Anyonic excitations in fast rotating Bose gases

A. Lakhoua,[1] M. Lassaut,[1] T. Masson,[2] and J. C. Wallet[2]

[1]*Groupe de Physique Théorique, Institut de Physique Nucléaire, F-91406 Orsay Cedex, France*
[2]*Laboratoire de Physique Théorique, Bâtiment 210, 91405 Orsay Cedex, France*

The role of anyonic excitations in fast rotating harmonically trapped Bose gases in a fractional quantum Hall state is examined. Standard Chern-Simons anyons as well as "nonstandard" anyons obtained from a statistical interaction having Maxwell-Chern-Simons dynamics and suitable nonminimal coupling to matter are considered. Their respective ability to stabilize attractive Bose gases under fast rotation in the thermodynamical limit is studied. Stability can be obtained for standard anyons while for nonstandard anyons, stability requires that the range of the corresponding statistical interaction does not exceed the typical wavelength of the atoms.

The experimental realization of Bose-Einstein condensation (BEC) of atomic gases [1] has given rise to a rich variety of phenomena and motivated numerous investigations focused on ultracold atomic Bose gases in rotating harmonic traps. BEC confined to two dimensions have been created [2] and their response under rotation has been studied. Basically, the rotation of a BEC produces vortices in the condensate [3,4]. When the rotation frequency increases, the BEC state is destroyed and for sufficiently high frequency, a state corresponding to fractional quantum Hall effect (FQHE) [5] is expected to possibly occur [6]. In particular, when the rotation frequency is tuned to the characteristic frequency of the harmonic confining potential in the radial plane, FQHE states have been predicted to become possible ground states for the system [7]. This observation has been followed by studies focused on the FQHE for bosons with short range interactions [8,9]. FQHE states involve anyonic excitations [10]. Starting from the standard Chern-Simons (CS) realization for anyons [10], it has been shown in [11] that in the thermodynamical limit, a two-dimensional (2D) harmonically trapped rotating Bose gas with *attractive* interactions can be stabilized (in a FQHE state) thanks to its anyonic (quasiparticle) excitations [12].

The above mentioned description of anyons is not unique. Another realization has been investigated in [13] where the minimal coupling of the statistical gauge potential to matter, which is constrained to have Maxwell-Chern-Simons (MCS) dynamics, is supplemented by a Pauli-type coupling, as recalled below. Differences do exist between CS and MCS anyons. The latter stem from a statistical interaction having finite range (whereas CS interaction has zero range) and have an additional attractive mutual interaction (absent for CS anyons). These differences cannot be observed in electronic quantum Hall systems [5] but may show up in fast rotating attractive Bose gases. We point out that the existence of this attractive mutual interaction may have an impact on the ability of MCS anyonic excitations to stabilize an attractive Bose gas in a FQHE state. We find that stability requires that the range of the MCS statistical interaction does not exceed the typical wavelength for the atoms. The description of Quantum Hall Fluids within the MCS Landau-Ginzburg theory stemming from the MCS description of anyons is also discussed.

Consider an interacting Bose gas at zero temperature in a rotating harmonic trap with strong confinement in the direction of the rotation axis so that the system is actually two dimensional. The Hamiltonian in the rotating frame [14] and the corresponding action quoted here for further convenience are [15]

$$H = \sum_{A=1}^{N} \frac{1}{2m}[\mathbf{p}_A - \mathbf{A}(\mathbf{x}_A)]^2 + W + V + \cdots, \quad (1)$$

$$S_0 = \int_t \left(\sum_{A=1}^{N} \frac{m}{2}\dot{\mathbf{x}}_A^2 - W - V \right) + \int_x \mathbf{A}(\mathbf{x}) \cdot \mathbf{J}(\mathbf{x},t) + \cdots, \quad (2)$$

