Experimental construction of optical multiqubit cluster states from Bell states

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Cluster states serve as the central physical resource for one-way quantum computing. We here present an experimental demonstration of the efficient cluster-state construction scheme proposed by Browne and Rudolph. In our experiment, three-photon cluster states with high purity are created from two Bell states via a qubit "fusion" operation, showing a strong violation of a three-particle Mermin inequality of $|\langle A\rangle| = 3.10\pm 0.03$. In addition, the entanglement properties of the cluster states are examined under σ_z and σ_x measurements on a qubit. This scheme could be extended to any desired number of qubits and represents an essential step for the optical one-way quantum computation.

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There has been considerable interest in optical approaches to quantum computation due to the photon's intrinsic robustness against decoherence and the relative ease with which it can be manipulated with high precision. Remarkably, by exploiting the nonlinearity induced by measurement, Knill, Laflamme, and Milburn (KLM) pioneeringly showed that efficient quantum computation is possible with linear optics [1]. Impressive as it is, however, significant obstacles still stand in the way of scalable quantum computation with the KLM scheme mainly due to its requirement of enormous resource and difficulty in experimental implementation.

Surprisingly, Raussendorf and Briegel proposed a conceptually new quantum computation model [2]. They showed that universal quantum computation can be done by one-qubit measurements on a specific entangled state, the cluster state [3]. With the cluster states prepared, information is then written onto, processed, and read out from the cluster by one-particle measurement only. Underlying this unique computation model are the cluster states, serving as the entire physical resource. Much effort is being made towards the construction of cluster states. Proposals and experiments using neutral atoms trapped in the periodic potential of an optical lattice with controlled collisions between neighboring atoms have been reported [4]. In the optical regime, Nielsen [5] showed that optical cluster states can be efficiently created using nondeterministic gates from the KLM scheme. Recently, a much more simple and powerful linear optical quantum computation scheme was proposed by Browne and Rudolph [6]. They showed how cluster states may be efficiently generated from pairs of maximally entangled photons in a scalable way using a technique called qubit "fusion." This scheme is very attractive since it completely moves away from the use of teleportation to boost the success probability of nondeterministic gate and is significantly less demanding not only in resource requirement, but also in complexity of experimental implementation.

While all the previous experiments [7–10] followed the KLM way, we here take a first step towards the cluster state model of quantum computation. In this letter, we report the first experimental demonstration of constructing linear multiqubit cluster states from pairs of Bell states. As the most fundamental step in Browne and Rudolph's scheme, we produced a three-photon cluster states by a Type-I fusion of two pairs of maximally polarized entangled photons. We then provide sufficient experimental evidence to confirm that the cluster states we obtained possess genuine three-particle entanglement with high purity, thus excluding any possibility of hybrid models [11] and opening the possibility to experimentally investigate various quantum information processing schemes with linear optics. We also examined the entanglement properties of the remaining two photons after a measurement on one of its qubits in basis $\{\sigma_z,\sigma_x\}$.

Let us first review Browne and Rudolph's efficient linear optical quantum computation scheme. The primary resource used are two photon Bell states which are relatively easier to obtain, e.g., probabilistically from single photons. Given a supply of Bell states, arbitrarily long linear cluster states can be generated efficiently using an operation called Type-I qubit fusion [see Fig. 1(a)]. This operation, the same as a parity check [12,13], is implemented by mixing two photons in a polarizing beam splitter (PBS) and accepting the output in mode 3′ only for those cases in which polarization-sensitive detector $D2$ receives one and only one photon. Here is the most simple and fundamental case: from two pairs of Bell states to a three-qubit cluster state. A two-qubit cluster states encoded in polarization which is equivalent to a Bell state can be written as

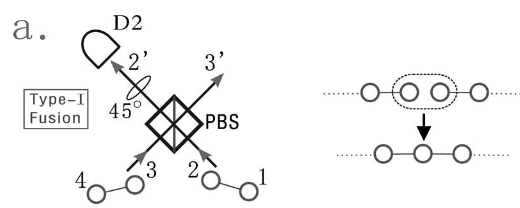

FIG. 1. (a). The nondeterministic qubit fusion operation. $D2$ stand for a polarization discriminating photon detector. Two photons of different spatial modes are mixed in a PBS, the output in mode 3′ is accepted only when $D2$ receives exactly one photon. (b). A successful Type-I fusion combines two linear clusters of length n and m into a new one of length $(n+m-1)$.

$$|\phi\rangle_{ij} = |H\rangle_i|H\rangle_j + |V\rangle_i|V\rangle_j =_{l.u.} |H\rangle_i|+\rangle_j + |V\rangle_i|-\rangle_j.$$

