Heisenberg optical near-field microscope

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It is proven that in molecular interferometry a which-path information can be recorded using detection of spontaneously emitted photons of long wavelengths in the optical near field of the apertures. This does not, however, contradict Heisenberg's uncertainty principle: It is shown that even if the momentum transfer in this experiment is negligible, the entanglement between the detector molecule and photon is responsible for the loss of coherence.

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I. WAVE PARTICLE DUALITY AND ENTANGLEMENT

Wave particle duality [1–4] is considered being at the heart of quantum mechanics and has been discussed by prominent physicists since the early time of Einstein, Bohr, and Heisenberg, to the best of our knowledge. On the basis of Bohr's principle of complementarity [5], it is indeed universally accepted that the observation of two properties, such as the position and momentum, requires mutually exclusive experimental arrangements. However, the origin of such a property is still being debated. This is well illustrated in the canonical situation which is referred in the literature as the double apertures interferometer of Young: One says that the determination of the density of probability in the aperture plane and in the interference plane cannot be obtained by using the same particles. The justification usually presented is based on Heisenberg's uncertainty principle [6,7] and involves an irremediable exchange of momentum between the analyzed object and the measuring apparatus.

Over the past decades the primacy of such recoil argument has been, however, contested in favor of a more general entangling process by which one means the quantum correlation between the quantum system and its environment [8–14]. While Heisenberg emphasized the perturbation of the object, many decoherence theorists would now emphasize that the information transfer from the quantum system to the environment is the key to understanding the loss of fringe visibility in a quantum interference experiment. And in that sense the decoherence theory may also be regarded as a mathematical formulation of Bohr's complementarity. It has been, in particular, emphasized [15] that an atom, after emitting a long wavelength photon in a cavity, located close to one of the two Young's aperture, can generate a recordable which-path information without transfer of a significant momentum. This stirred-up considerable controversy and a debate on the genuine meaning of "momentum transfer" for which-path experiments [16–40].

In counterpart one can imagine the so-called Heisenberg microscope which-path experiment in which a spontaneously emitted photon allows us to define the hole crossed by the atom during is motion. In this configuration, constituting a perfect application of Heisenberg's recoil mechanism, the path determination is only possible if the wavelength λ_γ of the photon is smaller than the separation d between the two pinholes A and B.

Considering this example one can wonder if there is something to add to Heisenberg's explanation.

In the present paper we answer this question positively and we discuss a related Gedanken-experiment in the context of a molecule or an atom interferometry [11,13,22,41–45] using near-field nanooptics. We analyze the required idealized experimental arrangement for obtaining unambiguous path information in Young's double-slit experiment, when near-field optical microscopy is used to detect the emitted photons close to the apertures. The Gedanken experiment is based on two ideas: First, the photon wavelength would be chosen to be much larger than the separation of the two pinholes ($\lambda_\gamma > d$), to guarantee that any recoil and backaction can be safely neglected. Second, we consider the use of a near-field scanning optical microscope (NSOM) [46–48] which is known to achieve a position resolution even down to $\lambda_\gamma/20$. This would then certainly allow us to resolve the position of the molecules inside the double-slit arrangement. And we will discuss in the following how these two basic ideas relate to the concepts of recoil, complementarity, and entanglement.

In the following after a discussion of decoherence by photon emission detected in the optical far field (Secs. II and III A) we are going to discuss the implications of near-field optics for the fringe visibility in molecule interferometry (Sec. III B) in order to prove that a recoil-free which-path determination is possible in the vacuum using long wavelength photons.

II. DECOHERENCE AND SPONTANEOUS EMISSION

In order to simplify we suppose an ideal two level molecule crossing the plane $z=0$ containing the screen with two narrow pinholes of width negligible compared to d. Before the screen ($z \leqslant 0$) the motion of the molecule is described by a wave train infinitely extended in the x,y direction impinging normally on the aperture (see Fig. 1). In the absence of any photon emission, the molecular state behind the holes immediately evolves into a sum of two diffracted waves: $\Psi(\mathbf{r},t)=\Psi_A(\mathbf{r},t)+\Psi_B(\mathbf{r},t)$, where we neglect any further

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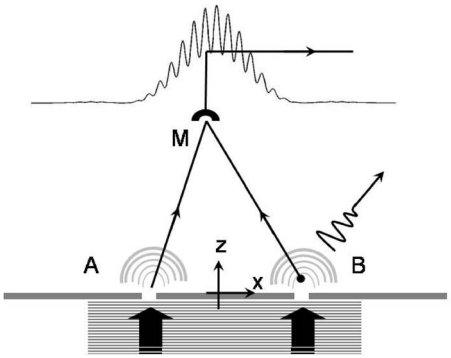

FIG. 1. Sketch of Young's double-hole experiment for molecule emitting spontaneously one photon in the vicinity of the apertures A or B.

phase shifts, such as, for instance, due to the van der Waals interaction with the grating walls. We write

$$\Psi_{A,B}(\mathbf{r},t) = e^{im(\mathbf{r}-\mathbf{r}_{A,B})^2/(2\hbar t)}/t^{3/2}, \tag{1}$$

where $\mathbf{r}_{A,B}=\pm\hat{\mathbf{x}}d/2$ are the holes positions on the x axis, and m the molecule mass. If the incoming wave possesses a sufficiently long spatial coherence length $\delta z \gg \lambda_{dB}$ (λ_{dB} is the de Broglie wavelength for the matter wave) we expect to find molecular interference fringes in a distance from the hole with a visibility $V_0=(I_{\max}-I_{\min})/(I_{\max}+I_{\min})$ close to 1. When we include the possibility of photon emission, while the molecule is still in the vicinity of the double pinhole, one has to write down a joint wave function for both the molecule and the photon. Assuming [49–52] the nonrelativistic approximation (see Appendix A) and that the de-excitation occurs sufficiently fast behind the pinhole this joint wave function will actually be an entangled state which carries the position ambiguity of the molecule over to an ambiguity of the photon state:

$$|\Psi_J\rangle \simeq \int d^3\mathbf{r} \sum_{\mathbf{k},\epsilon} \Psi\left(\mathbf{r}+\frac{\hbar\mathbf{k}t}{m},t\right) \times g_{\mathbf{k},\epsilon}\frac{e^{-i\omega t}}{\omega-\omega_\gamma+i\Gamma/2}\cdot e^{-i\mathbf{k}\cdot\mathbf{r}}|\mathbf{k},\epsilon,\mathbf{r}\rangle. \tag{2}$$