where $\mathbf{J}(\mathbf{x},t) = \Sigma_A \dot{\mathbf{x}}_A(t)\delta[\mathbf{x} - \mathbf{x}_A(t)]$, the external gauge potential $A_i(\mathbf{x}_A) = m\omega\epsilon_{ij}x_A^j$ yields the Coriolis force, $W = \Sigma_{A=1}^{N}(m/2)(\omega_T^2 - \omega^2)|\mathbf{x}_A|^2$ $((\omega/2\pi)$ [resp $(\omega_T/2\pi)$] is the rotation (resp trapping) frequency), $V = \Sigma_{A<B}V(\mathbf{x}_{AB})$, $V(\mathbf{x}_{AB})$ is the two-body potential felt by the bosons in the plane. It can be well approximated by $V(\mathbf{x}_{AB}) = g_2\delta(\mathbf{x}_{AB})$ since the scattering between ultracold bosons is dominated by the *s* wave [14]. Here, g_2 can be viewed as an effective coupling constant encoding basically the effects of the harmonic axial confinement (with trapping frequency $\omega_z/2\pi$ and localization length $l_z = \sqrt{1/m\omega_z}$ along the rotation axis on the initial three dimensional (3D) scattering properties of the atoms [16]. This will be discussed later on. In (1) and (2), the ellipses denote possible multibody interaction terms. We now consider the "limit" $\omega \simeq \omega_T$ for which the trapping and centrifugal potential (nearly) balance each other.

The standard CS description of anyons [10] obtained from (1) and (2) is achieved by minimally coupling to S_0 a statistical CS gauge potential $a_\mu : S_0 \rightarrow S_1 = S_0 + \int_x(\eta/4)\epsilon_{\mu\nu\rho}a^\mu f^{\nu\rho} + a_\mu J^\mu$ where the CS parameter η is dimensionless, $J_\mu = (\rho; \mathbf{J})$ with $\rho = \rho(\mathbf{x},t) = \Sigma_A\delta[\mathbf{x} - \mathbf{x}_A(t)]$ and $\mathbf{J}$ as given above $[f_\mu \equiv \frac{1}{2}\epsilon_{\mu\nu\rho}f^{\nu\rho}, f_{\mu\nu} = \partial_\mu a_\nu - \partial_\nu a_\mu, f_0$ (resp f_i) is the statistical magnetic (resp electric) field]. The equations of motion for a_μ take the field-current identity form $f_\mu = -(1/\eta)J_\mu$ (i), ensuring that particle-statistical magnetic flux composite anyonic objects are formed [10]. Particle-flux coupling leads to Aharonov-Bohm type interactions. The anyonic character of the wave functions for the quasiparticles is determined by

the Aharonov-Bohm phase [10] $\exp(i\int_C \mathbf{a}\cdot d\mathbf{x})=\exp(i/\eta)$ (C is some closed curve) that is induced when one quasiparticle moves adiabatically around another one, equivalent to a *double* interchange of identical quasiparticles in the wave function. Then, their statistics are controlled by η: they have Fermi (Bose) statistics when $\eta=1/[(2k+1)2\pi]$ ($[1/(2k)2\pi]$), $k\in\mathbb{Z}$. The statistics are anyonic otherwise. This leads to the CSLG action [17] underlying the analysis presented in [11]. Since the statistical interaction is mediated by a CS gauge potential, its range is zero.

Another realization for anyons has been proposed and discussed in [13]. It is obtained by coupling a MCS statistical gauge potential a_μ to S_0 through minimal and nonminimal (Pauli-type) coupling term, the strength of this latter being fixed to a specific value [13], namely: $S_0\rightarrow S_2=S_0+\int_x -(1/4e^2)f_{\mu\nu}f^{\mu\nu}+(\eta/4)\epsilon_{\mu\nu\rho}a^\mu f^{\nu\rho}+a_\mu J^\mu-(1/\eta e^2)f_\mu J^\mu$ (e^2 has mass dimension 1), where the coupling constant of the Pauli term (last term in S_2) has been already fixed to the specific value. Then, the equations of motion for a_μ can be written as $-(1/e^2)\epsilon_{\alpha\mu\rho}\partial^\mu[f^\rho+(1/\eta)J^\rho]+\eta[f_\alpha+(1/\eta)J_\alpha]=0$ which is solved by (i). This latter observation has been used as a starting point to construct an effective theory reproducing the usual anyonic behavior [13]. The Aharonov-Bohm phase defining the statistics of the resulting quasiparticles still verifies $\exp(i\int_C \mathbf{a}\cdot d\mathbf{x})=\exp(i/\eta)$ so that η again controls the statistics. However, the resulting MCS anyons have an additional attractive contact mutual interaction [13]. Combining S_2 with the second quantization machinery, one obtains the corresponding MCSLG action

