One-step implementation of a multiqubit controlled-phase-flip gate

Xiu-Min Lin,[1,2,*] Zheng-Wei Zhou,[1,†] Ming-Yong Ye,[1] Yun-Feng Xiao,[1] and Guang-Can Guo[1,‡]
[1]*Key Laboratory of Quantum Information, Department of Physics, University of Science and Technology of China, Hefei 230026, People's Republic of China*
[2]*School of Physics and Optoelectronics Technology, Fujian Normal University, Fuzhou 350007, People's Republic of China*
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We propose a scheme for implementing quantum computation with N atoms trapped in a single-sided cavity. In our scheme, a multiqubit controlled-phase-flip (MCPF) gate as well as a quantum controlled phase flip gate is performed by only one step. The numerical simulations show that the protocol is robust to practical noise. In addition, the scheme can also be extended to achieve other multiqubit controlled unitary gates, and implement MCPF gate on atoms trapped in different cavities.

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I. INTRODUCTION

Quantum computing has attracted much interest since quantum algorithms are exponentially better than the best-known classical solutions [1]. The recent studies show that quantum algorithms need the techniques to efficiently implement general n-qubit gates [2]. Although multiqubit gates can be implemented by concatenating a sequence of two-qubit controlled-phase-flip (CPF) gates and single-qubit gates [3,4], the complexity of quantum circuits will have a great increase with a large number of multiqubit gates intervening. For instance, Barenco *et al.* showed that the simulation of N-qubit controlled-phase-flip gate with $N-1$ control qubits requires at least $N-1$ CPF gates [3]. Therefore, reducing the number of required physical logical gates is quite a significant task for practical quantum computation [5]. For a physical system, if we can easily implement general n-qubit gates as well as universal single-qubit and two-qubit logical gates, it will bring immense advantages to reduce the complexity of practical quantum computation.

Recently, an interesting scheme through cavity-assisted interaction has been proposed to realize the CPF gate between the single-photon pulses [6]. In this scheme, photons act as qubits. As we know, photons have some special advantages, such as the long coherence time and being easy to manipulate with linear optical methods to realize single-qubit logical gates. But, on the other hand, they are inconvenient to store. These features make photons more appropriate to be used as flying qubits to connect different nodes in a quantum network. Hence, in our proposal, atoms trapped in the optical cavity are adopted as storing qubits while photons as flying qubits. Although similar treatments have been found in a large number of references [7], our proposal has some peculiar advantages. At first, for N identical atoms trapped in an optical cavity, we can implement m-qubit CPF gates for the arbitrary $m(m<N)$ atoms in this optical cavity. Moreover, these multiqubit controlled-phase-flip (MCPF) gates can be realized just by one step, which may cause a great reduction of the complexity for some quantum circuits since it becomes dispensable that some multiqubit gates decompose into a sequence of universal single-qubit and two-qubit logical gates. The numerical simulations show that these MCPF gates are robust to certain practical sources of noise, such as randomness in the atom's position and atomic spontaneous emission. Furthermore, considering the scalability of quantum computation, we provide a further method to implement MCPF gate for the atoms located in different optical cavities.

The paper is arranged as follows: In Sec. II, we describe the fundamental model of the MCPF gate in detail. Based on the relation between input and output modes in cavity QED systems, we put forward a qualitative illustration for implementing a MCPF gate. In Sec. III, we do a numerical simulation in the situation in which three atoms are trapped in an optical cavity. The fidelity and success probability of the MCPF gate are also discussed. Sec. IV gives several applications of this model. Finally, we summarize the results in Sec. V.

