer than expected [1–3]. The metastable $dt\mu_{J\nu}$ molecules exhibit a series of three-body resonances embedded in the $t\mu+d$ scattering continuum just below the $t\mu(2s)+d$ threshold. Such levels are autodissociating or resonance states, because they exist in the continuum above the dissociation limit $d\mu(1s)+t$ or $t\mu(1s)+d$. The resonance states formed during the cascade of the $dt\mu_{J,\nu}$ cycle can decay into highly energetic $d\mu(1s)$ or $t\mu(1s)$ atoms [4,5]. The decay into $d\mu(1s)$ is expected to increase the $P_{1s}^{d\mu}$ fraction of muons reaching the ground state of $d\mu$ atoms and holds the potential for removal of the persisting disagreements between the experiment and theory regarding the precise values of $P_{1s}^{d\mu}$ [6]. The formation of metastable $dt\mu_{J,\nu}$ can be one of the fastest processes depleting $2s$ state of $t\mu$ atoms in muon catalyzed fusion (μCF). Our task in this work is the calculation of the resonance state for the metastable ion, $dt\mu_{J,\nu}$, interacting only via the Coulomb force. The nonrelativistic Schrödinger equation for $dt\mu_{J\nu}$ systems is written. The approach, using the variational method and the related suitable wave functions which contains N basis functions allows one to obtain N different eigenvalues of the Hamiltonian which can be considered as, "energies" of real and/or quasistationary states (so called, "ghost states"). Most of these states, however, cannot be seen in experiments and they do not produce any experimental consequences. One of our predicted states which exist in the Vesman region is approximately an actual resonance state and can be seen in experiments. To assess the results of calculations with the possibility of their experimental observation, the corresponding lifetime for Vesman resonances (inverse width of these states) are calculated. The three-body theory and our used method are given in Sec. II leading to the numerical results and conclusions in Sec. III.

II. THREE-BODY THEORY AND METHOD

We employ wave functions for the metastable nonsymmetric three-body system (consisting deuterium and tritium as nuclei with one muon) and perform a numerical calculation, using the variational method. In our calculations, nuclear motions are decoupled from muonic motions, i.e., the given wave functions for three-body systems are assumed to have the following form:

$$\Phi_{g,u}^{J,\nu}(\vec{r},\vec{R}) = \sum_M A_M Y_{J,M} \chi^{J,\nu}(R) \Psi_{g,u}(\vec{r},\vec{R}), \tag{1}$$

where $\Psi_{g,u}(\vec{r},\vec{R})$ is obtained from

$$H_0 \Psi_{g,u}(\vec{r},\vec{R}) = E_{g,u}(R) \Psi_{g,u}(\vec{r},\vec{R}) \quad \text{and} \tag{2}$$

$$H_0 = -\frac{\hbar^2}{2M_{t\mu}} \nabla_{\vec{r}}^2 + V_C,$$

and $\Psi_{g,u}(\vec{r},\vec{R})$ and $E_{g,u}$ are the eigenfunctions and the eigenvalues for the Hamiltonian for fixed nuclei, H_0, respectively. Here, $\vec{r}$ is the position vector of muon, $M_{t\mu}$ is the reduced mass of tritium and muon, and the internuclear distance R is just a parameter. $V_C = -e^2/|\vec{r}+\beta\vec{R}| - e^2/|\vec{r}-\gamma\vec{R}| + e^2/R$, where $\beta = 0.400\,38$ and $\gamma = 1-\beta$. A_M is a normalization factor. J, ν, and M are rovibrational and magnetic quantum numbers for the three-body molecule, respectively. $Y_{J,M}$ is the angular part of the $dt\mu_{J\nu}$ nuclear wave function. The radial nuclear wave function for states with angular momentum J, $\chi^{J,\nu}(R)$, is solved using $E_{g,u}(R)$ as the effective potential for the nuclear motion

$$-\frac{\hbar^2}{2M_{dt}}\left(\frac{d^2}{dR^2} + \frac{2}{R}\frac{d}{dR}\right)\chi^{J,\nu}(R) + \left(\frac{\hbar^2}{2M_{dt}}\frac{J(J+1)}{R^2} + E_{g,u}(R) - \varepsilon\right)\chi^{J,\nu}(R) = 0. \tag{3}$$