$$S=\int_x i\phi^\dagger \mathcal{D}_0\phi - \frac{1}{2m}|\mathcal{D}_i\phi|^2 - g_2(\phi^\dagger\phi)^2 - U(\phi)$$
$$-\frac{1}{4e^2}f_{\mu\nu}f^{\mu\nu}+\frac{\eta}{4}\epsilon_{\mu\nu\rho}a^\mu f^{\nu\rho}, \qquad (3)$$

$$\mathcal{D}_0=\partial_0-ia_0+\frac{if_0}{\eta e^2}, \qquad (4a)$$

$$\mathcal{D}_i=\partial_i-i(a_i+A_i)+\frac{if_i}{\eta e^2}. \qquad (4b)$$

In (3), $\phi\,[=\phi(\mathbf{x},t)]$ is the order parameter, $\phi^\dagger\phi=\rho$, $U(\phi)$ denotes the Landau-Ginzburg (LG) potential for multibody interactions, $U(\phi)=g_3|\phi|^6+\cdots$ ($g_3>0$ as in [11]). The nonminimal coupling terms have been included in (4). The statistical MCS gauge potential has a finite mass [18] $M=|\eta|e^2$, so that the statistical interaction has a finite range $\Lambda_{st}=1/M$. The Chern-Simons Landau Ginzburg (CSLG) action [17] is obtained from (3) and (4) by taking the limit $e^2\rightarrow\infty$. While CSLG action is believed to encode the low energy physics of QHF, it appears that MCSLG action also reaches this goal. This is discussed later on. For the moment, we assume that the system is in a FQHE state ($\eta\neq 0$) described by (3) and (4). The equations of motion derived from (3) are

$$i\mathcal{D}_0\phi+\frac{1}{2m}\mathcal{D}_i\mathcal{D}_i\phi-2g_2(\phi^\dagger\phi)\phi=\frac{\delta U(\phi)}{\delta\phi^\dagger}, \qquad (5a)$$

$$-\frac{1}{e^2}\epsilon_{\alpha\mu\rho}\partial^\mu\left(f^\rho+\frac{1}{\eta}\mathcal{J}^\rho\right)+\left(f_\alpha+\frac{1}{\eta}\mathcal{J}_\alpha\right)=0, \qquad (5b)$$

where $\mathcal{J}_0=\rho$, $\mathcal{J}_i=(i/2m)[\phi^\dagger\mathcal{D}_i\phi-(\mathcal{D}_i\phi)^\dagger\phi]$ and $\mathcal{D}_\mu$ is defined in (4) while anyonic configurations are obtained from (5b) when $f_\mu=-(1/\eta)\mathcal{J}_\mu$ [19]. From the field's conjugate momenta $\Pi_{\phi^\dagger}=\Pi_{a_0}=0, \Pi_\phi=i\phi^\dagger, \Pi_{a_i}=(1/e^2)f_{0i}-(\eta/2)\epsilon_{ij}a^j+(1/\eta e^2)\epsilon_{ij}\mathcal{J}^j$, one obtains the Hamiltonian

$$H=\int_x \frac{1}{2e^2}\left(f_0+\frac{1}{\eta}\rho\right)^2+\frac{1}{2e^2}\Theta f_i^2+\frac{1}{2m}|D_i\phi|^2$$
$$+\left(g_2-\frac{1}{2\eta^2 e^2}\right)\rho^2+U(\phi), \qquad (6)$$

where $D_i=\partial_i-i(a_i+A_i)$ and $\Theta=1-(\rho/m\eta^2 e^2)$. The positivity of the model requires $\rho\leq m\eta^2 e^2$ [20]. Restoring $\hbar$ and c, this translates into $\rho\leq\rho_{lim}$, $\rho_{lim}=|\eta|(\Lambda/\Lambda_{st})1/\Lambda^2$ where $\Lambda=\hbar/mc$ is a typical (deBroglie) wavelength for the atoms and $\Lambda_{st}=(\hbar/|\eta|\tilde{\mu}c)(e^2=\tilde{\mu}c^2)$. This condition should be fulfilled by current experimental values, provided $\Lambda_{st}\lesssim\mathcal{O}(\Lambda)$, that we now assume [21]. For ^{7}Li, ^{23}Na, ^{87}Rb, one obtains, respectively, $\rho_{lim}\simeq|\eta|(\Lambda/\Lambda_{st})10^{29}, 10^{30}, 10^{31}$ cm^{-2}, which, for possibly reachable current quantum Hall states, should be larger by several orders of magnitude than the experimental values reached by the matter density so that $\rho/m\eta^2 e^2\ll 1$.

