Simple scheme for generating an *n*-qubit *W* state in cavity QED

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A simple scheme is proposed to generate an *n*-qubit *W* state in cavity QED. Conditioned on no photon leakage from the cavity, the *n*-qubit *W* state can be generated by resonant interaction between atoms and the cavity if the cavity is initially prepared in the single-photon state and all the atoms are in the ground states. We check the time evolution of the system involving decay, and show that, since the required interaction time is very short, with present cavity QED techniques, the success probability of our scheme is almost unity.

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I. INTRODUCTION

Entanglement is one of the most striking features of quantum mechanics. Entangled states not only help quantum mechanics win over local hidden theory [1], but also have applications in quantum teleportation [2], quantum cryptography [3], quantum dense coding [4], high-precision frequency measurement [5], and so on. Recently, much interest has been devoted to multiparticle entangled states, the classification and properties of which are more complex than that of the bipartite entangled states. In Ref. [6], the authors show that there exist two inequivalent classes of tripartite entangled states, i.e., the Greenberger-Horne-Zeilinger (GHZ) class [7] and the *W* class. They cannot be converted to each other even under stochastic local operations and classical communication. One of the interesting properties of the *W* state [such as $1/\sqrt{3}(|001\rangle+|010\rangle+|100\rangle)$] is that if one particle is traced out, there remains entanglement of the remaining two particles, or if one particle is measured in basis $\{|0\rangle, |1\rangle\}$, then the state of remained two particles is either in a maximally entangled state or in a product state.

Four-particle entanglement in ion trap [8] and three-particle entanglement [9] in cavity QED have been demonstrated experimentally. More recently, five-photon entanglement has also been reported [10]. As far as we konw, a three-particle *W* state in ion trap [11] has been realized, but there is no report of experimental realization of the *W* state in cavity QED, while there have been several papers discussing the preparation of *W* state in cavtiy QED [12–15]. In Ref. [12], atoms, interacting with a nonresonant cavity, can be entangled through virtual excitation of the cavity mode, which can loosen the quality requirement on the cavity. However, the scheme requires that the atom-cavity coupling strength be much smaller than the detuning between atomic transition frequency and cavity frequency, which restricts the operation speed. Reference [13] proposed how to generate both three-atom *W* state and three-cavity *W* state. In the case of the three-atom *W* state (or three-cavity *W* state), each atom (or an atom) interacts with the cavity (or each cavity) sequentially with different interaction time. In Ref. [15], the *W* state is investigated by adiabatic passages and decoherence-free subspace.

The general form of the *W* state for *n* particles is $W_n=1/\sqrt{n}|n-1,1\rangle$, where $|n-1,1\rangle$ denotes all the totally symmetric states involving $n-1$ zeros and 1 one. In this paper, we present an alternative scheme to realize an *n*-qubit *W* state via cavity QED. The scheme only requires that *n* identical atoms interact simultaneously and resonantly with a single-mode, high-*Q* microwave cavity. After an appropriate interaction time, the atoms will be entangled in a *W* state provided that there is no photon leakage from the cavity. Compared with Ref. [12], our scheme is much faster due to the resonant interaction instead of the virtual excitation of the cavity mode. Moreover, after all atoms have been prepared in ground states and cavity in a one-photon state, one step of our implementation can achieve the *W* state, which is much simpler and more straightforword than in Ref. [13], in which the preparation of three-atom *W* state needs the three atoms going through the cavity one by one, with each atom interacting with the cavity for a different period of time. Furthermore, the success probability in our scheme increases with the atom number, in contrast to the decreasing probability of success in Ref. [12].

II. ENTANGLEMENT WITHOUT DECAY

We first consider an ideal model of *n* identical two-level atoms interacting resonantly with a single-mode cavity field in the absence of any decay. The Hamiltonian (assuming $\hbar=1$) in the interaction picture reads

$$H=\sum_{j=1}^{n} g(a^{\dagger}\sigma_j^{-}+a\sigma_j^{+}), \tag{1}$$

where $\sigma_j^{+}=|1\rangle_{jj}\langle 0|$, $\sigma_j^{-}=|0\rangle_{jj}\langle 1|$, with $|1\rangle_j$ ($|0\rangle_j$) being the excited (ground) state of the *j*th atom. *g* is the atom-cavity coupling strength. $a^{\dagger}$, *a* are, respectivly, the creation and annihilation operators for the cavity mode. Assuming that the cavity is initially in a one-photon state $|1\rangle$ and all the *n* atoms