Here H and V denote horizontal and vertical polarizations, $|\pm\rangle = 1/\sqrt{2}|H\rangle \pm |V\rangle$; i and j index the photon's spatial modes. Given two pairs of Bell states, $|\phi\rangle_{12}$ and $|\phi\rangle_{34}$, we then superpose photons 2 and 3 in a PBS. Since the PBS transmits horizontal and reflects vertical polarization, detecting one and only one photon in $D2$ makes sure that both photon 2 and photon 3 are horizontally polarized or vertically polarized. By a further measurement performing on output $2'$ in the + and or − basis, photon 1, 3, 4 will be in a three-photon cluster state

$$|\phi^{\pm}\rangle_{134} = |+\rangle_1|H\rangle_3|+\rangle_4 \pm |-\rangle_1|V\rangle_3|-\rangle_4$$

depending on the result of detector $D2$. This is equivalent to a three-qubit Greenberger-Horne-Zeilinger (GHZ) state [14] under local unitary transformation.

As has been discussed in detail by Browne and Rudolph in Ref. [6], the creation of three-photon cluster states is the most fundamental element in their linear optical quantum computation scheme. With a success probability of 50%, Type-I fusion combines two linear cluster state of lengths n and m into a new one of length $(n+m-1)$. Any linear cluster states with desired length can be efficiently created with this method given the necessary resource of Bell states. Further, using a similar technique called Type-II fusion, which is nothing more than a Type-I fusion with an additional σ_x measurement on the qubit leaving the device, we can also generate arbitrary two-dimensional clusters from those obtained linear cluster states, leading to the construction of a powerful square lattice cluster state that would allow the simulation of an arbitrary quantum network directly by single-qubit measurements alone [2,6].

We then implement an experimental demonstration of the core module of this proposal: construction of a three-qubit cluster state from two Bell states. Obviously, given perfect photon pairs and number-discriminating photon detectors, the scheme described above can be realized optimally without post selection. However, we note that, although these techniques are not yet available, one can still perform an experimental demonstration based on post selection.

A schematic drawing of our experimental setup is shown in Fig. 2. An ultraviolet pulsed laser from a mode-locked Ti:sapphire laser (pump power 450 mW, center wavelength 394 nm, pulse duration 200 fs, repetition rate 76 MHz) passed through β-barium borate (BBO) crystal twice to generate two maximally entangled photon pairs in modes 1-2 and mode 3-4. After proper birefringence compensation and local unitary transformations with half wave plate (HWP) and nonlinear crystals, two two-qubit cluster states are produced as the primary source with a visibility of 94%.

We then superpose the photons 2 and 3 at the PBS. Their path lengths are adjusted such that they arrive simultaneously. To achieve good spatial and temporal overlap, the outputs are spectrally filtered (FWHW=2.8 nm) and monitored by fiber-coupled single-photon detectors. The filtering process stretches the coherence time to about 740 fs, substantially larger than the pump pulse duration [15]. There

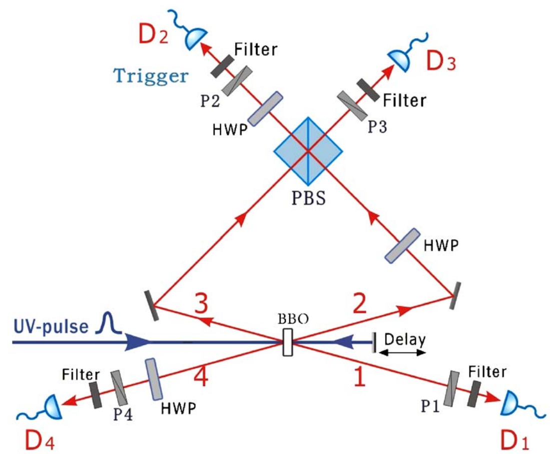

FIG. 2. (Color online) Experimental setup for generating a three-photon cluster state from two pairs of maximally entangled photons produced by Type-II spontaneous parametric down conversion. The half wave plate (HWP) used in paths 2 and 4 are used to locally transform the photon from H/V basis to $+/-$ basis and the four polarizers P1, P2, P3, P4 are used for necessary polarization analysis. In the experiment, we managed to obtain an average twofold coincidence of 2.2×10^4 s^{-1}.

processes effectively erase any possibility of distinguishing the two photons and thus lead to interference.