Compared to the CSLG Hamiltonian [11], H involves additional terms from the Maxwell part of (3) and the nonminimal coupling in (4). The interaction energy $g_2\rho^2$ receives contributions from $(1/2e^2)f_0^2$ and the Pauli coupling in (4a): both combine to yield the first term in (6) together with the additional attractive (magnetic) contribution $-\rho^2/2\eta^2 e^2$ to $g_2\rho^2$. This, depending on the relative magnitude of Λ and Λ_{st}, may somehow alter the conclusion obtained in [11]. We set $\phi=\gamma e^{i\varphi}\equiv\sqrt{\rho}e^{i\varphi}$, $\hat{a}_i=\partial_i\varphi-(a_i+A_i)$, $\eta>0$. For N atoms in the trap, one has $\int_\mathbf{x}\phi^\dagger\phi=N$. Using $f_\mu=-(1/\eta)\mathcal{J}_\mu$, assuming that fields vanish at infinity, the static energy stemming from (6) is

$$H=\omega N+\int_\mathbf{x}\frac{1}{2m}\left(\partial_i\gamma-\epsilon_{ij}\frac{\hat{a}^j\gamma}{\sqrt{\Theta}}\right)^2+(g_2-g_0)\rho^2$$
$$+\int_\mathbf{x} m\eta^2 e^2\sum_{k=3}^\infty (\eta e^2 C_{k-1}+2\omega C_k)\left(\frac{\rho}{m\eta^2 e^2}\right)^k+U(\gamma). \qquad (7)$$

In (7), $\omega>0$, the positive constants C_k's are given by $(1-\rho/m\eta^2 e^2)^{1/2}=1-\sum_{k=1}^\infty C_k(\rho/m\eta^2 e^2)^k$ [$C_k=\Gamma(k-\frac{1}{2})/2k!\sqrt{\pi}$ where Γ is the Euler function] and we have defined

$$g_0=-\frac{\hbar^2}{2m\eta}\left[1-\frac{\Lambda_{st}}{\Lambda}\left(1-\frac{\hbar\omega}{2mc^2}\right)\right]\simeq-\frac{\hbar^2}{2m\eta}\left(1-\frac{\Lambda_{st}}{\Lambda}\right) \qquad (8)$$

($\hbar$ and c reinstalled), where the rightmost relation stems from $\hbar\omega\ll mc^2$ which holds for current experimental values for ω [taking $\omega/2\pi\sim\mathcal{O}(10)$–$\mathcal{O}(10^3)$ Hz as a benchmark]. In the same way, the term $\sim\omega C_k$ in (7) can also been neglected [since $\hbar\omega\ll\eta\tilde{\mu}c^2$ in view of $\Lambda_{st}\lesssim\mathcal{O}(\Lambda)$].

The quartic interaction terms in (7) can be eliminated provided g_2 is chosen to be

$$g_2 = g_0. \tag{9}$$

Then, neglecting for the moment in (7) the small terms $\sim \mathcal{O}(\gamma^6)$ as in [11], the ground state of (7) is obtained for those configurations satisfying $[\partial_i \gamma - \epsilon_{ij}(\hat{a}^j \gamma / \Theta^{1/2})] = 0$, $i = 1, 2$. This, combined with (5a), (8), and (9), further expanding the various contributions depending on Θ in powers of $\rho / m \eta^2 e^2$ and using the fact that $\rho / m \eta^2 e^2 \ll 1$ yields $\partial_i^2 \gamma - (\partial_i \gamma)^2 / \gamma = -\gamma [\gamma^2 / \eta (1 + \omega / \eta e^2) + 2m\omega]$ where, in the RHS, $\omega / \eta e^2 \ll 1$ still holds so that this latter equation can be accurately approximated by

$$\partial_i^2 \gamma - \frac{(\partial_i \gamma)^2}{\gamma} = -\gamma \left(\frac{\gamma^2}{\eta} + 2m\omega \right). \tag{10}$$