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are in the ground states, i.e., $|0\rangle_{1,2,\ldots,n}$, we have the evolution of the system as follows:

$$\Psi(t) = \cos(\sqrt{n}gt)|0\rangle_{1,2,\ldots,n}|1\rangle - i\sin(\sqrt{n}gt)W_n|0\rangle. \quad (2)$$

By choosing $\sqrt{n}gt=\pi/2$, i.e., $t=\pi/(2g\sqrt{n})$, we can get the n-atom W state. This result can be easily understood from the property of Eq. (1): The excitation number is unchanged. The initial one photon is shared by n atoms and each atom has an equal propability to absorb the photon. As only one atom can succeed, the n-atom W state is naturally generated.

III. ENTANGLEMENT INCLUDING DECAY

We introduce the cavity decay from now on. As long as there is no photon decay from the cavity, the evolution of the system is governed by the non-Hermitian Hamiltonian (assuming $\hbar=1$)

$$H' = H - i\frac{\kappa}{2}a^\dagger a, \quad (3)$$

where κ is the cavity decay rate. If the initial state of the atom-cavity system is $|0\rangle_{1,2,\ldots,n}|1\rangle$, then the evolution of the system is

$$\Psi'(t) = \exp\left(-\frac{\kappa t}{4}\right)\left\{\left[\cos\left(\frac{\lambda t}{4}\right) - \frac{\kappa}{\lambda}\sin\left(\frac{\lambda t}{4}\right)\right]|0\rangle_{1,2,\ldots,n}|1\rangle - i\frac{4g\sqrt{n}}{\lambda}\sin\left(\frac{\lambda t}{4}\right)W_n|0\rangle\right\}, \quad (4)$$

with $\lambda=\sqrt{16ng^2-\kappa^2}$. The probability that no photon has leaked out of the cavity during the evolution is

$$P = \exp\left(-\frac{\kappa t}{2}\right)\left\{\left[\cos\left(\frac{\lambda t}{4}\right) - \frac{\kappa}{\lambda}\sin\left(\frac{\lambda t}{4}\right)\right]^2 + \frac{16ng^2}{\lambda^2}\sin^2\left(\frac{\lambda t}{4}\right)\right\}. \quad (5)$$

If the interaction time τ is chosen to satisfy $\tan(\lambda\tau/4)=\lambda/\kappa$, we can obtain

$$\Psi'(\tau) = -i\frac{4g\sqrt{n}}{\lambda}\exp\left(-\frac{\kappa\tau}{4}\right)\sin\left(\frac{\lambda\tau}{4}\right)W_n|0\rangle, \quad (6)$$

and the corresponding success probability of getting the n-qubit W state is

$$P_{suc} = \frac{16ng^2}{\lambda^2}\exp\left(-\frac{\kappa\tau}{2}\right)\sin^2\left(\frac{\lambda\tau}{4}\right). \quad (7)$$

To carry out our scheme, we first need to prepare the single-photon cavity state, which can be done by sending an auxiliary atom being in state $|1\rangle_{aux}$ to go through an initially vaccum cavity state; after an appropriate interaction time, the resonant interaction between the atom and the cavity will leave the cavity in state $|1\rangle$. Then, all the atoms initially prepared in the ground states (i.e., $|0\rangle_{1,2,\ldots,n}$), are sent into the single-mode high-Q microwave cavity simultaneously with the same velocity. The desired interaction time τ can be achieved by choosing an appropriate velocity of the atoms.