Here we used the transition energy $\hbar\omega_\gamma$ and the life time of the excited state $\Gamma^{-1}\ll t$. Photon states are expanded in the plane wave basis using the notation of [53] and $g_{\mathbf{k},\epsilon}$ is a coupling constant [53]. Inserting $\Psi_{A,B}$ into Eq. (1) and neglecting terms in k^2 leads to

$$|\Psi_J\rangle \simeq \int d^3\mathbf{r}[\Psi_A(\mathbf{r},t)|\gamma_A(t)\rangle + \Psi_B(\mathbf{r},t)|\gamma_B(t)\rangle]|\mathbf{r}\rangle, \tag{3}$$

where

$$|\gamma_{A,B}(t)\rangle = \sum_{\mathbf{k},\epsilon} g_{\mathbf{k},\epsilon}\frac{e^{-i\omega t}}{\omega-\omega_\gamma+i\Gamma/2}e^{-i\mathbf{k}\cdot\mathbf{r}_{A,B}}|\mathbf{k},\epsilon\rangle \tag{4}$$

represents the photon which is produced by a pointlike radiating dipole $\boldsymbol{\mu}(t)=\boldsymbol{\mu}_{ge}e^{-i\omega_\gamma t}e^{-\Gamma t/2}$ located in A or B if $\boldsymbol{\mu}_{ge}$ is the transition dipole. The molecular intensity collected at the screen being proportional to $G^{(1)}(\mathbf{r},t)=\mathrm{Tr}[\hat{\rho}|\mathbf{r}\rangle\langle\mathbf{r}|]$ (with the density operator $\hat{\rho}=|\Psi_J\rangle\langle\Psi_J|$) we deduce

$$G^{(1)}(\mathbf{r},t) \propto 1+\mathcal{V}(k_\gamma d)\cos[mxd/(\hbar t)], \tag{5}$$

where the fringe visibility

$$|\mathcal{V}(k_\gamma d)| = (G^1_{\max}-G^1_{\min})/(G^1_{\max}+G^1_{\min}) \tag{6}$$

is defined by

$$\mathcal{V}(k_\gamma d) = \frac{\mathcal{F}(k_\gamma\mathbf{d})}{\mathcal{F}(0)}, \tag{7}$$

with

$$\mathcal{F}(k_\gamma\mathbf{d}) = \int e^{i\mathbf{k}\cdot\mathbf{d}}d^3\mathbf{k}\frac{(|\boldsymbol{\mu}_{ge}|^2-|\boldsymbol{\mu}_{ge}\cdot\mathbf{k}|^2/k^2)}{(\omega-\omega_\gamma)^2+\Gamma^2/4}. \tag{8}$$

If instead of a pure state we consider an average on the orientation of the transition dipole we have

$$\mathcal{F}(k_\gamma\mathbf{d}) = \left\langle \int e^{i\mathbf{k}\cdot\mathbf{d}}d^3\mathbf{k}\frac{(|\boldsymbol{\mu}_{ge}|^2-|\boldsymbol{\mu}_{ge}\cdot\mathbf{k}|^2/k^2)}{(\omega-\omega_\gamma)^2+\Gamma^2/4}\right\rangle. \tag{9}$$

For an isotropic distribution of transition dipoles we deduce

$$|\mathcal{V}| = |\sin(k_\gamma d)/(k_\gamma d)|e^{-\Gamma d/(2c)} \simeq |\sin(k_\gamma d)/(k_\gamma d)|. \tag{10}$$

This equation [54] formalizes the also intuitive fact that photons of shorter wavelengths provide a better path resolution inside the interference device: For $k_\gamma d\gg 1$, i.e., for the case of a short photon wavelength the quantum interference vanishes and reciprocally $|\mathcal{V}|\simeq 1$ for $k_\gamma d\lesssim 1$.

This result is in agreement with the complementarity principle since the fringe visibility depends on the amount of information that is, in principle, available to an outside observer. It is consistent with Heisenberg's back-action argument, since those photons that provide a better path information also impart a stronger recoil. And finally it is derived by the decoherence theory, which means it is based on the entanglement between the molecule and the emitted photon. Equation (10) has indeed been experimentally well confirmed in several interferometry experiments both with atoms [11,44,45] and with large molecules [41,43].

It can be observed that in Eq. (3) the spatial wave functions $\Psi_{A,B}(\mathbf{r},t)$ associated with the center of mass are not affected since they factorize from the photon states (this factorization is a very good approximation in the nonrelativistic regime) one can then wonder if there is effectively a momentum transfer in this analysis. The answer is naturally yes and this can be seen by expanding the photon states in the basis $|\mathbf{k},\epsilon\rangle$

$$|\gamma_{A,B}\rangle = \sum_{\mathbf{k},\epsilon}\gamma^{A,B}_{\mathbf{k},\epsilon}|\mathbf{k},\epsilon\rangle. \tag{11}$$

Equation (3) is then equivalent to

$$|\Psi_J\rangle \simeq \int d^3\mathbf{r}\sum_{\mathbf{k},\epsilon}[\Psi_A(\mathbf{r},t)\gamma^A_{\mathbf{k},\epsilon}+\Psi_B(\mathbf{r},t)\gamma^B_{\mathbf{k},\epsilon}]|\mathbf{k},\epsilon,\mathbf{r}\rangle. \tag{12}$$

The first order signal $G^{(1)}(\mathbf{r},t)=\mathrm{Tr}[\hat{\rho}|\mathbf{r}\rangle\langle\mathbf{r}|]$ is consequently

$$G^{(1)}(\mathbf{r},t) \propto \sum_{\mathbf{k},\epsilon} [|\gamma^A_{\mathbf{k},\epsilon}|^2 + |\gamma^B_{\mathbf{k},\epsilon}|^2 + \gamma^{B,*}_{\mathbf{k},\epsilon}\gamma^A_{\mathbf{k},\epsilon} e^{im\mathbf{r}/\hbar t\cdot\mathbf{d}} + \text{c.c.}]. \tag{13}$$

If we consider now the result $G^{(1)}_{FF}(\mathbf{r},t)$ expected for the Heisenberg (far-field) experiment we find

$$G^{(1)}_{FF}(\mathbf{r},t) \propto \sum_{\mathbf{k},\epsilon} |\gamma^{(0)}_{\mathbf{k},\epsilon}|^2 \left[1 + \cos\left(\frac{m\mathbf{r}}{\hbar t}\cdot\mathbf{d} + \mathbf{k}\cdot\mathbf{d}\right)\right], \tag{14}$$

where $|\gamma^{(0)}_{\mathbf{k},\epsilon}|^2$ is there to find a photon with wave vector $\mathbf{k}$ and polarization ϵ [44,55]. The momentum transferred to the photon by the atom or molecule affects the coherence of the recorded signal. This is clearly visible from the fact that $G^{(1)}_{FF}$ is the sum of patterns with unit visibility shifted by an amount $\mathbf{k}\cdot\mathbf{d}$. Each individual pattern is unable to erase the fringes but the sum of all these patterns can do it. However this transfer of momentum is not visible on the single aperture pattern which is *not broadened* significantly. The consequence of this analysis is the following: In the far-field experiment the x component of the photon wave vector can vary in the interval $\delta k_x \sim 2\pi/d$ (since we consider the regime $\lambda_\gamma \leqslant d$). For this reason the fringes wash out and this is exactly Heisenberg's argumentation [5–7].