When ω goes to zero, the solutions of (10) reduces smoothly [22] to the solutions of the Liouville equation

$$\Delta \ln \rho = -\frac{2}{\eta} \rho. \tag{11}$$

These latter can be parametrized as $\rho(z) = 4\eta |h'(z)|^2 [1 + |h(z)|^2]^{-2} (z = x_1 + ix_2)$ for any holomorphic function $h(z)$. The particular choice $h(z) = (z_0/z)^n (z = re^{i\theta})$, $n \in \mathbb{N}$, gives rise to radially symmetric vortex-type solutions $\rho(r) = 4\eta n^2 / r^2 [(r_0/r)^n + (r/r_0)^n]^{-2}$ where r_0 is some arbitrary length scale and $\phi = \sqrt{\rho} e^{in\theta}$.

From (7)–(9), one realizes that the initial two-body coupling term $\sim g_2$ can be compensated by the statistical interaction. In view of (7), for fixed $\eta > 0$, the particular value g_0 (8) represents the limiting value for g_2 below which the Bose gas cannot be stabilized by the statistical interaction. In the CS case, $\Lambda_{st} = 0$ and g_0 is negative. Then, when $g_2 = g_0$, corresponding to an initial attractive Bose gas, the system behaves as a free anyon gas whose ground state (neglecting the small $|\phi|^6$ terms) is exactly described by (10) giving rise to nonsingular finite energy matter distribution [22]. Then, one would conclude as in [11] that an attractive Bose gas may be stabilized against collapse in the thermodynamical limit by anyonic excitations stemming from a CS (zero range) statistical interaction. This conclusion is somehow altered for "nonstandard" MCS anyons. In this case, g_0 receives an additional positive contribution from the (statistical) magnetic energy and the magnetic coupling in (4a) and the sign of g_0 depends now on the relative magnitude of Λ and Λ_{st} [see (8)]. When $g_2 = g_0$, the system behaves again as a free MCS anyon gas whose ground state (neglecting again small γ^6 terms) is now accurately described by (10). However, this situation can be reached by an initial attractive Bose gas only if $\Lambda_{st} < \Lambda$ corresponding to negative values for g_0. A MCS statistical interaction with range exceeding the typical wavelength Λ for the atoms could not protect an attractive Bose gas from collapse.

Now, let us assume that $\Lambda_{st} < \Lambda$ and discuss (9) from a physical viewpoint. In (most of) current experiments, 2D atomic systems are obtained by tightening the axial confinement applied to the initial (harmonically trapped) 3D system, which restricts the dynamics of the atoms along the axial

direction to zero point oscillations [16]. Kinematically, the system becomes 2D while the effective coupling constant g_2 for the interparticle interaction depends closely on the motion of the atoms in the axial direction [16]. As shown in [16],

$$g_2 = \frac{4\pi \hbar^2}{m} \left[\sqrt{2\pi} \left(\frac{l_z}{a_s} \right) + \ln \left(\frac{B}{\pi} \frac{\hbar \omega_z}{\epsilon} \right) \right]^{-1}$$

$[B \simeq 0.915, \epsilon$ is the typical energy for the relative motion of the atoms and a_s is the (3D) s-wave scattering length] where $\epsilon \ll \hbar \omega_z$ must hold. This regime (called quasi-2D regime in [16]), relevant here, seems to be accessible to experiments [2,16]. Then, (9) holds provided a_s is tuned to the negative value

$$a_s = -\sqrt{2\pi} l_z \left[\frac{8\pi \eta}{1 - \frac{\Lambda_{st}}{\Lambda}} + \ln \left(\frac{B}{\pi} \frac{\hbar \omega_z}{\epsilon} \right) \right]^{-1}.$$

Typical values obtained for $\hbar \omega_z / \epsilon \sim \mathcal{O}(10^2 - 10^3)$ are $|a_s| / l_z \sim \mathcal{O}(10^{-1})$ [resp $|a_s| / l_z \sim \mathcal{O}(10^{-1} - 10^{-2})$] for a CS (resp MCS) statistical interaction (assuming possibly reachable current quantum Hall states and $\Lambda_{st} \lesssim \frac{1}{2} \Lambda$). Let us discuss the possible effect of the inclusion of the next to leading order interaction terms $\sim |\phi|^6$ on the stability of the system [23], assuming that (9) holds. In the CS case, stability requires $g_3 > 0$, as indicated in [11]. In the MCS case, the $|\phi|^6$ term in $U(\phi)$ receives an additional *positive* (repulsive) contribution coming from the infinite sum in (7). Then, stability in the MCS case is obtained provided $g_3 + 1/8m^2 \eta^3 e^2 > 0$. The actual computation of g_3, which here must correspond to the quasi-2D regime, is still lacking and would need to extend the analysis reported in [16] to the case of higher order interactions within the quasi-2D regime.