II. THE FUNDAMENTAL MODEL AND THEORETICAL ANALYSIS

The schematic setup of the fundamental model is shown in Fig. 1 N identical alkali atoms, for example, ^{85}Rb atoms, are trapped in a resonant single-mode optical cavity, and separated from each other by more than one optical wavelength so that the dipole-dipole interaction can be neglected. A single-photon pulse with right-circular (R) polarization enters the cavity, after reflection from the one-sided cavity, and the MCPF gate for N atoms trapped in the cavity

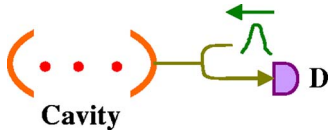

FIG. 1. (Color online) Schematic setup to implement the multiqubit controlled-phase-flip (MCPF) gate with N trapped atoms, where the input single-photon pulse is R polarization, and D is single-photon detector.

*Electronic address: xmlin@mail.ustc.edu.cn
†Electronic address: zwzhou@ustc.edu.cn
‡Electronic address: gcguo@ustc.edu.cn

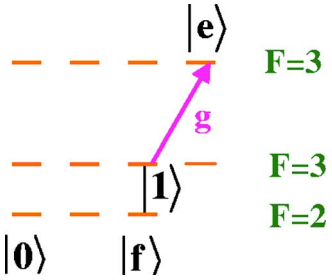

FIG. 2. (Color online) The relevant level structure and transition of rubidium atom. The states $|0\rangle$ and $|f\rangle$ ($|1\rangle$) correspond to the Zeeman sublevels of the $F=2(F=3)$ ground hyperfine level $5S_{1/2}$ while $|e\rangle$ corresponds to the excited hyperfine level of $5P_{3/2}$. $|1\rangle \rightarrow |e\rangle$ is resonantly coupled to the cavity mode a_R with coupling constant g.

is implemented. Figure 2 shows the relevant atomic levels and transitions. The states $|0\rangle$, $|f\rangle$, and $|1\rangle$ correspond to $|F=2,m=0\rangle$, $|F=2,m=2\rangle$, and $|F=3,m=2\rangle$ of $5S_{1/2}$, respectively, while $|e\rangle$ corresponds to $|F=3,m=3\rangle$ of $5P_{3/2}$. The qubit is represented by the atomic states $|0\rangle$ and $|1\rangle$. The transition $|1\rangle \rightarrow |e\rangle$ is resonantly coupled to the cavity mode a_R, which has R polarization and is resonantly driven by the input single-photon pulse with R polarization. In the rotating wave approximation and the rotating frame at the frequency ω_{e1} (i.e., the atomic transition frequency of $|1\rangle \rightarrow |e\rangle$), the whole Hamiltonian of the system of N atoms and cavity, including the coupling to the cavity output, has the following form (setting $\hbar=1$):

$$H = \sum_{j=1}^{N} (g_j a_R |e\rangle_j\langle 1| + g_j a_R^\dagger |1\rangle_j\langle e|) + \delta a_R^\dagger a_R + \int_{-\infty}^{\infty} \omega d\omega b_R^\dagger(\omega) b_R(\omega) + i\sqrt{\frac{\kappa}{2\pi}} \int_{-\infty}^{\infty} d\omega [a_R b_R^\dagger(\omega) - a_R^\dagger b_R(\omega)], \quad (1)$$

where g_j represents the coupling rate of the jth atom to cavity field, κ is the cavity decay rate, δ (here $\delta=0$, but we keep it for the following analysis) denotes the detuning of the cavity field mode a_R from the atomic transition, and $b_R(\omega)$ with the standard relation $[b_R(\omega), b_R^\dagger(\omega')]=\delta(\omega-\omega')$ denote the one-dimensional free-space modes which couple to the cavity mode a_R. Here, based on the fact that only terms which are almost resonant are important, the range $(-\infty,\infty)$ of the ω integration replacing $(-\omega_{e1},\infty)$ is a good approximation [8]. The Heisenberg equations of motion for $b_R(\omega)$ and a_R are

$$\dot{b}_R = -i\omega b_R + \sqrt{\frac{\kappa}{2\pi}} a_R(t) \quad (2)$$

and

$$\dot{a}_R(t) = -i[a_R(t), H_I] - i\delta a_R - \sqrt{\frac{\kappa}{2\pi}} \int_{-\infty}^{\infty} b_R(\omega) d\omega, \quad (3)$$