M_{dt} is the reduced mass of t and d nuclei. ε is the eigenvalue that is measured from the first excited state of $t\mu$.We define the wave functions $\Psi_{g,u}(\vec{r},\vec{R})$as

$$\Psi_{g,u}(\vec{r},\vec{R}) = f_{g,u}(\zeta_1 \Psi_1(\vec{r},\vec{R}) \pm \zeta_2 \Psi_2(\vec{r},\vec{R})). \tag{4}$$

ζ_1 and ζ_2 parameters are calculated using the variational method. Our calculation shows that for any ζ_1 and ζ_2 values, the $f_{g,u}$ is independent of R. For example for $\zeta_1=\zeta_2=1$ our calculated value for $f_{g,u}$ for any nuclear radius (0–1.434

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TABLE I. The calculated energies ε (eV) and widths per $\hbar$, $\Gamma_{()}$ (s^{-1}), for resonance (1,9) and (0,8) and ghost (0,0), (0,1), and (1,1) states in the Vesman and thermal regions, respectively, and their comparison with energies previously reported, $E_b^{\nu J}$ (eV) [7].

(J,ν)	ε	$\Gamma_C(\times 10^{11})$	$\Gamma_{Aug}(\times 10^{13})$	$\Gamma_\gamma(\times 10^{10})$	$\Gamma_f(\times 10^{9})$	$\Gamma_r(\times 10^{13})$	$\Gamma_{ent}(\times 10^{14})$	$E_b^{\nu J}$
(1,9)	−0.319	1.001	1.062	3.022	6.026	1.072	2.750	−0.324
(0,8)	−0.827	2.510	6.809	4.239	5.428	6.834	2.223	−0.832
(0,0)	−217.741					0.111	0.879	−217.889
(0,1)	−139.804					1.977	0.700	−139.728
(1,1)	−134.978					1.003	0.713	−135.362

$\times 10^{-8}$ cm) is constant and equal to 0.7071. At the large nucleus radii R, the role of the wave function of $\mu t(2s)$, $(\phi_{2,0,0})$, appears in the wave functions Ψ_g and Ψ_u. Therefore, we write the wave functions $\Psi_1$and Ψ_2 as

$$\Psi_1(\vec{r},\vec{R}) = \left(1 - \frac{|\vec{r}+\beta\vec{R}|}{2a_\mu}\right)\tau^{-1/2}\exp\left(-\frac{|\vec{r}+\beta\vec{R}|}{2a_\mu}\right), \quad (5)$$

$$\Psi_2(\vec{r},\vec{R}) = \Psi_1(\vec{r},\beta\vec{R}\rightarrow -\gamma\vec{R}),$$

where $a_\mu \simeq 2.655\times 10^{-10}$ cm and $\tau = 0.139\times 10^{-26}$ cm^3. The behavior of the functions $E_g(R)$ and $E_u(R)$ show that $dt\mu_{J,\nu}$ can have resonance states only for $E_g(R)$. In other words, $E_u(R)$ does not have an absolute minimum value. The function $E_g(R)$ is minimized at $R=R_0$ so that

$$R_0 = 1.8\times 10^{-10}\ \text{cm} \quad \text{and} \quad E_b = -0.6283\times 10^{-9}\ \text{erg}. \quad (6)$$

E_b is the minimum energy. Equation (3) is the Schrödinger equation for the motion of nuclei in the effective potential, $E_g(R)$. ε is calculated and rounded up to three significant figures by the Rung-Kutta 45 method, using a variational procedure. The calculated energies of $dt\mu_{J\nu}$ levels for (0,0), (0,1), (0,8), (1,1), and (1,9) states are compared to those previously reported [7], given in Table I. In the previous method, the structure of $dt\mu_{J\nu}$ has been studied using the coupled rearrangement channel method with Gaussian basis functions. It means that different forms for the potential terms and the related wave function are used. In our calculations, the crucial quantities for determining the resonance formation cross section for the muonic molecule are the widths for the process

$$t\mu(2s) + (D_2)_{k_i,v_i} \rightarrow [(dt\mu)_{J\nu} - dee]_{k_r v}, \quad (7)$$

where k_i and k_r are the rotational quantum numbers. The second index is the vibrational quantum number. Therefore, we calculate the widths of the states which can be related to the process (7).The cross section for $dt\mu_{J,\nu}$ formation reaction is given by the Breit-Wigner relation

$$\sigma(\epsilon_i) = \frac{\pi}{K^2_{t\mu(2s)}}\frac{\Gamma_{ent}\Gamma_r}{(\epsilon_i-\epsilon_{res})^2 + \frac{1}{4}(\Gamma_{ent}+\Gamma_r)^2}. \quad (8)$$