We would like to emphasize that this discussion was only centered on the process of decoherence by photon emission and that the quantum state given by Eq. (3) is actually independent of any particular which-path detector. This seems to be a logical way of treating the problem since in the words of R. P. Feynman "Nature does not know what you are looking at, and she behaves the way she is going to behave whether you bother to take down the data or not" [7]. However N. Bohr pointed out already that we cannot ignore the physical presence of the detector in the analysis since "As regards the specification of the conditions for any well-defined application of the formalism, it is moreover essential that the whole experimental arrangement be taken into account" [5]. In order to discuss complementarity we must now actually take down the data and see what happens.

III. INTERFEROMETRY WITH CORRELATED PHOTON DETECTION

A. Which-path experiment in the optical far field of the molecules

As we explained, Eq. (10) would also suggest that the emission of photons of the long wavelength does not influence the molecular interference in any detrimental way, no matter whether one cares to correlate the molecular and photonic degrees of freedom. In order to control if this is indeed true one should, as proposed by Heisenberg, introduce an optical instrument such as a microscope detecting the photon in coincidence with the molecule arrival. In other terms we should consider a two-particle (second order) instead of a single-particle (first order) interferometry experiment. Because the detector is sensible to the electric field intensity it is convenient to introduce the (vectorial) probability amplitude for finding the photon in the space-time point $P=(t,\mathbf{r}_\gamma)$, conditioned by the probability amplitude for registering the molecule at position $M=(t,\mathbf{r}_m)$

$$\mathbf{A}(M,P) \propto \Psi_A(M)\mathbf{E}_A(P) + \Psi_B(M)\mathbf{E}_B(P), \tag{15}$$

where $\mathbf{E}_{A,B}(P)$ denotes the electric field at the detector, which is caused by the photon originating in A or B. If we register the emitted photon at a given location $(\mathbf{r}_\gamma,t)$, the photon electric field [53,56] can be written as $\mathbf{E}_{A,B}(\mathbf{r}_\gamma,t) = \langle 0|\hat{\mathbf{E}}^{(+)}(\mathbf{r}_\gamma)|\gamma_{A,B}\rangle$, where $\hat{\mathbf{E}}^{(+)}$ is the positive frequency part of the electric field operator. In agreement with the definition of the photoelectric effect $G^{(2)}(M,P)=|\mathbf{A}(M,P)|^2=\text{Tr}[\hat{\rho}\hat{\mathbf{E}}^{(-)}(\mathbf{r}_\gamma)\hat{\mathbf{E}}^{(+)}(\mathbf{r}_\gamma)|\mathbf{r}_m\rangle\langle\mathbf{r}_m|]$ is proportional to the molecular intensity collected on the screen in M when the photon *is recorded in coincidence* in P by an isotropic detector. In the present case $\mathbf{E}_{A,B}(\mathbf{r}_\gamma,t)$ is also given by a classical expression [57] for a pointlike dipole. Using Eq. (4) one indeed obtains [58]

$$\mathbf{E}_{A,B}(\mathbf{r}_\gamma,t) = -\frac{1}{c^2}\frac{\partial^2 \mathbf{\Pi}_{A,B}(\mathbf{r}_\gamma,t)}{\partial t^2} + \nabla(\nabla\cdot\Pi_{A,B}(\mathbf{r}_\gamma,t)). \tag{16}$$

$\mathbf{\Pi}_{A,B}(\mathbf{r}_\gamma,t)=\boldsymbol{\mu}(t-R_{A,B}/c)/R_{A,B}$, with $R_{A,B}=|\mathbf{r}_\gamma-\mathbf{r}_{A,B}|$ is the Hertz vector associated with the electric dipole [57]. This field can equivalently be written as

$$\begin{aligned}\mathbf{E}_{A,B}(\mathbf{x}_\gamma,t) = {}& \chi_\gamma^2\left(\frac{\mathbf{R}_{A,B}}{R_{A,B}}\times\mathbf{\Pi}_{A,B}(\mathbf{x}_\gamma,t)\right)\frac{\mathbf{R}_{A,B}}{R_{A,B}} \\ & + \left[3\frac{\mathbf{R}_{A,B}}{R_{A,B}}\left(\frac{\mathbf{R}_{A,B}}{R_{A,B}}\cdot\mathbf{\Pi}_{A,B}(\mathbf{x}_\gamma,t)\right) - \mathbf{\Pi}_{A,B}(\mathbf{x}_\gamma,t)\right] \\ & \times\left(\frac{1}{R^2_{A,B}} - \frac{i\chi_\gamma}{R_{A,B}}\right),\end{aligned} \tag{17}$$

where $\chi_\gamma=(\omega_\gamma - i\Gamma/2)/c$. This expression shows characteristic dependencies in $1/R^3$ (near field), $1/R^2$ (intermediate field), and $1/R$ (far field).

In this coincidence measurement the recorded intensity can be expressed as

$$\begin{aligned}G^{(2)}(M,P) = {}& |\Psi_A(M)\mathbf{E}_A(P)|^2 + |\Psi_B(M)\mathbf{E}_B(P)|^2 \\ & + \Psi^*_B(M)\Psi_A(M)\mathbf{E}^*_B(P)\mathbf{E}_A(P) + \text{c.c.}.\end{aligned} \tag{18}$$

Using Eq. (1) the fringe visibility of the molecular pattern which is defined as

$$V_{\text{Cond}}(P) = (G^{(2)}_{\max,P} - G^{(2)}_{\min,P})/(G^{(2)}_{\max,P} + G^{(2)}_{\min,P}) \tag{19}$$

becomes

$$V_{\text{Cond}}(P) = \frac{2|\mathbf{E}_A(P)\cdot\mathbf{E}^*_B(P)|}{|\mathbf{E}_A(P)|^2 + |\mathbf{E}_B(P)|^2}. \tag{20}$$

It should be noted that the photon, which is moving much quicker than the molecule, will be absorbed a long time before the molecule even leaves the aperture region. However, Eq. (36) is valid even if the two events P and M are not simultaneous since the absorption of the photon transfers the entanglement of the system photon and/or molecule to the system detector and/or molecule. With Heisenberg's micro-

scope the optical collection system would operate in the optical far field and would be focused on the double pinhole. If we then place a pointlike photon detector at the maximum of the optical diffraction pattern which is associated with the image of the hole A or B we obtain

$$V_{FF} = 2|f(k_\gamma d)|/[1 + f(k_\gamma d)^2]. \quad (21)$$

The imaging function taking into account diffraction [57] by the objective is

$$f(k_\gamma d) = 2J_1(k_\gamma d\mathrm{NA})/(k_\gamma d\mathrm{NA}), \quad (22)$$

where NA is the numerical aperture of the lens [57]. This function is proportional to the electric field of the source observed in the image plane as given by classical optics. It can be observed that the time of the detection can be factorized from the field and that the visibility is consequently independent of the detection time. The important conclusion is that again one finds the result of Eq. (10) which leads to $V_{FF} \simeq 1 \simeq |\mathcal{V}|$ for $k_\gamma d \ll 1$. This only confirms once again the mutual exclusion of which-path information and fringe visibility.