IV. DISCUSSION AND CONCLUSION

We briefly discuss the experimental possibility of our proposal considering Rydberg atoms with principal quantum numbers 50 and 51 and the radiative lifetime $T_r=3\times10^{-2}$ s. The atom-cavity coupling strength is described by $g(z,r)=g_0\exp(-r^2/w^2)\cos(2\pi\nu z/c)$ [16], where $g_0=25\times2\pi$ kHz, w is the waist, r is the atom position in the direction parallel to cavity mirrors with $r=0$ on the cavity axis, ν is the cavity frequency, and z denotes the direction along the cavity axis. If we assume that all the atoms go along the r direction through antinodes, then the atom-cavity coupling strength for each atom is identical, i.e., $g=25\times2\pi$ kHz [17]. For the present case, the cavity frequency ν is 51.1 GHz and the cavity quality factor Q is 3×10^8 [18]. So we have $\kappa=(2\pi\nu)/Q\simeq170.3\times2\pi$ Hz. In Fig. 1, we plot interaction time τ versus the number of atoms n, where the interaction time is of the order of 10^{-6} s (much shorter than T_r, so we can safely neglect atomic spontaneous decay) and is sharply decreased with the increase of the atom number. In Fig. 2, we show the dependence of success probability P_{suc} on cavity decay rate κ in different cases from 3 to 20 atoms. For a certain atom number, the success probability decreases with the increase of κ. For a certain κ, P_{suc} increases with the atom number n, and it may be understood from Fig. 1 that the interaction time is sharply decreased with the increase of the atom number. Given $\kappa\simeq170.3\times2\pi$ Hz, the success probability is above 99.6%. With the data in Ref. [16], there are nine antinodes in the z direction, only up to five of which have the same coupling strength. So we can carry out our proposal with n no more than 5 with their apparatus if we choose all the atoms to go through antinodes.

As mentioned above, there have been some proposals for preparing a W state in cavity QED. For example, atoms in Ref. [12] get entangled in a nonresonant cavity through virtually exciting the cavity mode, in which the implementation time is very long due to the weak effective atom-cavity coupling. Moreover, to prepare an n-atom W state, Ref. [12] needs $(n+1)$ atoms when $n>4$, and the success probability is approximately proportional to the inverse of atom number $(n+1)$. In contrast, our proposal only needs n atoms for an n-atom W state without any auxiliary measurement, and can be carried out with much higher success probability, which could lower the repeated times in experiments. In Ref. [13], it is obvious that the interaction time increases with the atom number for a series of atoms with one-by-one interaction with the cavity; thus if n is too large, the cavity decoherence (harmful to fidelity) should be seriously considered. With more steps to take, the total experimental imperfections would be enlarged, because there would be inevitably some operational imperfection or unexpected harmful effects in each step. Furthermore, due to the different interaction times for different atoms with the cavity, it is somewhat difficult to control the delay time between nearest-neighbor atoms. In contrast, our scheme includes only one step of resonant in-