where the Hamiltonian

$$H_I = \sum_{j=1}^{N} (g_j a_R(t) |e\rangle_j\langle 1| + g_j a_R^\dagger(t) |1\rangle_j\langle e|) \quad (4)$$

depicts the coherent interaction between the atoms and the cavity mode a_R. We now define input and output field operators by [8]

$$b_R^{in}(t) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-i\omega t} b_0(\omega) d\omega \quad (5)$$

and

$$b_R^{out}(t) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-i\omega(t-t_1)} b_1(\omega) d\omega, \quad (6)$$

where $b_0(\omega)$ $[b_1(\omega)]$ is the value of $b_R(\omega)$ at $t=0$ $(t=t_1, t_1 > t)$ and has the same commutation relations as $b_R(\omega)$, so $b_R^{in}(t)$ and $b_R^{out}(t)$ satisfy the following commutation relations:

$$[b_R^{in}(t), b_R^{in+}(t')] = \delta(t-t'), \quad (7)$$

$$[b_R^{out}(t), b_R^{out+}(t')] = \delta(t-t'). \quad (8)$$

From Eqs. (2)–(6), we can learn that the single-sided cavity input $b_R^{in}(t)$ and output $b_R^{out}(t)$ are connected with the cavity mode $a_R(t)$ through the following relations [6,8]:

$$\dot{a}_R(t) = -i[a_R(t), H_I] - \left(i\delta + \frac{\kappa}{2}\right) a_R(t) - \sqrt{\kappa} b_R^{in}(t) \quad (9)$$

and

$$b_R^{out}(t) = b_R^{in}(t) + \sqrt{\kappa} a_R(t). \quad (10)$$

Eqs. (9) and (10) describe the evolution of the joint state of atoms and photon pulse.

Before solving exactly the problem through numerical simulations, we first give a rough theoretical analysis to understand the basic physics of the model. For convenience, the coupling rates of all atoms to the cavity field mode are taken to be the same $g_j=g$, but in Sec. III we will simulate the result for the case of unequal g_j. We use the notation $|n,N-n\rangle_a$ to denote the state of the N-atom system in which n atoms are in $|0\rangle$ and $N-n$ atoms in $|1\rangle$. Similar to Ref. [6], we have the following two cases: (i) If the N-atom system is in the state $|N,0\rangle_a$, the Hamiltonian H_I does not work. Similarly, we define the Fourier components of the intracavity field by

$$a_R(t) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-i\omega t} a_R(\omega) d\omega. \quad (11)$$

By substituting Eqs. (5), (6), and (11), into (9) and (10) we can obtain

$$b_R^{out}(t) \approx \frac{i\delta - \kappa/2}{i\delta + \kappa/2} b_R^{in}(t) \quad (12)$$

when nonresonant terms contribute very little, which is a valid approximation if the input pulse shape changes slowly with time t compared with the cavity decay rate κ. Therefore, in the case of resonant interaction $\delta=0$, we have

$b_R^{out}(t) \approx -b_R^{in}(t)$. (ii) If the N-atom system is in the state $|n, N-n\rangle_a$ $(0 \leqslant n \leqslant N-1)$, for the case of strong coupling [9], the
effective detunings of two dressed cavity modes from the input pulse are $\delta = \pm\sqrt{N-n}g$, respectively. So we have $b_R^{out}(t) \approx b_R^{in}(t)$ in the case that $g \gg \kappa$. When the detector D in Fig. 1 is triggered, the state of the whole system including N atoms, cavity, and free space is collided into the state described by the first term of right-hand sides of latter Eqs. (19) or (27). Consequently, we conclude that the state of the whole system acquires the phase π or 0 because $b_R^{out}(t) \approx -b_R^{in}(t)$ or $b_R^{out}(t) \approx b_R^{in}(t)$. The whole input-output process can be described by