From such a Heisenberg experiment and similar others, one could propose, as an hypothesis, the so-called Heisenberg mechanism: It is impossible to determine through which hole A or B a molecule passes without exchanging with the measuring device a minimal transverse momentum $\sim h/d$, sufficient to disturb mechanically the molecule and to erase completely the interference pattern. In other terms, to paraphrase R. Feynman [7], "there is no way to go around the Heisenberg principle." Such an archetypal explanation has been, however, criticized by Scully *et al.* [15] who argued that the uncertainty relation *does not* always enforces recoils kicks sufficient to wash out the fringes. Since that point has stirred up considerable controversy in the literature [16–40] we are not going to discuss this problem here. We will only remark that both the Heisenberg and the Scully *et al.* experiment can be understood by considering only the photon emission process. The observer is in both cases unnecessary since the fringes are destroyed even if we do not actually detect the photon.

We are now going to consider an experiment in which oppositely the presence of the observer is necessary to interpret the data and extract the which-path information.

B. Which-path experiment in the optical near field of the molecules

From the precedent discussion concerning Heisenberg's microscope we could deduce that Rayleigh's and Abbé's theory of optical microscopy would certainly deny the possibility that the molecular path could be resolved under the circumstances $\lambda_\gamma \gg d$. But it is well known that near-field scanning optical microscopy can circumvent this diffraction limit (see, for example, [47,48], and references therein). It could be then expected that with a NSOM the formulas (21) or (10) are not the end of the story. The essential element in a NSOM is a metal coated tapered optical fiber tip with a subwavelength aperture located at the fiber apex. It collects light only from a very small volume around the aperture [59–63]. Nowadays, detectors with down to 50 nm resolution can be routinely obtained this way. Since the electric near field of the emitted photon given by Eq. (9) is of dipolar character it decreases strongly with the distance R from the emitting molecule,

$$\mathbf{E}_{A,B}(\mathbf{r}_\gamma) \simeq 3\mathbf{R}_{A,B}(\mathbf{R}_{A,B} \cdot \boldsymbol{\mu})/|\mathbf{R}_{A,B}|^5 - \boldsymbol{\mu}/|\mathbf{R}_{A,B}|^3, \quad (23)$$

where we have omitted all time dependence. As before, the molecular interference visibility conditioned on the detection of a near-field photon is given by a formula as Eq. (20). However, the NSOM is not an isotropic but a vectorial detector behaving essentially as a point dipole aligned along the direction $\mathbf{e}$ [61,64–68]. This is equivalent to consider instead of a NSOM tip an ideal point detector, located in the near field of the molecule. Theories [60,61] show that the optical tip acts like an induced electric point dipole whose polarizability reads as $\sim a^3$ where a is the radius of the NSOM aperture. It should be noted that in the regime of very small NSOM aperture this polarizability is (like in Mie's theory [57,60]) completely independent of the photon wavelength. It can be added that we can, in principle, similarly define an induced magnetic point dipole associated with the tip [59,67]. However since in the optical near field the magnetic field emitted by the molecule is typically smaller than the electric field by a factor $\sim d/\lambda_\gamma$ [60] we will without any losses in generality neglect this term in the following. Additionally, it can be observed that the possibility of identifying the detector to a point dipole shows that the present analysis is not limited to a NSOM detector. It is indeed a characteristic of nano objects much smaller than λ_γ. The same analysis can be, for example, used to describe the principle of a which-path detection by a second molecule or the scattering by a metal nanoparticle [69–71] located close to one of the apertures A and B.

The visibility conditioned on the photon detection reads now

$$V_{\mathrm{Cond}}(P) = \frac{2|\mathbf{e} \cdot \mathbf{E}_A(P)\mathbf{e}^* \cdot \mathbf{E}_B^*(P)|}{|\mathbf{e} \cdot \mathbf{E}_A(P)|^2 + |\mathbf{e} \cdot \mathbf{E}_B(P)|^2}. \quad (24)$$

This visibility is in the near-field limit independent of the photon wavelength. However it is strongly dependent on the position of the detector as well as on the direction of the molecular dipole and on the nature of the detector (scalar or vectorial).

In order to show the strong dependence of the molecular-fringes visibility with the position of the near-field detector we calculate it when the detector is on the x axis. We find the orientation of the molecule dipole

$$V_{NF} = 2R_A^3 R_B^3/(R_A^6 + R_B^6). \quad (25)$$

If the detector is located in the middle between both holes the visibility is maximal. This can be immediately seen both from Eq. (25) ($V_{NF}=1$ for $R_B=R_A$), and from the simple reasoning that the symmetry of the arrangement does not allow us to extract any path information. If, however (see Fig. 1), one shifts the detecting tip closer to one of the holes, $\epsilon = R_B \ll R_A \simeq d$, the conditional interference contrast can be strongly suppressed $V_{NF} \simeq 2(\epsilon/d)^3 \ll 1$. The position distin-

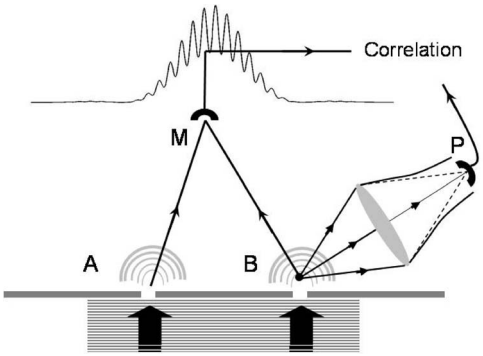

FIG. 2. Young's double-hole which-path experiment with far-field detection of the emitted photon using a microscope objective focalized on the apertures A and B.

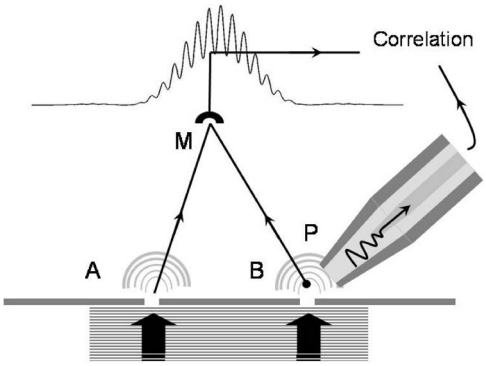

FIG. 3. Principle of the near-field Heisenberg microscope. The photon emitted by the molecule in the aperture A or B can be recorded by the NSOM tip in P and can be correlated with the molecule arrival in M.

guishing capability of the detector can be evaluated by considering the probability to record an event at the detector as a function of the molecule's position in the plane of the double pinhole. The probability for a detected photon to originate from hole B instead of hole A is given by

$$P_B = E_B^2/(E_A^2 + E_B^2) \simeq 1 - \epsilon^6/d^6 \gg P_A \simeq \epsilon^6/d^6. \quad (26)$$

The distinguishability can be defined [1] both in the far-field and the near-field detection to be

$$D = |P_A - P_B| = |E_A^2 - E_B^2|/(E_A^2 + E_B^2). \quad (27)$$

In the near field this yields

$$D_{NF} = |R_A^6 - R_B^6|/(R_A^6 + R_B^6) \simeq 1 - 2\epsilon^6/d^6 \sim 1, \quad (28)$$

which is in marked contrast to the far-field expression

$$D_{FF} = |1 - f(k_\gamma d)^2|/[1 + f(k_\gamma d)^2] \simeq 1 - f(k_\gamma d)^2 \sim 0. \quad (29)$$

To illustrate these equations, let us take the example of two holes which are sufficiently narrow, say smaller than 50 nm and separated by $d=500$ nm. We further assume that the molecule emits light at a wavelength of $\lambda_\gamma = 5$ μm, i.e., at a wavelength exceeding the pinhole separation by a factor of 10.