ϵ_i and ϵ_{res} are the collision and Vesman resonance energies of the $t\mu(2s)$ muonic atom relative to the D_2 target, respectively. Since several of the resonances are located within the dissociation energy of D_2 ($\simeq$4.5 eV) below the $t\mu(2s)$ threshold, we consider the Vesman formation mechanism, whereby the excess energy is transferred to the rovibrational degrees of freedom of the host D_2 molecule in the process (7). The Vesman resonance energy ϵ_{res} satisfies the energy conservation conditions as

$$\epsilon_{res} + \varepsilon = \Delta E_{rovib} + \Delta E_{hf}, \quad \epsilon_{res} < 4.5\ \text{eV}, \quad (9)$$

where ΔE_{rovib} is the difference between the rovibrational levels of hybrid molecule $[(dt\mu)_{J\nu}-dee]_{k_r v}$ and the D_2 molecule, while ΔE_{hf} is the difference in hyperfine splitting for $t\mu$ and $dt\mu$ levels. Γ_r (proportional to inverse lifetime) is the reactive scattering width, given by the rate of Auger deexcitations of the metastable molecule (Γ_{Aug}), Γ_f is the fusion width, while Γ_C and Γ_γ are the widths for Coulomb and radiative decay into $d\mu(1s)+t$ or $t\mu(1s)+d$. So that the reactive scattering width is given as

$$\Gamma_r = \Gamma_{Aug} + \Gamma_f + \Gamma_C + \Gamma_\gamma. \quad (10)$$

The entrance width Γ_{ent} is determined by the following formula:

$$\Gamma_{ent} = \frac{4m_{t\mu(2s)}K_{t\mu(2s)}}{\hbar^2(4\pi)^2}\sum_M \int d\Omega |V_{ir}(\vec{K}_{t\mu(2s)})|^2, \quad (11)$$

where $m_{t\mu(2s)}$ is the reduced mass of $t\mu(2s)+D_2$ system. $K_{t\mu(2s)}$ (with angles Ω) is the wave number of the $t\mu(2s)$ muonic atom relative to the D_2 target. The sum on M means a sum on the magnetic quantum numbers related to the angular part of wave functions. Because the process (7) is a rearrangement one in which a deuterium is coupled to $t\mu$ in the resonance state and to other deuterium in the channel state, no perturbation expression (or Morse-Feshbach-type formula) applies. Instead the width involves the transition matrix element, $|V_{ir}|^2$, calculated between the wave function in the initial state i of the system $t\mu(2s)+(D_2)_{k_i,v_i}$ and the final state r of $[(dt\mu)_{J\nu}-dee]_{k_r v}$, assuming a pointlike pseudonucleus, $dt\mu_{J,\nu}$, for the hybrid molecule we have

$$|V_{ir}(\vec{K}_{t\mu(2s)})|^2 = |\langle \Phi_g^{J,\nu}(\vec{r},\vec{R})\,\bar{\vartheta}_{k_r}|V_{t\mu-d}|\vartheta_{k_i}\phi_{2,0,0} \times \exp(i\vec{K}_{t\mu(2s)}\cdot\vec{R}')\rangle|^2, \quad (12)$$

where $V_{t\mu-d} = e^2/R - e^2/|\vec{r}-\gamma\vec{R}|$ and $\bar{\vartheta}_{k_r}$ is the Born-Oppenheimer wave function of the hybrid molecule. ϑ_{k_i} is

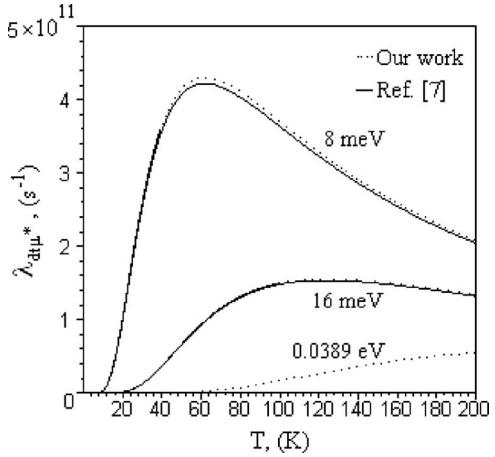

FIG. 1. The calculated resonance formation rate vs temperature, T, for the resonance energies $\epsilon_{res} \simeq 0.008$, 0.016, and 0.0389 eV, and their comparison with previously reported (see Ref. [7]).