QHF, usually described in the low energy regime by a CSLG theory [17], can also be described by a MCSLG theory as defined in (3) and (4). To compare to the CS case [17], it is convenient to add a chemical potential term to (3), $S' = S + \int_x \mu \phi^\dagger \phi (\mu > 0)$ and set $U(\phi) = 0$ in all. Equation (5a) becomes $iD_0 \phi + (1/2m) D_i D_i \phi - 2g_2 (\phi^\dagger \phi) \phi = \mu \phi$ while the relevant anyonic configurations are still obtained when $f_\mu = -(1/\eta) \mathcal{J}_\mu$ solving (5b). The equations of motion admit the uniform (constant) density solution minimizing the energy $H' = H - \int_x \mu \rho$, given by

$$\phi = \sqrt{n_0}, \tag{12a}$$

$$\mathbf{a} + \mathbf{A} = 0, \tag{12b}$$

$$a_0 = 0, \tag{12c}$$

$$n_0 = \frac{\mu}{2\hat{g}_2}, \tag{12d}$$

where $\hat{g}_2 = g_2 - 1/2\eta^2 e^2$, similar to the usual uniform density solution supported by the CSLG action for QHF [17]. Equation (12b) implies that the external magnetic field is screened by the statistical magnetic field. This combined with $f_0 = -(1/\eta) \rho$, yields $\nu = 2\pi \eta$ where ν is the filling factor. When statistical transmutation occurs as expected in Hall systems,

$\nu = 1/2k+1$, $k \geq 0$. Introducing polar coordinates (r, θ) and setting $\phi = \sqrt{\rho} e^{in\theta}$, $\mathbf{a} = \mathbf{e}_\theta[\alpha(r)/r]$, $a_0 = a_0(r)$ $[\mathbf{e}_\theta = (\sin\theta, -\cos\theta)$, $n \in \mathbb{Z}$ is the winding number], one finds that H' admits static finite energy vortex solutions satisfying the boundary conditions $\rho \sim n_0$, $\alpha(r) \sim m\omega r^2$ for $r \to \infty$, $\rho \to 0$, $\alpha(r) \to -n$ for $r \to 0$. The vortex effective magnetic flux $\Phi \equiv \int_\mathbf{x}(f_0 + F_0)$ is then $\Phi = -2\pi n(F_0 = \epsilon_{ij}\partial^i A^j)$. Using $n_0 = \eta F_0$, $f_0 = -\rho/\eta$, Φ is related formally to the "vortex charge" $Q = \int_\mathbf{x}(\rho - n_0)$ through $Q = -\eta\Phi = 2\pi\eta n$. When statistical transmutation [10] occurs, $Q = n/2k+1$ so that the vortices can be interpreted as (the analog of) the fractionally charged Laughlin quasiparticles, as it is the case in the CS description of QHF [17]. By expanding S' around (12) up to the quadratic order in the fields fluctuations, fixing the gauge freedom, integrating out the fluctuations of a_μ, one obtains the low energy effective action for the matter. It yields the following dispersion relation with a gap

$$\omega_0^2(\mathbf{p}) = \left(\frac{\mathbf{p}^4}{4m^2} + \mathbf{p}^2\frac{\mu}{m} + \frac{n_0^2}{m^2\eta^2}\right)(1 - 2\Theta_0^2) + \mathbf{p}^2(\Theta_0 + 2\Theta_0^2),$$

$$(13)$$

$\Theta_0 = n_0/m\eta^2 e^2$ $[\omega_0^2(\mathbf{p}) \simeq \mathbf{p}^4/4m^2 + \mathbf{p}^2(\mu/m) + n_0^2/m^2\eta^2$ since $\Theta_0 \ll 1]$, so that S' describes an incompressible fluid.