The classical microscope shall be in a sufficient distance to fulfill the far-field criterion (see Fig. 2), whereas the NSOM tip is assumed to be placed in a distance $\epsilon=50$ nm to the right of the right hole, corresponding to a separation of 550 nm from the left hole (Fig. 3). We can then compute both the expected molecular fringe visibility V and molecular position distinguishability D for both detector arrangements. For the optical far-field setup we find—as expected—a high fringe contrast of $V_{FF}=0.9988$ combined with a very low distinguishability $D_{FF}=0.049$. In contrast to that, we compute an almost vanishing visibility of $V_{NF}=0.0044$ and a very high distinguishability of $D_{NF}=0.999\,99$ for the case of a near-field detection of the emitted photons. The near-field detector would thus be able to distinguish the particle's position with very high certainty in the domain $d < \lambda_\gamma/2$, where any far-field microscope would fail. In this context it can be observed that the same experiment but with the detector in the far field (where $E_{A,B} \sim 1/R_{A,B}$) would give a visibility close to 1 and then no distinguishability. Secondly we can remark that for the chosen ratio $d/\lambda_\gamma = 10^{-1}$, the intensity of the photon magnetic field at the tip location is 100 times smaller than the intensity of the photon electric field at the same location. This justifies our approximation of neglecting such magnetic contributions.

In our treatment a specific formulation of the complementarity principle [1,2], namely,

$$V^2 + D^2 = 1 \quad (30)$$

is respected both for the far field and the near field. However, in the NSOM case we consider the limit $\lambda_\gamma \gg d$ and we deduce then from Eq. (14) that the experiment is essentially *recoil free* It is, therefore, wrong to believe that a which-path experiment using the spontaneous emission of long wavelength photons in vacuum is impossible.

It is important to remark that the duality relation Eq. (30) is valid only for a pure quantum state [1]. For a mixture of oriented molecular dipoles and a scalar detector we find

$$V = 2|\langle \mathbf{E}_A \cdot \mathbf{E}_B \rangle|/(\langle \mathbf{E}_A^2 \rangle + \langle \mathbf{E}_B^2 \rangle),$$

$$D = |\langle \mathbf{E}_A^2 \rangle - \langle \mathbf{E}_B^2 \rangle|/(\langle \mathbf{E}_A^2 \rangle + \langle \mathbf{E}_B^2 \rangle), \quad (31)$$

and

$$D^2 + V^2 \leqslant 1, \quad (32)$$

where $\langle [\ ] \rangle$ denotes the average of molecular dipole orientations. Similarly with the vectorial NS detector the visibility and distinguishability reads as

$$V = \frac{2|\langle \mathbf{e} \cdot \mathbf{E}_A \mathbf{e}^* \cdot \mathbf{E}_B^* \rangle|}{\langle |\mathbf{e} \cdot \mathbf{E}_A|^2 \rangle + \langle |\mathbf{e} \cdot \mathbf{E}_B|^2 \rangle},$$

$$D = \frac{|\langle |\mathbf{e} \cdot \mathbf{E}_A|^2 \rangle - \langle\ |\mathbf{e} \cdot \mathbf{E}_B|^2 |\rangle|}{\langle |\mathbf{e} \cdot \mathbf{E}_A|^2 \rangle + \langle\ |\mathbf{e} \cdot \mathbf{E}_B|^2 |\rangle}, \quad (33)$$

which leads once again to Eq. (32). In particular, Eqs. (25)–(28) are always true in whatever the distribution considered. We calculated, for the case of an isotropic distribution, the visibility V_{NF} as a function of the detector position in the plane x, z. Figures 4–7 show the result of such calcu-

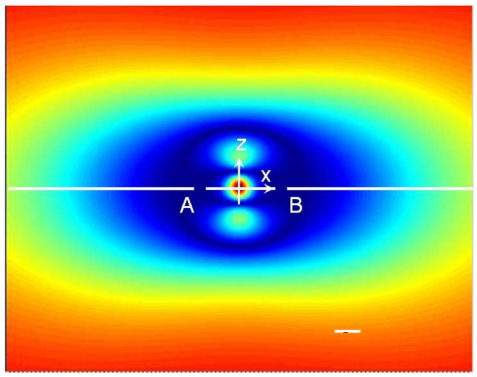

FIG. 4. (Color online) Visibility V_{NF} calculated as a function of the detector position in the plane x,z for an isotropic distribution of dipoles in the case of a scalar point detector. The aperture plane ($z=0$) is represented by white lines.

lations for, respectively, a scalar detector, a vectorial detector oriented in the x, y or z direction. In every case the visibility takes its minimum close to the apertures A and B and tend to $V=1$ far away from them. Finally we can observe that even if the present paper focalizes only on the double-pinhole experiment we can make the same reasoning for a double-slit experiment. In that case we consider the problem with 2D dipoles and the visibility given by Eq. (19) becomes $V_{NF}=2R_A^2R_B^2/(R_A^4+R_B^4)$.

IV. DISCUSSION CONCERNING COMPLEMENTARITY AND QUANTUM ERAZING

We stated above that we could detect which-path information without significantly perturbing the molecular motion if one was able to record long wavelength radiation with optical near-field methods. But even in a Gedanken experiment one has to consider the inherent limitations of such a detection scheme. These are the finite size, finite momentum, and the low efficiency of any such detector, which are imposed by the working principle itself rather than by technical constraints.

It is a fundamental requirement for a good NSOM tip to have a very small aperture typically of $s=2a\sim 50-100$ nm

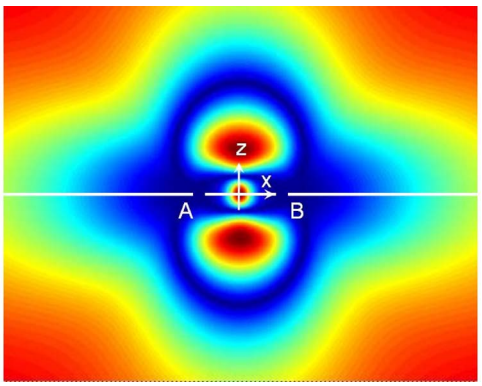

FIG. 5. (Color online) Visibility V_{NF} calculated as a function of the detector position in the plane x,z for an isotropic distribution of dipoles in the case of a vectorial detector oriented along x.