To experimentally verify the three-photon cluster state, we first show that, upon a trigger of $D2$, the threefold coincidence only includes $+H+$ and $-V-$ components, but no others. This is done by comparing the counts of all eight possible polarization combination $+H+,\cdots,-V-$. The experimental results in the $(+/-,H/V,+/-)$ basis [see Fig. 3(a)] show that the signal-to-noise ratio defined as the ratio of any of the desired threefold events ($+H+$ and $-V-$) to any of the six other undesired ones is about 29:1 on average. Second, we further perform a polarization measurement in the "diagonal" basis $(H/V,+/-,H/V)$ to demonstrate that the two terms $+H+$ and $-V-$ are indeed in a coherent superposition. Transforming $|\phi^{+}\rangle_{134}$ into the diagonal basis $(H/V,+/-,H/V)$, we note that only components $(H+H, H-V, V+V, V-H)$ occur, other combinations $(H-H, H-V, V+H, V-V)$ do not occur. As a test for coherence, we compare the $H+H$ and $H-H$ count rates as a function of the pump delay mirror position. We see in Fig. 3(b) that, at zero delay, the unwanted component is suppressed with a visibility of 0.78 ± 0.03, which is sufficient to violate the Bell-type inequality imposed by local realism [16].

However, the data presented above are still not sufficient enough to confirm genuine entanglement of all three particles. This has been shown in Ref. [11] that such data can be explained by a hybrid model in which only less than three particles are entangled. In order to exclude such a hybrid model and produce the three-photon GHZ state in the form $|HHH\rangle+|VVV\rangle$, we first did a local transformation of the cluster state and performed four series of measurements in the $\sigma_x\sigma_x\sigma_x$, $\sigma_x\sigma_y\sigma_y$, $\sigma_y\sigma_y\sigma_x$ and $\sigma_y\sigma_x\sigma_y$ direction. We then tested the three-particle Bell inequality of the form derived by Mermin [17] with the result

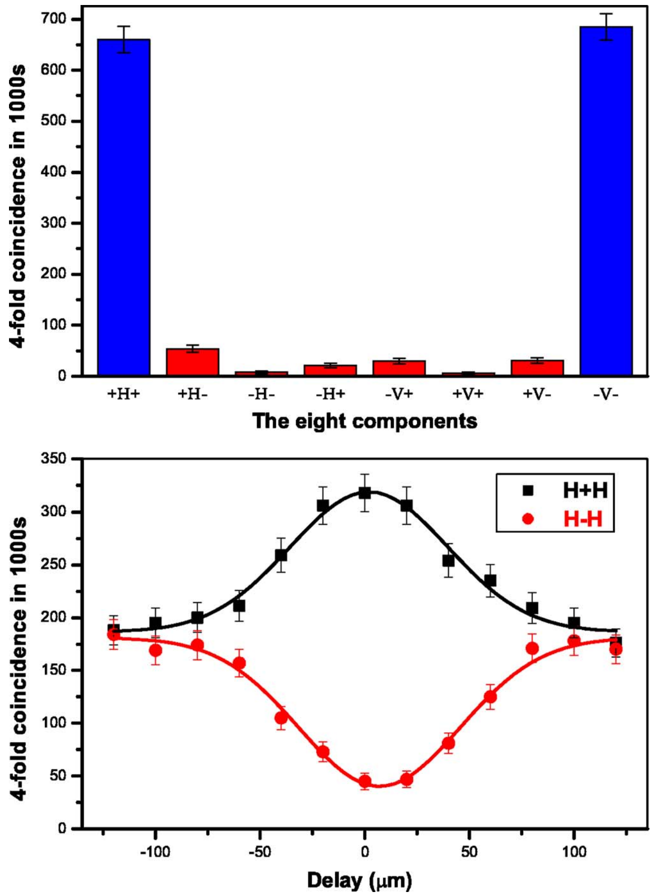

FIG. 3. (Color online) (a) Experimental data under eight different polarization settings. Two desired terms $+H+$ and $-V-$ are prominent while other six are strongly depressed on average to around 3% of these desired ones. (b) Experimental data in the diagonal basis showing the two components are in a coherent superposition. Maximum interference occurs at zero delay between the two incoming photons.

$$|\langle A\rangle| = 3.10 \pm 0.03,$$

where

$$A = \sigma_x\sigma_y\sigma_y + \sigma_y\sigma_x\sigma_y + \sigma_y\sigma_y\sigma_x - \sigma_x\sigma_x\sigma_x.$$

Clearly this shows a strong violation of the Mermin inequality: $|\langle A\rangle| \leq 2$ imposed by local realism by 34 standard deviations, which also confirms that the state we produced are indeed genuine tripartite entanglement [18]. Compared with the previous experiments of Bouwmeester *et al.* [19] and Pan *et al.* [20], our three-photon entanglement source distinguishes itself from those not only by its intensity, which is about 40 times more brighter, but also by its high purity, which would make it possible to perform a lot of quantum information processing tasks, such as quantum secreting sharing and the third-man cryptography [21,22].