Although CS and MCS frameworks both permit one to describe the expected low energy properties of QHF, MCS anyons must be considered as anyonic objects distinct from the standard CS anyons, since in particular the former feel an additional contact attraction. This can be understood from the mechanism [13] responsible for the formation of each type of these particle-flux anyonic quasiparticles. This reflects itself in each case into the origin of the field-current identity (i), which must hold in order to generate composites with anyonic statistics. Recall that (i) expresses the fact that, for a distribution of point-like particles, only localized (statistical) fields can appear. This is automatically fulfilled in the CS case (see S_1) since the CS statistical interaction has zero range. In the MCS case for which the interaction has finite range, (i) results from an exact cancellation between electric and Pauli-type coupling effects [13], which amounts to fix the Pauli coupling constant to a specific value (see [24]). This leads to the appearance of an additional attractive contact interaction among the resulting MCS anyons which, apart from this, behave essentially as CS anyons. The question is to examine if some effects specific to the MCS framework, in particular those related to the above contact attraction, may be experimentally observed. Clearly, the "electronic" quantum Hall systems are excluded: the fact that the repulsive Coulomb interaction among electrons cannot be manipulated combined with the specific features of the experimentally accessible observables make these systems only sensitive to the properties both shared by CS and MCS anyons in the long wavelength limit. In fast rotating Bose gases, the interaction among atoms can be manipulated, offering a way to study the possible stabilization of initially attractive Bose gases in a FQHE state through their anyonic excitations. As we have shown, stability may be somehow conditioned by the actual nature of the statistical interaction. Basically, anyons of MCS origin loose their ability to stabilize the system as the range of the MCS interaction grows. An interesting proposal to measure the statistical phase of anyons has been presented in [25]. This, combined with an experimental implementation of the analysis presented here may provide a deeper insight into the physical features of the anyonic composite quasiparticles.

[1] M. H. Anderson *et al.*, Science **269**, 198 (1995); K. B. Davis, M. Q. Mewes, M. R. Andrews, N. J. Van Druten, D. S. Darfee, D. M. Kurn, and W. Ketterle, Phys. Rev. Lett. **75**, 3969 (1995); W. Ketterle, Phys. Today **52**, 30 (2000).

[2] A. Görlitz *et al.*, Phys. Rev. Lett. **87**, 130402 (2001).

[3] M. R. Matthews, B. P. Anderson, P. C. Haljin, D. S. Hall, C. E. Wieman, and E. A. Cornell, Phys. Rev. Lett. **83**, 2498 (1999).

[4] K. W. Madison, F. Chevy, W. Wohlleben, J. Dalibard, Phys. Rev. Lett. **84**, 806 (2000); F. Chevy, K. W. Madison, and J. Dalibard, Phys. Rev. Lett. **85**, 2223 (2000); J. R. Abo-Shaeer, C. Raman, J. M. Vogels, and W. Ketterle, Science **292**, 476 (2001).

[5] *The Quantum Hall Effect*, edited by R. E. Prange and S. M. Girvin (Springer-Verlag, Berlin, 1990).

[6] When the trapping and centrifugal potential balance each other, the bosons feel only the Coriolis force. Then, the system is formally equivalent to that of 2D (bosonic) particles in a magnetic field.

[7] N. K. Wilkin, J. M. F. Gunn, R. A. Smith, Phys. Rev. Lett. **80**, 2265 (1998); N. R. Cooper and N. K. Wilkin, Phys. Rev. B **60**, R16279 (1999); N. K. Wilkin and J. M. F. Gunn, Phys. Rev. Lett. **84**, 6 (2000); N. R. Cooper, N. K. Wilkin, and J. M. F. Gunn, *ibid.* **87**, 120405 (2001).

[8] A. D. Jackson, G. M. Kavoulakis, B. M. Helson, and S. M. Reimann, Phys. Rev. Lett. **86**, (2001) 945; B. Paredes, P. Fedichev, J. I. Cirac, and P. Zoller, *ibid.* **87**, 010402 (2001); T.-L. Ho, *ibid.* **87**, 060403 (2001); U. R. Fischer and G. Baym, *ibid.* **90**, 140402 (2003).