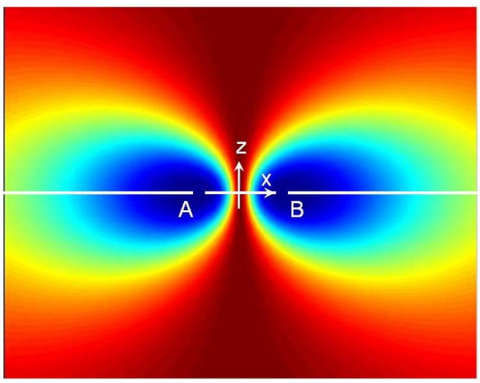

FIG. 6. (Color online) Visibility V_{NF} calculated as a function of the detector position in the plane x,z for an isotropic distribution of dipoles in the case of a vectorial detector oriented along y.

diameter. Photon transmission through such a narrow constriction is described by Bethe-Bouwkamp's diffraction theory [61] and one finds that the transmission scales with $T\sim(k_\gamma s)^4$. And it will generally [47,48,65–67] range between $T=10^{-3}-10^{-5}$. In addition to that the finite detector size implies that it would cover only a finite solid angle $d\Omega/\Omega\sim 0.1$. Most molecules arriving at the interference screen will thus not have given any information to the photon detector. However, as long as the experiment exploits the observed correlations only, the finite transmission may increase the run-time of the experiment but it will not degrade the quality of the expected experimental result. We should then remark that the possibility of extracting a finite signal $G^{(2)}(M,P)>0$ with a NSOM detector imposes necessarily $|\mathcal{V}|<1$. For consistency an ideal which-path experiment with $V_{NF}=0$ requires that the probability of detecting a photon is always smaller or equal to $\mathcal{M}=1-|\mathcal{V}|$. In the present example $\mathcal{M}\simeq 0.06\gg T$ and there is no problem.

In this context, one might also argue that it is not important at all that the photon is really detected by a counter or even a human observer. The standard decoherence [8] argument only requires that the emitted photon takes the environment into a superposition of two distinguishable states. So even if the transmission is low and the detector does not fire, we might have to consider what happens upon absorption or

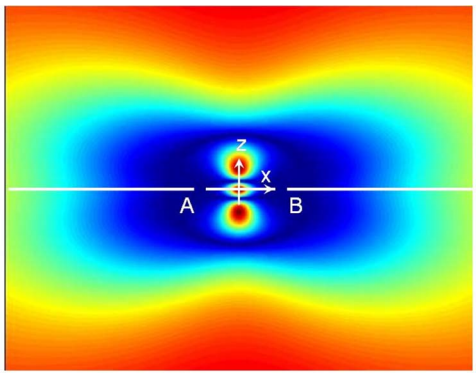

FIG. 7. (Color online) Visibility V_{NF} calculated as a function of the detector position in the plane x,z for an isotropic distribution of dipoles in the case of a vectorial detector oriented along z.

reflection of a photon by the NSOM tip. However, it turns out that generally also these processes are not relevant, as one can see from a simple argument based on the position-momentum uncertainty with respect to the state of the NSOM tip: In order to make any which-path statement we require the tip position to be localized within $\Delta x_{\mathrm{tip}} \ll d$, which immediately imposes a momentum uncertainty on it of the order of $\Delta p_{\mathrm{tip}} > \hbar / \Delta x_{\mathrm{tip}}$. This has to be compared to the recoil which an absorbed or reflected photon would impart on the tip, which is $\Delta p_{\gamma,a} = h/\lambda_\gamma$ and $\Delta p_{\gamma,r} = 2h/\lambda_\gamma$, respectively. We thus see that if the wavelength λ_γ is much larger than the maximum path separation d, the momentum of the absorbed photon will not even suffice to carry the detector's momentum state beyond its own intrinsic uncertainty $\Delta p_\gamma \simeq \Delta p_{\mathrm{tip}}$. Absorption or reflection on the NSOM tip, therefore, intrinsically do not allow us to correlate the emitted photon with the molecular signal: the effect of the photon on the environment is just too small.

One might finally argue that there could be a back action of the NSOM tip on the evanescent field. The additional potentials imposed by the presence of nearby walls will, of course, also shift and distort the molecular interference pattern. But none of the phase shifts caused by the tip will lead to decoherence since they are static and it is assumed that they are independent of the internal molecular state. Furthermore, the reflected field of the dipole image in the metallic tip contains the same photon momentum characteristic as the field emitted by the molecule. Although the spectral emission probability of the molecule will be changed [72,73], the molecular center-of-mass motion is not expected to be affected. On the other hand, if we wanted to get a detector "click" with certainty for every passing molecule, the molecules would have to emit such a huge number of infrared photons that momentum diffusion would again be a valid explanation for the disappearance of the molecular fringe contrast.

These points are essential in understanding the particularity of the NSOM experiment proposed here. At the difference of all the previous proposed which-path experiments exploiting the entanglement between a molecule and a photon the coincidence measurement is here crucial. Indeed, in the experiment by Scully *et al.*, as well as in the Heisenberg's far-field experiment, the which-path information is already present in the photon state and the fringes wash out even if the photon is not recorded. Here oppositely the which-path information is not visible without considering the two-particle correlation.

This experiment is also somewhat different from earlier experimental realization, where the photon correlation was used for the restoration instead of the destruction of interference fringes [15,74,75]. Indeed in the so-called quantum eraser presented originally by Scully and Drühl [74] it is, in principle, possible to wipe out the which-path information gained with the detector and to recover the interference. This in turn shows definitely the superiority of the entangling argument over a semiclassical recoil explanation for the loss of coherence. In this sense we can say with Scully *et al.* [15] that there is a way around the position-momentum uncertainty obstacle: In the quantum eraser it is the correlation between molecular and photonic events which is responsible for the restoration of the interference erased by the which-path detector. In comparison, our new proposal using a NSOM can be seen like the counterpart of the quantum eraser experiment since here the which-path information is revealed only after that data for the detection of molecules and photons are compared. It is, however, important to observe that the symmetry between these two experiments is not total. Indeed, in the quantum eraser experiment almost all pairs of particles can participate in the experiment but here (as we already said before) only a small fraction of the photon-molecule pair can be involved. The inefficiency of the photon detector is then here a necessity for the success of the NSOM experiment. It must, however, be noted that count rate limitations will render the described "minimally invasive" NSOM path measurement; an almost impossible enterprize with large molecules, and still very difficult to render these measurements, with even the best available atomic beams (see, however; [76]).

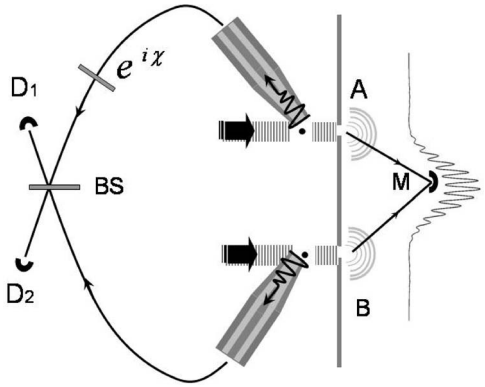

FIG. 8. Quantum eraser experiment using two NSOM detectors.