An interesting entanglement property of a linear cluster state is that, measurements in σ_z and σ_x basis on a qubit of a cluster state have totally different effects on the remaining qubits. It has been shown in Refs. [2,6] that a σ_z eigenbasis measurement removes the qubit from the cluster and breaks all bond between that qubit and the rest of the cluster; while a σ_x measurement on a linear cluster removes the measured qubit and combines the adjacent qubits into a redundantly encoded qubit. These properties are critical for understanding the cluster-state construction [6] and the cluster model of quantum computation [2]. We thus examined the entanglement properties of the two remaining photons under a σ_z measurement and a σ_x measurement on the "mid" qubit, i.e., upon the trigger of detecting a $|H\rangle$ photon or a $|+\rangle$ photon, respectively, in $D3$. We analyzed the polarization correlations between photons 1 and 4 by keeping polarizer 1 fixed and varying the angle of polarizer 4. The experimental results are shown in Figs. 4(a) and 4(b) corresponding to measurement in the σ_z and σ_x basis, respectively. As Fig. 4 shows, the experimental data are in good agreement with theoretical prediction.

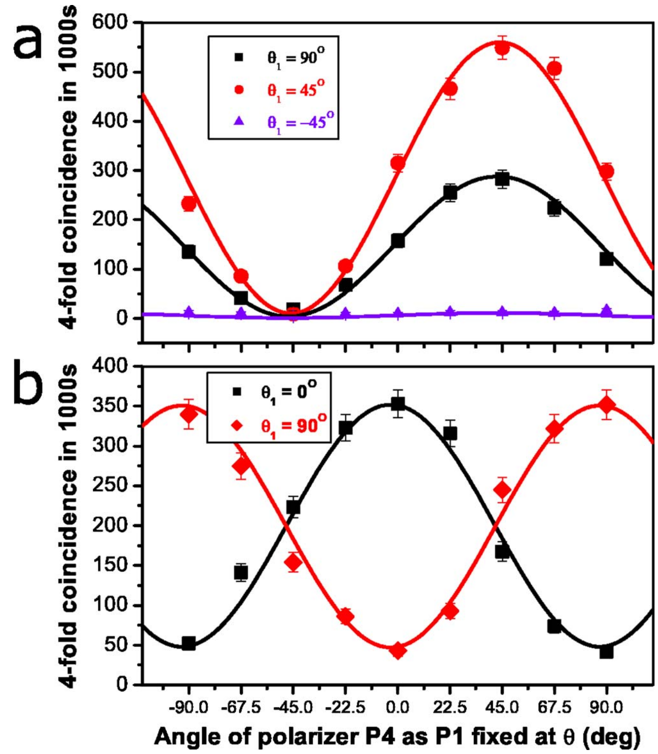

FIG. 4. (Color online) Experimental results showing polarization correlation between photon 1 and 4, under a σ_z and σ_x measurement of photon 3. (a). Data obtained under a σ_z measurement. The coincident counts when $P1$ was set at $-45°$ were so strongly suppressed that they are close to zero; while counts when $P1$ was set at 45° are the most prominent, twice as when $P1$ was set at 90° and with a fringe of visibility 0.93 ± 0.03. The experimental results clearly agree that the state obtained is $|+\rangle_1 \otimes |+\rangle_4$. (b). Data obtained under a σ_x measurement. The two sinusoidal curves with a visibility of 0.79 ± 0.03 demonstrate that photons 1 and 4 are in an entangled state as $|+\rangle_1|+\rangle_4 + |-\rangle_1|-\rangle_4$.

In summary, we have demonstrated the process of constructing a linear three-photon cluster state from two Bell states. In principle these method can be extended to any desired number of particles given enough Bell states, holding the promise of constructing an optical one-way quantum computer efficiently. It should be noted that in the experiment we achieved a high fidelity taking the advantage of simple linear optical devices, which would make it more favorable for reliable quantum computation. The three-photon GHZ state we created in this experiment could be ready to be used for implementation of some quantum communication schemes like quantum secret sharing and third-man cryptography. After verification of the obtained three-photon cluster state, we also demonstrate that a σ_z measurement on a qubit of the obtained three-photon cluster state breaks the bond between the remaining photons; while a σ_x measurement does not, but instead combines them into a redundantly encoded qubit. Possible future work could include production and characterization of larger cluster state and use the obtained cluster state to implement some interesting quantum computation tasks. By exploiting the photon's intrinsic flying nature, we could also envision that this experimental technique maybe applicable in distributed quantum computation and "quantum internet" [23]. When combined with recent advances in trapping neutral atoms in an optical lattice [4] and atom-photon entanglement [23], we could also dream of a photon-assisted atomic one-way quantum computer that can efficiently implement distributed quantum information processing. We expect this work will stimulate further work towards feasible quantum computation. In any event, the experimental results present here may provide a first step towards that goal.

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