[9] M. Manninen, S. Viefers, M. Koskinen, and S. M. Reimann, Phys. Rev. B **64**, 245322 (2001); J. Sinova, C. B. Hanna, and A. H. MacDonald, Phys. Rev. Lett. **89**, 030403 (2002); V. Schweikhard, I. Coddington, P. Engles, V. P. Mogendorff, and E. A. Cornell, *ibid.* **92**, 040404 (2004); N. Regnault, and T. Jolicoeur, *ibid.* **91**, 030402 (2003); Phys. Rev. B **69**, 235309 (2004).

[10] A. Lerda, *Anyons: Quantum Mechanics of Particles with Fractional Statistics*, Lecture Notes in Physics, Vol. 14 (1992); A. Khare, *Fractional Statistics and Quantum Theory* (World Scientific, Singapore, 1997).

[11] U. R. Fischer, Phys. Rev. Lett. **93**, 160403 (2004); see also U. R. Fischer, *ibid.* **94**, 208904 (2005).

[12] In the thermodynamical limit, a system of attractive untrapped bosons is unstable against collapse; see P. Noziéres and D. Saint James, J. Phys. (Paris) **43**, 1133 (1982).

[13] J. Stern, Phys. Lett. B **265**, 119 (1991); I. I. Kogan, *ibid.* **262**, 83 (1991); Y. Georgelin and J. C. Wallet, Mod. Phys. Lett. A **7**, 1149 (1992); Phys. Rev. D **50**, 6610 (1994).

[14] F. Dalfovo, S. Giorgini, L. P. Pitaevskii, and S. Stringari, Rev. Mod. Phys. **71**, 463 (1999); A. J. Leggett, *ibid.* **73**, 307 (2001).

[15] Our conventions are $\hbar=c=1$ when not explicitly written; indices $A,B=1,\ldots,N$ label bosons; $i,j=1,2$ label coordinates in the plane, $\mathbf{x}\equiv(x_1,x_2)$, $\epsilon_{12}=1$, $\mathbf{x}_{AB}=\mathbf{x}_A-\mathbf{x}_B$; space-time metric is $g_{\mu\nu}=\mathrm{diag}(+,-,-)$, $\mu,\nu,\ldots=0,1,2$, $\epsilon_{012}=1$, $\partial_i^{(A)}\equiv\partial/\partial x_A^i$, $\partial_0\equiv\partial_t$, $\int_x\equiv\int dt d\mathbf{x}$, $\int_{\mathbf{x}}\equiv\int d\mathbf{x}$, $\int_t\equiv\int dt$.

[16] See, e.g., D. S. Petrov, D. M. Gangardt, and G. V. Shlyapnikov, J. Phys. IV **116**, 5 (2004) and references therein; See also M. D. Lee, S. A. Morgan, M. J. Davis, and K. Burnett, Phys. Rev. A **65**, 043617 (2002).

[17] S.-C. Zhang, T. H. Hansson, and S. Kivelson, Phys. Rev. Lett. **62**, 82 (1989).

[18] S. Deser, R. Jackiw, and S. Templeton, Ann. Phys. **140**, 372 (1982).

[19] It can be realized that the general (static) solutions of (5b) such that $f_\mu\neq-(1/\eta)\mathcal{J}_\mu$ have infinite energy.

[20] S. Latinski, and D. Sorokin, Mod. Phys. Lett. A **6**, 3525 (1991).

[21] $\Lambda/\Lambda_{st}\ll1$ corresponds to a long ranged statistical interaction which is unlikely. It is related either to the small e^2 regime (MCS theory in strong coupling regime) for which a description of anyons as given by (3) is questionable or to $\eta\ll1$ (large denominator FQHE state).

[22] Z. F. Ezawa, M. Hotta, and A. Iwazaki, Phys. Rev. D **44**, 452 (1991).

[23] Inclusion of small $|\phi|^6$ terms would not alter noticeably the shape of the density profiles discussed above; see also 2nd of [11].

[24] For a generic Pauli coupling $\alpha f_\mu J^\mu$ (see S_2), a (static) pointlike particle produces electric and magnetic fields spread over a domain of size $\sim\Lambda_{st}$; when $\alpha=-1/\eta e^2$, the magnetic field becomes localized while the electric field is totally screened by Pauli coupling contributions.

[25] See B. Paredes, J. J. Garcia-Ripoll, P. Zoller, J. I. Cirac, J. Phys. IV **116**, 135 (2004), and references therein.