One could observe concerning the last remarks, that if restoring fringes by selecting half of the events, like in the Scully and Drühl proposal, might be somewhat paradoxical, destroying fringes with a very small efficiency would be a far less astonishing result. However, such an argumentation would miss the central point since the destruction of the fringes is associated here with a physical which-path information (complementary of the interference information) which is hidden before the quantum correlation (in the same sense that in the quantum eraser experiment the interference information is hidden). This is then not comparable to a dephasing process in which the information is lost in the environment.

In this context it is worth mentioning that one could always exploit the freedom offered to the experimentalist to erase the which-path information gained previously by using a quantum eraser protocol. Such a quantum eraser could be implemented, in principle, using two NSOM tips like those in Fig. 8. The light coming from the two detectors can be collected using optical fibers and the contributions of each fiber mixed using a beam splitter (BS) [77,78]. Oscillation fringes can be recovered from the correlation photon molecule depending on the difference of path lengths in this interferometer. Let us suppose that the beam splitter is symmetric and characterized by its reflectivity $R = i/\sqrt{2}$ and transmissivity $T = 1/\sqrt{2}$. Let χ be an additional phase shift in one

of the two arms. The probability amplitude for finding the photon in D_1 or D_2 conditioned to the probability amplitude for registering the molecule at position M is given by

$$G_{D1}(M) = [(\mathbf{e}\cdot\mathbf{E}_A)\Psi_A(M) + (\mathbf{e}\cdot\mathbf{E}_B)\Psi_B(M)]\frac{ie^{i\chi}}{\sqrt{2}} + [(\mathbf{e}'\cdot E'_A)\Psi_A(M) + (\mathbf{e}'\cdot\mathbf{E}'_B)\Psi_B(M)]\frac{1}{\sqrt{2}},$$

$$G_{D2}(M) = [(\mathbf{e}\cdot\mathbf{E}_A)\Psi_A(M) + (\mathbf{e}\cdot\mathbf{E}_B)\Psi_B(M)]\frac{e^{i\chi}}{\sqrt{2}} + [(\mathbf{e}'\cdot E'_A)\Psi_A(M) + (\mathbf{e}'\cdot\mathbf{E}'_B)\Psi_B(M)]\frac{i}{\sqrt{2}}. \tag{34}$$

Here $\mathbf{e}$ and $\mathbf{e}'$ characterize the dipoles orientation of the two NSOM tips. Similarly $\mathbf{E}_{A,B}$ and $\mathbf{E}'_{A,B}$ denote the photon field associated with the photon coming from A and B and detected by the dipoles $\mathbf{e}$ and $\mathbf{e}'$. Clearly the fringes visibility depends strongly on the tips positions. To simplify we suppose the symmetric situation where we can write $(\mathbf{e}\cdot\mathbf{E}_{A,B}) = (\mathbf{e}'\cdot\mathbf{E}'_{B,A})$. The molecular fringes visibility conditioned on photodetection in D_1 or D_2 read

$$V_{D1} = \frac{2|\Phi_A|\cdot|\Phi_B|}{|\Phi_A|^2 + |\Phi_B|^2},$$

$$V_{D2} = \frac{2|\Phi'_A|\cdot|\Phi'_B|}{|\Phi_A|^2 + |\Phi_B|^2}, \tag{35}$$

where

$$\Phi_A = (\mathbf{e}\cdot\mathbf{E}_A)ie^{i\chi} + (\mathbf{e}\cdot\mathbf{E}_B),$$

$$\Phi_B = (\mathbf{e}\cdot\mathbf{E}_B)ie^{i\chi} + (\mathbf{e}\cdot\mathbf{E}_A),$$

$$\Phi'_A = -(\mathbf{e}\cdot\mathbf{E}_A)ie^{i\chi} + (\mathbf{e}\cdot\mathbf{E}_B),$$

$$\Phi'_B = -(\mathbf{e}\cdot\mathbf{E}_B)ie^{i\chi} + (\mathbf{e}\cdot\mathbf{E}_A). \tag{36}$$

In the particular case where $ie^{i\chi}=1$ we deduce $V_{D1,D2}=1$ and we obtain

$$G_{D1}(M) = \frac{1}{\sqrt{2}}[\Phi_A + \Phi_B][\Psi_A(M) + \Psi_B(M)],$$

$$G_{D2}(M) = \frac{i}{\sqrt{2}}[\Phi_A - \Phi_B][\Psi_B(M) - \Psi_A(M)], \tag{37}$$

associated with, respectively unit-visibility fringes and antifringes patterns. By either removing or not removing the beam splitter one can then, in principle, select which complementary information to observe and this even a long time after the molecules were detected [75]. This once again shows that it is the entanglement and information which are the central issues here and not a hypothetical semiclassical stochastic mechanism based on random photon kicks. It is important to remark that with two NSOM tips the presence of the double pinholes is even not necessary. Since we work in the coincidence regime the two tips play the role of two molecular virtuals sources (this is is reminiscent of the Karl Popper experiment [79]).

As a conclusion we insist once again on the fact that the near-field which-path experiment proposed here can be called "essentially recoil free" in analogy to [15], because the detected photon wavelength is larger than the slit separation. It is actually the entanglement between the molecular center-of-mass state and the photon and the information one has obtained about the photon state which produce the path measurement and which destroys the interference fringes; if we only register those molecules for which we also get a photon click. The present work represents, to the best of our knowledge, the first description of a which-path experiment in the optical near-field regime. This paper is just a discussion and constitutes only a small exploration into the subject of quantum optics in the near-field regime. More work has to be done to understand this domain which is for, the major part, *terra incognita* [80,81].

ACKNOWLEDGMENTS

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APPENDIX A: SPONTANEOUS EMISSION BY A MOVING WAVE PACKET

We consider the total hamiltonian associated with a two-levels atom, e.g.,

$$\hat{\mathbf{H}} = \frac{(\hbar\mathbf{P})^2}{2m} + E_e|e\rangle\langle e| + E_g|g\rangle\langle g| + \sum_\alpha \hbar\omega_\alpha \hat{a}^\dagger_\alpha \hat{a}_\alpha - \sum_\alpha \sqrt{2\pi\hbar\omega_\alpha}[\boldsymbol{\mu}_{ge}\cdot\mathbf{E}_\alpha(\mathbf{x})|e\rangle\langle g|\hat{a}_\alpha + [\boldsymbol{\mu}_{ge}\cdot\mathbf{E}_\alpha(\mathbf{x})]^* \times|g\rangle\langle e|\hat{a}^\dagger_\alpha], \tag{A1}$$

where $E_e = E_g + \hbar\omega_\gamma > E_g$, $\boldsymbol{\mu}_{ge} = \langle g|\hat{\mu}|e\rangle$, and α is a orthogonal basis for the transverse photon field. Here we use the plane wave basis for a photon of momentum $\hbar\mathbf{k}$ and of polarization $\boldsymbol{\epsilon}_{\mathbf{k},i}$ $(i=1,2)$: $\mathbf{E}_\alpha(\mathbf{x}) = \boldsymbol{\epsilon}_{\mathbf{k},i}e^{i\mathbf{k}\cdot\mathbf{x}}/\sqrt{V}$ (Box normalization).