$$|N,0\rangle_a|R\rangle \rightarrow -|N,0\rangle_a|R\rangle, \tag{13}$$

or

$$|n,N-n\rangle_a|R\rangle \rightarrow |n,N-n\rangle_a|R\rangle, \tag{14}$$

where we have discarded the state of cavity since it is always in the vacuum state, and $|R\rangle$ denotes the state of free-space photon. If the atoms are initially in a superposition state $(\lambda_N|N,0\rangle_a + \Sigma_{n=0}^{N-1}\lambda_n|n,N-n\rangle_a)$, this input-output process is

$$\left(\lambda_N|N,0\rangle_a + \sum_{n=0}^{N-1}\lambda_n|n,N-n\rangle_a\right)|R\rangle \rightarrow \left(-\lambda_N|N,0\rangle_a + \sum_{n=0}^{N-1}\lambda_n|n,N-n\rangle_a\right)|R\rangle, \tag{15}$$

where λ_n is superposition coefficient. After the detector D is triggered, the atomic state is collided into the state $(-\lambda_N|N,0\rangle_a + \Sigma_{n=0}^{N-1}\lambda_n|n,N-n\rangle_a)$. Thus we have realized the desired MCPF gate $U_N^{CPF} = e^{i\pi|N,0\rangle_a\langle N,0|}$ on atoms, which has $N-1$ control qubits. In other words, only if all of the atoms are initially in $|0\rangle$, the state of the N-atom system acquires the phase π after one single-photon pulse is reflected from the cavity; otherwise, the state of the N-atom system acquires the phase 0. Actually, the following numerical simulation results show that the MCPF gate works remarkably well even if $g \sim \kappa$.

With the same setup, we can also implement a two-qubit CPF gate U_2^{CPF} on any two atoms trapped in the cavity, which is important for universal quantum computation. Assume that the jth atom is known as the control qubit and the ξth atom as the target qubit; we firstly completely transfer the occupations of the state $|1\rangle$ of all atoms except the jth and ξth to those of the state $|f\rangle$, which is feasible with present technologies [10,11]. Then the single-photon input pulse with R polarization enters the cavity. After the detector D clicks, the occupations of the state $|f\rangle$ of atoms are again transferred to those of the state $|1\rangle$. Thus a CPF gate described by

$$U_2^{CPF} = U_2^{j\xi} = e^{i\pi|2,0\rangle_a\langle 2,0|} \tag{16}$$

between two desired atoms is performed. Following a similar consideration, we can easily implement the number of qubit-adjustable MCPF as we will, if only to alter the state $|1\rangle$ of the needless qubits to the state $|f\rangle$.

III. NUMERICAL SIMULATIONS AND BRIEF DISCUSSION

Next we adopt the Hamiltonian method to solve exactly the systemic dynamics [8,12,13]. After considering atomic spontaneous emission noise, Eq. (1) becomes

$$H' = \sum_{j=1}^{N}\left(-i\frac{\gamma}{2}|e\rangle_j\langle e| + g_j a_R|e\rangle_j\langle 1| + g_j a_R^\dagger|1\rangle_j\langle e|\right) + \int_{-\omega_b}^{+\omega_b}\omega d\omega b_R^\dagger(\omega)b_R(\omega) + i\sqrt{\frac{\kappa}{2\pi}}\int_{-\omega_b}^{+\omega_b}d\omega[a_R b_R^\dagger(\omega) - a_R^\dagger b_R(\omega)], \tag{17}$$

where γ is the spontaneous emission rate from the state $|e\rangle$, and we have taken an approximate condition that all the modes outside of the bandwidth $[\omega_{e1}-\omega_b, \omega_{e1}+\omega_b]$ have negligible contributions to related dynamics due to the strongly detuning. Assuming that the N-atom system is initially in the state

$$|\Psi(0)\rangle = \prod_{j=1}^{N}(\alpha_j|0\rangle_j + \beta_j|1\rangle_j), \tag{18}$$

with $|\alpha_j|^2 + |\beta_j|^2 = 1$, which can be prepared by Raman pulse sequences [10] or stimulated Raman adiabatic passage [11], it can be divided into the two following cases:

(i) When the N-atom system is initially in the state $|N,0\rangle_a$, the whole state of the system of N atom, cavity, and free space can be expanded into the following superposition at t time

$$|\Phi(t)\rangle = |N,0\rangle_a|0\rangle_c \otimes |\varphi(t)\rangle_f + d(t)|N,0\rangle_a|1\rangle_c|\text{vac}\rangle_f, \tag{19}$$

where

$$|\varphi(t)\rangle_f = \int_{-\omega_b}^{+\omega_b}c(\omega,t)b_L^\dagger(\omega)|\text{vac}\rangle_f d\omega \tag{20}$$

denotes the state of the single-photon output pulse, $|\text{vac}\rangle_f$ stands for the vacuum state of the free-space modes, $|m\rangle_c$ represents the state with m photons in cavity. According to the Schrödinger equation

$$i\partial_t|\Phi(t)\rangle = H'|\Phi(t)\rangle, \tag{21}$$

the coefficients $c(\omega,t)$ and $d(t)$ satisfy the following evolution equations:

$$\dot{c}(\omega,t) = -i\omega c(\omega,t) + \sqrt{\frac{\kappa}{2\pi}}d(t), \tag{22}$$

$$\dot{d}(t) = -\sqrt{\frac{\kappa}{2\pi}}\int_{-\omega_b}^{+\omega_b}c(\omega,t)d\omega. \tag{23}$$

For the numerical simulations, the free-space field $b_h(\omega)$ need to be discretized by introducing a finite but small frequency interval $\delta\omega$ between two adjacent modes, and the state $|\varphi(t)\rangle_f$ becomes [13]

$$|\varphi(t)\rangle_f = \sum_{\mu=1}^{M} c_\mu(\omega_\mu,t) b_\mu^+(\omega)|\text{vac}\rangle_f, \tag{24}$$

where b_μ represents the μth mode of the free space, $\omega_\mu=(\mu-M/2)\delta\omega$, and $M=2\omega_b/\delta\omega$. Thus, Eqs. (22) and (23) are replaced by

$$\dot{c}_\mu = -i\omega_\mu c_\mu + \sqrt{\frac{\kappa\delta\omega}{2\pi}} d, \tag{25}$$

$$\dot{d} = -\sqrt{\frac{\kappa\delta\omega}{2\pi}} \sum_{\mu=1}^{M} c_\mu. \tag{26}$$

At the time $t=0$, the coefficients $c_\mu(\omega_\mu,0)$ is determined by the corresponding Fourier transformation of input pulse $f(t)$, and $d(0)=0$.

(ii) When the N-atom system is initially in the state $|n,N-n\rangle_a$ $(0\leqslant n\leqslant N-1)$, where we assume that the first n atoms of N atoms are in $|0\rangle$, and thus the whole state of the system of N atom, cavity, and free-space can be expanded into the following superposition at t time:

$$|\Phi(t)\rangle = |n,N-n\rangle_a|0\rangle_c \otimes |\varphi'(t)\rangle_f + p(t)|n,N-n\rangle_a|1\rangle_c|\text{vac}\rangle_f + q(t)\sum_{\lambda=n+1}^{N} \frac{1}{\sqrt{N-n}}|n,N-n-1,e_\lambda\rangle_a|0\rangle_c|\text{vac}\rangle_f, \tag{27}$$

where

$$|\varphi'(t)\rangle_f = \sum_{\mu=1}^{M} c'_\mu(\omega_\mu,t) b_\mu^\dagger(\omega)|\text{vac}\rangle_f, \tag{28}$$