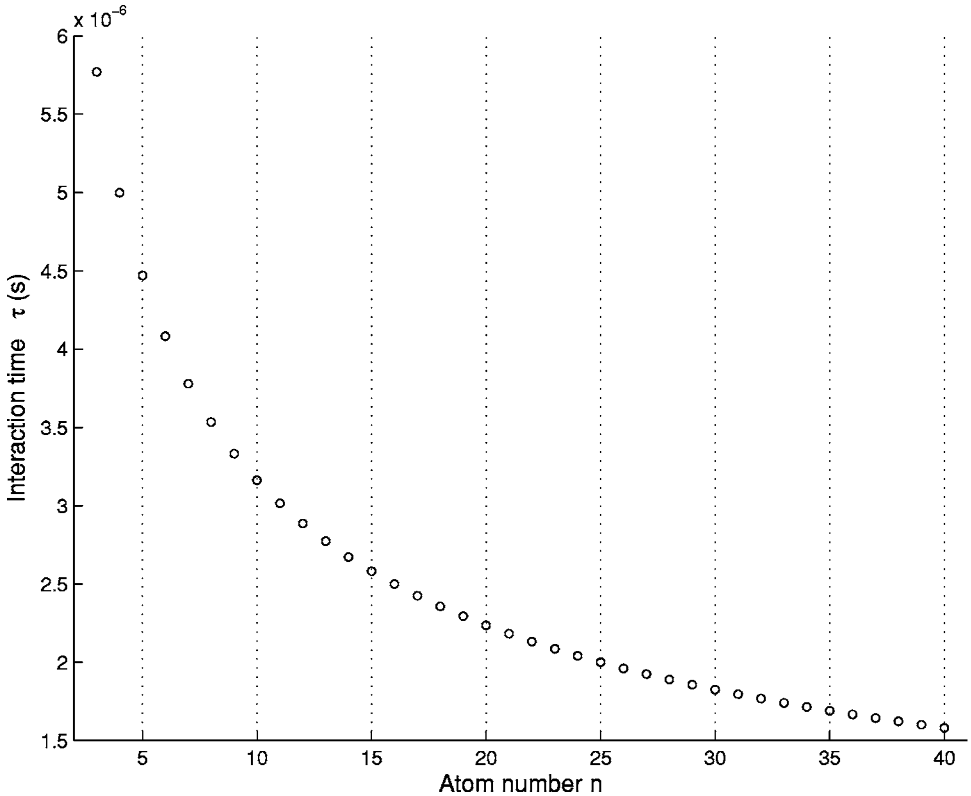

FIG. 1. The interaction time τ versus the number of atoms n with n ranging from 3 to 40, where $g=25\times 2\pi$ kHz, $\kappa=170.3\times 2\pi$ Hz.

teraction. The above-mentioned problems do not exist in our scheme.

The main experimental challenge for our scheme is to control n atoms to go through a cavity simultaneously with the same velocity. However, we don't think this requirement is more stringent experimentally than those in Refs. [12,13]. While we have not yet found any experimental report of more than one atom going through a cavity simultaneously [19], we expect more advanced cavity QED techniques to carry out our scheme.

Before ending our discussion, we have to point out that our analytical treatment of the W state preparation under the consideration of cavity decay is helpful for experimenal work with a comparatively low-Q cavity, as we can accu-

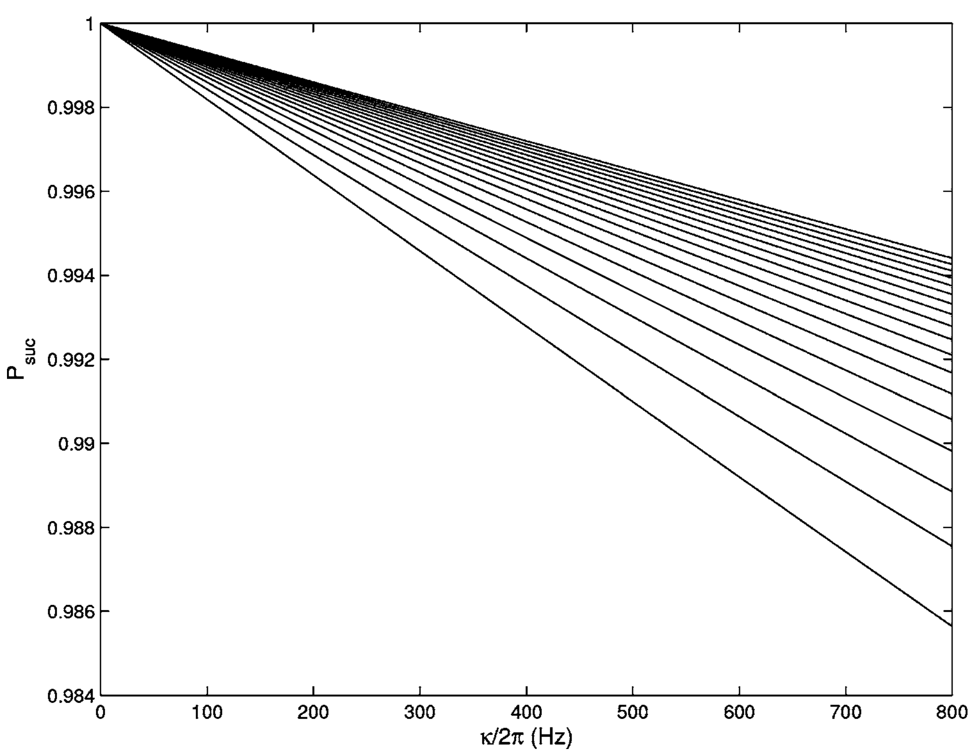

FIG. 2. The dependence of success probability P_{suc} on cavity decay rate κ, where $g=25\times 2\pi$ kHz, and from bottom to top, the curves correspond to the atom number varying from 3 to 20.

rately demonstrate the situation with large cavity decay.

In summary, we have presented a simple scheme for generating an n-qubit W state in cavity QED. Conditioned on no photon leakage from the cavity, the n-qubit W state can be generated very efficiently by resonant interaction between atoms and the cavity with initial single-photon cavity field and all atoms initially in ground states. Our scheme is faster and simpler than previous proposals and is reachable by the near future cavity QED techniques.

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