the wave function of $(D_2)_{k_i,v_i}$, $\vec{R}' = \vec{R} + 0.5\vec{R}_{D_2}$, and $\vec{R}_{D_2}$ is the distance vector between the two deuterium nuclei in the D_2 molecule. $\vec{R}'$ is the position vector of $t\mu(2s)$ relative to the D_2 molecule. Making a Taylor expansion of the initial D_2 wave function, ϑ_{k_i} along the coordinates of the hybrid molecule ($\vec{r}_{dt\mu_{J,v}-d}$), and attending to the muonic coordinate overlap which is only for small $t\mu-d$ distances, ($t\mu-d$ distances $R \sim 1.4\times 10^{-8}$ cm are approximated as infinite in our calculations), the first term of expansion will dominate in the matrix element

$$|V_{ir}|^2 \simeq |\langle \Phi_g^{J,v}(\vec{r},\vec{R})|V_{t\mu-d}|\phi_{2,0,0} \times \exp(if\vec{K}_{t\mu(2s)}\cdot\vec{R})\rangle\langle \bar{\vartheta}_{k_r}(R_h) \times |\vartheta_{k_i}(R_h)\exp(ig'\vec{K}_{t\mu(2s)}\cdot\vec{R}_h)\rangle|^2 \quad (13)$$

where $f=0.695$ and $g'=0.5$ are the mass dependent projection coefficients and $R_h \equiv |\vec{r}_{dt\mu^*_{J,v}-d}|$ (see Refs. [7,8]). A substantial simplification in the evaluation of the above expression is due to the sum rule

$$\sum_{k_r} |\langle \bar{\vartheta}_{k_r}(R_h)|\vartheta_{k_i}(R_h)\exp(ig'\vec{K}_{t\mu(2s)}\cdot\vec{R}_h)\rangle|^2 = 1, \quad (14)$$

where the sum over k_r implies summing over all electronic states. For example, the calculated $\Gamma_{ent}(\epsilon_i)$ for $J=1$ with resonance energy $\epsilon_i = \epsilon_{res} \simeq 0.262$ eV is approximately equal to 0.275×10^{15} s^{-1}. In nucleus radii $R > 1.434\times 10^{-8}$ cm (semiclassically), the $dt\mu_{J,v}$ system decay back to $t\mu(2s)+d$ state or decay to $t\mu(1s)+d$ or $d\mu(1s)+t$ states. Therefore, $t\mu-d$ distances equal to 1.434×10^{-8} cm are approximated to be infinite in our numerical calculations. The resonance process (7) is formed under the energy conservation law (9) and the following condition as $\Gamma_r/\Gamma_{ent} \to 0$. In other words, after the formation of the $[(dt\mu)_{Jv}-dee]_{k_r v}$ hybrid molecule, the mentioned reactive decay processes must occur very slowly. In this case, a formalism for calculating Γ_f for metastable states was developed in Ref. [3]. Preliminary extensions of those calculations indicate that Γ_f for any of resonance states (within the Vesman region) here concerned is less than 10^{10} s^{-1}. For the fusion width, we write

$$\Gamma_f/\hbar = N_0 A \varrho_f, \quad \varrho_f = \int |\Phi_{g,u}^{J,v}(\vec{r},\vec{R})|^2 \delta(\vec{R}) d\vec{R} d\vec{r}, \quad (15)$$

where ϱ_f is the probability for the nuclei to coincide, A is equal to 1.28×10^{-14} cm^3/s, related to the astrophysical factor for dt fusion and $N_0 = 4.25\times 10^{22}$/cm^3 is the liquid hydrogen density (LHD) [9]. The width of the radiative decay can be estimated by realizing that the muon of the $dt\mu_{J\nu}$ molecule is strongly clustered on tritium, deuterium only weakly interacting with the $t\mu$ atom. Γ_γ is thus well approximated by the radiative decay rates of $t\mu(2s)$ and $t\mu(2p)$. These mean that we can approximately calculate Γ_γ via the following formula:

$$\Gamma_\gamma = (1-\beta_{J\nu}^2)\Gamma_\gamma^{2p\to 1s}(t\mu), \quad \beta_{J\nu} = (\varepsilon - E_b^{\nu J})/\Delta E_{2s}, \quad (16)$$

where $\beta_{J\nu}$ is estimated by evaluating the energy ε including the vacuum polarization potential, related to the $2p$ orbital admixture in the $dt\mu$ wave function and $E_b^{\nu J}$ is given in Ref. [7]. ΔE_{2s} is the vacuum polarization shift of the free $t\mu(2s)$ atom. The width for Coulomb decay is written as

$$\Gamma_C = \frac{K_r m_r}{(2\pi)^2(\hbar)^2(2J+1)} \sum_M \int d\Omega_{\hat{K}_r} |\langle \eta_{1s} e^{-i\vec{K}_r\cdot\vec{R}} |H_r^I| \Phi_g(\vec{r},\vec{R}) \times \chi^{J,\nu}(R) Y_{J,M}\rangle|^2. \quad (17)$$

m_r and K_r are the reduced mass and the relative wave number of the released particles from the decay process of the $dt\mu$ molecule, respectively. H_r^I is a perturbation Coulomb potential, related to the variations of energy after performing of the mentioned decay reaction. For the $dt\mu_{J,\nu}$ states within the Vesman region, $\Gamma_\gamma \simeq 5\times 10^{10}$ s^{-1} and $\Gamma_C \simeq 10^{11}$ s^{-1} were calculated in Ref. [7]. Our results for the values of the widths show that the main contribution to the reactive width Γ_r comes from the Auger transition

$$[(dt\mu)_{J_i\nu_i} - dee] \to [(dt\mu)_{J_f\nu_f} - de]^+ + e^-, \quad \text{with width } \Gamma_{Aug}^{J_i\nu_i\to J_f\nu_f} = 2\pi\rho(E)\sum_f |\langle f|V_I|i\rangle|^2, \quad (18)$$

where $|f\rangle$ and $|i\rangle$ are the final and initial states of system and $\rho(E)$ is the density of final state for a given energy. The interaction operator V_I can be approximated by the following relations:

$$V_I = -e(\vec{r}_e\cdot\vec{d})r_e^{-3}, \quad d = |e\vec{r}_t + e\vec{r}_d - e\vec{r}_\mu| \simeq e\eta R, \quad (19)$$

where $\eta = 0.608$ and $\vec{d}$ is the electric dipole moment operator. $\vec{r}_t$, $\vec{r}_d$, and $\vec{r}_\mu$ are the coordinates of the particles t, d, and μ, respectively, with respect to center of mass of the $dt\mu_{J\nu}$ ion. We make the following estimation of the resonance Auger de-excitation width Γ_{Aug}^{res}:

$$\Gamma_{Aug}^{res} \simeq \frac{|\langle \chi^{J_f,\nu_f}(R)|R|\chi^{J_i,\nu_i}(R)\rangle|^2}{|\langle \chi^{0,1}(R)|R|\chi^{1,1}(R)\rangle|^2}\frac{K_b^e}{K_{res}^e}\Gamma_{Aug}^{11\to 01}. \quad (20)$$

K_b^e and K_{res}^e are the wave numbers of the ejected electron from bound and resonance states, respectively. For example, $(J_f,\nu_f)=(0,4)$ and $(J_i,\nu_i)=(1,9)$, being a state of the $dt\mu_{J\nu}$ within Vesman region [7]. Writing the initial and final states as a product of the electronic and muonic wave functions and assuming that the density of final states for the electron ejected is the same for the plane wave, $\Gamma_{Aug}^{11\to 01}$ comes as

$$\Gamma_{Aug}^{11\to 01} = \frac{K_b^e m_e e^4 \eta^2}{(2J_1+1)\pi\hbar^2}\sum_{M,M_1}\left|\langle \chi^{0,1}(R)|\vec{R}|\chi^{1,1}(R)Y_{1,M_1}(\hat{R})\rangle \times\left\langle \psi_f(\vec{r}_e)\left|\frac{\hat{r_e}}{r_e^2}\right|\psi_i(\vec{r}_e)\right\rangle\right|^2, \quad (21)$$

where J_1 is equal to one for bound state. $\psi_f(\vec{r}_e)$ and $\psi_i(\vec{r}_e)$ are the electronic wave functions of the Auger electron being ejected from the hybrid molecule in the initial and final states, respectively. The electronic matrix element, $|\langle\psi_f(\vec{r}_e)|(1/r_e^2)|\psi_i(\vec{r}_e)\rangle|^2$, is calculated by taking electronic wave functions as

$$\psi_i(\vec{r}_e) = 2Y_{0,0}(\hat{r_e})e^{-r_e}, \quad \psi_f(\vec{r}_e) = \frac{F_1(1/K_b^e, K_b^e r_e)}{K_b^e r_e}Y_{1,M}(\hat{r_e}). \quad (22)$$