$|n,N-n-1,e_\lambda\rangle_a$ denotes the state of the first n atoms in the state $|0\rangle$, the λth atom in the state $|e\rangle$, and other $N-n-1$ atoms in the state $|1\rangle$. Following the same discretized procedure above, we can gain the following evolution equations:

$$\dot{c}'_\mu = -i\omega_\mu c'_\mu + \sqrt{\frac{\kappa\delta\omega}{2\pi}} p, \tag{29}$$

$$\dot{p} = -\sqrt{\frac{\kappa\delta\omega}{2\pi}} \sum_{\mu=1}^{M} c'_\mu - i\sqrt{N-n}gq, \tag{30}$$

$$\dot{q} = -i\sqrt{N-n}gp - \frac{\gamma}{2}q, \tag{31}$$

with $p(0)=q(0)=0$, $c'_\mu(\omega_\mu,0)=c_\mu(\omega_\mu,0)$, where we have assumed $g_j=g$ for simplicity. For other initial states $|n,N-n\rangle_a$, we can acquire the similar equations.

In the following, we present numerical calculation results for $N=3$. Assume that the input pulse is taken to be Gaussian pulse with $f(t)\propto\exp[-(t-T/2)^2/(T/5)^2]$, Fig. 3 shows the phase variation between input pulse and output pulse of the cavity, and demonstrates that the phase factor is either $e^{i\pi}$ or e^{i0} depending on the atomic state $|3,0\rangle_a$ or $|n,3-n\rangle_a$ $(0\leqslant n\leqslant 2)$. Where we have taken the input pulse $f(t)$ duration $T=5$ μs and referred to the parameters of Ref. [14] for numerical simulation, i.e., $g\approx 6\kappa$, $(\kappa,\gamma)/2\pi\approx(2.8,6)$ MHz. The fidelity of the MCPF gate $U_3^{CPF}=e^{i\pi|3,0\rangle_a\langle 3,0|}$ depends on the initially atomic state. We define the minimum of the fidelity F as the quality factor Q of the MCPF gate

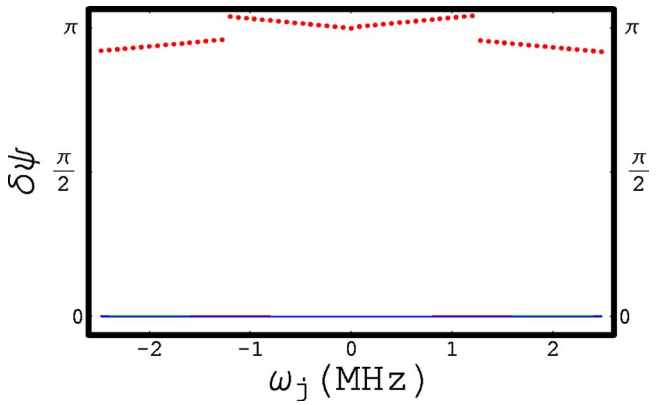

FIG. 3. (Color online) The phase variation $\delta\psi$ between input pulse and output pulses of cavity with three-atom in the state $|3,0\rangle_a$ (dotted curve), $|2,1\rangle_a$ (solid curve), $|1,2\rangle_a$ (solid curve), and $|0,3\rangle_a$ (solid curve), respectively. Here, we have taken $g=6\kappa$, $(\kappa,\gamma)/2\pi=(2.8,6)$ MHz, and $T=5$ μs.

$$Q = F_{\min} = |\langle\Psi_{ideal}|\Psi_{\text{real}}\rangle|^2, \tag{32}$$

where $|\Psi_{ideal}\rangle$ refers to the state of the atomic system in the ideal case after a single-photon pulse is reflected from the cavity; $|\Psi_{\text{real}}\rangle$ refers to the state of the atomic system by numerical simulations. The simulation shows that F has its minimum value $F_{\min}=Q\approx 0.989$ at $|\alpha_1|^2=|\alpha_2|^2=|\alpha_3|^2=1$, i.e., the initial state of atoms system is in the state $|3,0\rangle_a$, and the quality factor Q of the gate is independent of the variation of the coupling rate g. But, for the initial atomic state such as