The spontaneous emission process (beginning at $t=0$) supposes an initial state of the form

$$\int d^3\mathbf{p}\ \psi(\mathbf{p})e^{-ip^2t/(2m\hbar)}|\mathbf{p}\rangle e^{-iE_et/\hbar}|0\rangle|e\rangle, \tag{A2}$$

where $\mathbf{p}$ is the momentum basis of the molecule and $|0\rangle$ the photon vacuum state.

$$\int d^3\mathbf{p}\ \psi(\mathbf{p})e^{-ip^2t/(2m\hbar)}|\mathbf{p}\rangle = \int d^3\mathbf{x}\boldsymbol{\Psi}(\mathbf{x},t)|\mathbf{x}\rangle$$

is the spatial wave function of the atom or molecule in absence of spontaneous emission. The method of Weisskopf

and Wigner defines the evolution of such a state in time and we find for $t \geqslant 0$

$$|\psi(t)\rangle = |E(t)\rangle + |G(t)\rangle. \tag{A3}$$

The state $|E(t)\rangle$ equals

$$\int d^3\mathbf{p}\ \psi(\mathbf{p}) e^{-ip^2 t/(2m\hbar)} |\mathbf{p}\rangle |0\rangle |e\rangle e^{-iE_e t/\hbar} e^{-\Gamma t/2} \tag{A4}$$

and describes the exponential decay with the probability $e^{-\Gamma t}$ to remain in the exited state. The state $|G(t)\rangle$ expresses the entanglement matter field and can be written

$$\sum_{\mathbf{k},i} \int d^3\mathbf{p}\ \psi(\mathbf{p}) e^{-ip^2 t/(2m\hbar)} \gamma_{\mathbf{k},i}(t)$$

$$|\mathbf{P} - \hbar\mathbf{k}\rangle |\mathbf{k},i\rangle e^{-iE_g t/\hbar} |g\rangle, \tag{A5}$$

with

$$\gamma_{\mathbf{k},i}(t) = g_{\mathbf{k},i} \frac{e^{-i[\omega - \mathbf{k}\cdot\mathbf{p}/m + \hbar k^2/(2m)]t} - e^{-i\omega_\gamma t} e^{-\Gamma t/2}}{\omega - \omega_\gamma - \mathbf{k}\cdot\mathbf{p}/m + \hbar k^2/(2m) + i\Gamma/2}, \tag{A6}$$

and $g_{\mathbf{k},i} = -\sqrt{2\pi\hbar\omega/V}(\boldsymbol{\epsilon}_i \cdot \boldsymbol{\mu}_{ge})/\hbar$.

We suppose now $\mathbf{p}/m \cdot \mathbf{k} \ll \omega$ and $\hbar k^2/(2m) \ll \omega$ for $\omega_\gamma \sim \omega$ in the denominator. Such an approximation is justified in the nonrelativistic approximation and when we can neglect the Doppler effect. We can similarly neglect $\hbar k^2/(2m) \ll \omega$ in the numerator. However we do not omit the term $e^{i\mathbf{p}/m\cdot\mathbf{k}t}$ in the second exponential because it is of the same order of magnitude of $e^{i\mathbf{x}\cdot\mathbf{k}}$; appearing when we express the quantum state in the $\mathbf{x}$ representation. This leads effectively to

$$|G(t)\rangle = \int d^3\mathbf{x} |\mathbf{x}\rangle \Psi_G(\mathbf{x},t) e^{-iE_g t/\hbar} |g\rangle \tag{A7}$$

with

$$\Psi_G(\mathbf{x},t) = \Psi(\mathbf{x},t) \sum_{\mathbf{k},i} -g_{\mathbf{k},i} \frac{e^{-i\omega_\gamma t} e^{-\Gamma t/2}}{\omega - \omega_\gamma + i\Gamma/2} e^{-i\mathbf{k}\cdot\mathbf{x}} |\mathbf{k},i\rangle + \sum_{\mathbf{k},i} \Psi\left(\mathbf{x} + \frac{\hbar\mathbf{k}t}{m}, t\right) g_{\mathbf{k},i} \frac{e^{-i\omega t}}{\omega - \omega_\gamma + i\Gamma/2} e^{-i\mathbf{k}\cdot\mathbf{x}} |\mathbf{k},i\rangle. \tag{A8}$$

APPENDIX B: DIPOLE-FIELD AND SPONTANEOUS EMISSION BY A TWO LEVEL MOLECULE

The electric field defined by $\mathbf{E}_{A,B}(\mathbf{x}_\gamma,t) = \langle 0|\hat{\mathbf{E}}^{(+)}(\mathbf{x}_\gamma)|\gamma_{A,B}(t)$ is calculated using the operator expansion

$$\hat{\mathbf{E}}^{(+)}(\mathbf{x}_\gamma) = \sum_{\mathbf{k},i} \sqrt{\frac{2\pi\hbar\omega}{V}} \boldsymbol{\epsilon}_{\mathbf{k},i} \hat{a}_{\mathbf{k},i} e^{i\mathbf{k}\cdot\mathbf{x}_\gamma} \tag{B1}$$

and leads after the summation on the polarization states to

$$\mathbf{E}_{A,B}(\mathbf{x}_\gamma,t) = \int \frac{d^3\mathbf{k}}{(2\pi)^2} \omega \left(\boldsymbol{\mu}_{ge} - \frac{(\boldsymbol{\mu}_{ge}\cdot\mathbf{k})\mathbf{k}}{k^2}\right) \times e^{i\mathbf{k}(\mathbf{x}_\gamma - \mathbf{x}_{A,B})} \frac{e^{-i\omega_\gamma t} e^{-\Gamma t/2} - e^{-i\omega t}}{\omega - \omega_\gamma + i\Gamma/2}. \tag{B2}$$

This can written using the dyadic notation as

$$\mathbf{E}_{A,B}(\mathbf{x}_\gamma,t) = (-\overleftrightarrow{\delta}\nabla^2 + \nabla\nabla)\boldsymbol{\mu}_{ge} S(\mathbf{x}_\gamma - \mathbf{x}_{A,B}, t), \tag{B3}$$

where the integral

$$S(\mathbf{x}_\gamma - \mathbf{x}, t) = \int \frac{\omega d^3\mathbf{k} e^{i\mathbf{k}(\mathbf{x}_\gamma - \mathbf{x})}}{(2\pi)^2 k^2} \frac{e^{-i\omega_\gamma t} e^{-\Gamma t/2} - e^{-i\omega t}}{\omega - \omega_\gamma + i\Gamma/2} \tag{B4}$$

can be calculated in the complex plane and equals

$$S(\mathbf{x}_\gamma - \mathbf{x}, t) = \Theta(t - R/c) e^{-\Gamma(t - R/c)/2} e^{-i\omega_\gamma (t - R/c)}/R \tag{B5}$$

with $R = |\mathbf{x}_\gamma - \mathbf{x}|$. This formula for the electric field is equivalently written using the Hertz potential notation as in Eq. (16).

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