$Y_{1,M_1}(\hat{R})$ and $Y_{1,M}(\hat{r_e})$ are the angular part of the nuclear wave and electronic functions, respectively. The actual two center wave function, F_L, is the regular Coulomb function. The calculated values of $\Gamma_{Aug}^{11\to 01}$, K_b^e/K_{res}^e, $|\langle\chi^{0,1}(R)|e\eta R|\chi^{1,1}(R)\rangle|$, and $|\langle\chi^{0,4}(R)|e\eta R|\chi^{1,9}(R)\rangle|$ dipole moments are approximately 1.1×10^{12} s^{-1}, 3.78, 5.165×10^{-20} esu cm, and 8.256×10^{-20} esu cm, respectively. We calculate the widths for the $(1,9)$ and $(0,8)$ states which take place in Vesman region by the numerical integrations using the DCUHRE method and the ACM TOMS Algorithm 698. In the mentioned procedure, the accuracy of calculations is six digits. Our results, rounded up to three significant figures, are given in Table I. As $\Gamma_{Aug}^{19\to 04}=\Gamma_{Aug}^{res}\ll\Gamma_{ent}$, the exact value of Γ_{ent} irrelevant at the cross section or resonance formation rate simplifies as

$$\sigma(\epsilon_i) = \frac{2\pi^2}{K_{t\mu(2s)}^2}\Gamma_{Aug}^{19\to 04}\delta(\epsilon_i-\epsilon_{res}), \quad (23)$$

$$\lambda_{dt\mu^*}(T) = N\frac{\hbar K_{t\mu(2s)}(\epsilon_{res})}{m_{t\mu(2s)}}F(\epsilon_{res},T)\sigma(\epsilon_{res}),$$

where N is the density of the D_2 molecules in media. The muonic atoms $t\mu(2s)$ have a Maxwell distribution $F(\epsilon_i,T)$ at temperature T. The calculated curve of $\lambda_{dt\mu^*}(T)$ (normalized to LHD) versus T in the resonance collision energies $\epsilon_{res}\simeq 0.008$, 0.016, and 0.0389 eV are given in Fig. 1 and compared to the result given in Ref. [7]. The muonic atoms $t\mu(2s)$ can form the muonic molecular ions in the following reactions:

$$t\mu(2s) + D_2 \to [(dt\mu)^+ - de]^+ + e^-. \quad (24)$$

The above reaction is a straightforward process for muonic molecular ion $[(dt\mu)^+]$ and complex $([(dt\mu)^+-de]^+)$ formation which are formed via the emission of an Auger electron. The sum of the calculated energies of $dt\mu$ states (ghost states) do not satisfy the conditions of the Vesman resonances. The mentioned reaction is named the nonresonance formation process. The widths Γ_{ent} and Γ_r for $\epsilon_i\simeq 0.0253$ eV, thermal region, are calculated and given in Table I.

III. CONCLUSIONS

The nonsymmetric three-body problem for $dt\mu_{J\nu}$ is solved by related suitable wave function, using the variational method. The obtained results are compared with results of Ref. [7]. Our calculated energies for $(0,0)$ and $(0,1)$ states are in agreement within 0.068%, and 0.601%, 0.284%, and 1.543% for states $(0,8)$, $(1,1)$, and $(1,9)$, respectively. For precise assessment of the reliability of the given wave function, the formation rate are also calculated. The Eq. (20) shows that low and up states are proportional. In other words, the Auger transition process in up states strongly depends on its value for low states. For state $(J=1,\nu=9)$, the entrance width is much more than the reactive scattering width, Γ_{ent} and Γ_r are much less than resonance energies, therefore, the predicted state can be seen in experiments (see Ref. [7]). The results show that the used theory and method are good enough in calculating the actual resonance and ghost states for the $dt\mu_{J\nu}$ muonic molecular ion.

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