$$|\Psi(0)\rangle = \prod_{j=1}^{3} \frac{1}{\sqrt{2}}(|0\rangle_j + |1\rangle_j), \tag{33}$$

the fidelity change is about 10^{-4} for g varying from 2κ to 10κ. If each $g_j(j=1,2,3)$ of Eq. (17) is unequal, for example, $g_1=2\kappa$, $g_2=6\kappa$, $g_3=10\kappa$, the gate fidelity is about 99.86% [given an initial atomic state such as Eq. (33)]. This means that the gate operation works remarkably well even if the atoms are not localized in the Lamb-Dicke regime, which is very important on account of current experimental technology. The main noise in our scheme arises from atomic spontaneous emission, which leads to a vacuum-state output. However, the spontaneous emission noise does not affect the gate fidelity but lowers the success probability. The highest probability of spontaneous emission happens for the initially atomic state $|2,1\rangle_a$. The reason is that the effective coupling rate g' [i.e., $g'=\sqrt{N-n}g$, see Eqs. (30) or (31)] is enhanced, i.e., $\sqrt{2}g$ or $\sqrt{3}g$, for the initially atomic state $|1,2\rangle_a$ or $|0,3\rangle_a$. Figure 4 shows the highest probability P_s of spontaneous emission as a function of g/κ, which is well simulated by the empirical formula $P_s\approx 1/(1+g^2/\kappa\gamma)$. In the case $g=15\kappa$, P_s is only about 0.95%. This confirms that spontaneous emission is effectively restrained for strong coupling. Recently, an important advance has been achieved in trapping and

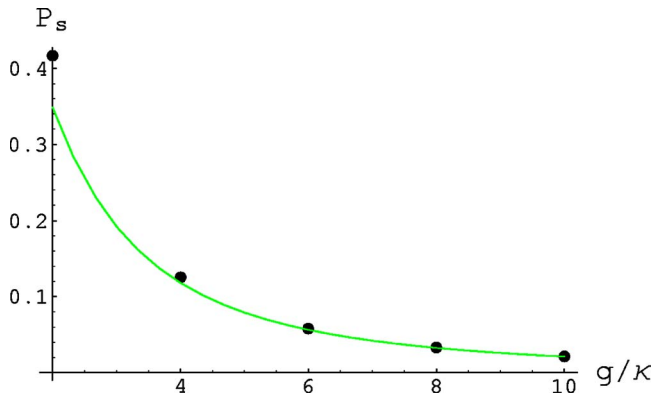

FIG. 4. (Color online) The highest probability P_s of spontaneous emission as a function of g/κ. The dots denote the results of numerical simulation, and the solid curve describes the empirical formula $P_s \approx 1/(1+g^2/\kappa\gamma)$. The parameters are the same as those in Fig. 3.

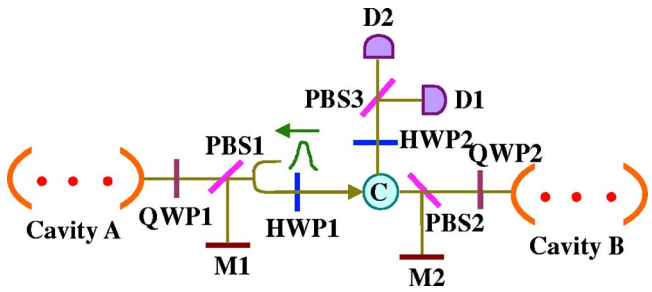

FIG. 5. (Color online) Schematic setup to implement the multiqubit controlled-phase-flip (MCPF) gate on distant atoms. Where PBS1, PBS2, and PBS3 symbolize polarization beam splitters, QWP1 and QWP2 are quarter-wave plates, HWP1 and HWP2 mark half-wave plates, M1 and M2 represent reflecting mirrors, D1 and D2 are single-photon detectors, and C denotes circulator. The optical paths from PBS1$\rightarrow$M1 (PBS2$\rightarrow$M2) and from PBS1$\rightarrow$QWP1, cavity A (PBS2$\rightarrow$QWP2, cavity B) are assumed to be equal.

cooling inside high-Q cavities in a regime of strong coupling [15–18]. In particular, Ref. [19] has reported trapping lifetimes in excess of 1 s, and the number of atoms trapped within the mode of an optical cavity can be determined in real time [20]. These technologic progresses will make it possible to implement experimentally the scheme in the future.

IV. APPLICATIONS

In this section, we will discuss some other applications of this model. First, we can take advantage of the MCPF gate to implement other important multiqubit gates by exerting some single-qubit operations—for example, a Toffoli gate. If the operation $U=H_3U_3^{CPF}H_3$ (where H_3 denotes the Hadamard operation on the third atom, which can be performed with classical laser pulses) is performed on the three-atom system trapped in a cavity, we can get the following outcomes: $|000\rangle\rightarrow|001\rangle$, $|001\rangle\rightarrow|000\rangle$, while other initially atomic states keep unchanged. Thus the Toffoli gate is realized by one step, while it needs six CPF gates in general quantum circuit [21]. Here, "one step" means that it just need one step of a physical process to realize the nonlocal operation. Of course, for realizing a Toffoli gate some secondary local operations are necessary. It does not allow for the quantity of local gate operations because, in our architecture of quantum computing, implementing local operations is easy to handle compared with performing nonlocal operations. This scenario also takes place in other model of quantum computing. Thus, we may further reduce the "actual" complexity of quantum computing to the quantity of nonlocal operations in quantum circuits. Our present scheme exactly makes a great reduction for MCPF gate. As we know, to construct an n-qubit Toffoli gate ($n>3$), $2^{n-2}+2^{n-3}-2$ Toffoli gates are needed [4], while in our proposal, concatenating exponential time nonlocal operations reduce to a single actual operation. Although there is no theory which shows that this reduction will essentially lead to the decline of computational complexity of quantum algorithms, some special algorithms can obtain great advantages from this reduction [22].

A more significant application of the model is that it can be used to implement MCPF gate on atoms trapped in different cavities. Based on current experimental technology, the above model is not readily scaled up to large-scale quantum computation, which requires that many atoms be separately addressed within a tiny optical cavity. Therefore, it is very important for scalable quantum computing to implement the connection between cavities. The setup for realizing the idea is shown in Fig. 5. Two single-mode cavities A and B trap N and M ^{85}Rb atoms, respectively, which are initially in the state described by Eq. (18). The polarization beam splitters PBS1, PBS2, and PBS3 transmit only horizontal (h) polarization component and reflect the vertical (v) polarization component. The quarter-wave plates QWP1 and QWP2 perform the transformation between R polarization and h polarization photons. The v polarized component of input pulse is reflected without shape and phase changes by the mirrors M1 and M2. Based on the results above, after reflection from cavity A (B), the h polarized component of input pulse acquires a phase of $e^{i\pi}$ if the N- (M-) atom system is initially in the state $|N,0\rangle_{aA}$ ($|M,0\rangle_{aB}$); however, the phase of h photon pulse keeps unchanged if the N- (M-) atom system is initially in the state $|n_A,N-n_A\rangle_{aA}$ ($|n_B,M-n_B\rangle_{aB}$), where $n_A\neq N$ ($n_B\neq M$). Two half-wave plates HWP1 and HWP2 change the state $|h\rangle$ into $1/\sqrt{2}(|v\rangle-|h\rangle)$ or the state $|v\rangle$ into $1/\sqrt{2}(|v\rangle+|h\rangle)$. The unidirectional coupling between the two single-sided cavities is achieved by circulator. The input single-photon pulse with initial state $|\phi\rangle_p=1/\sqrt{2}(|h\rangle+|v\rangle)$ pass through the following way: PBS1$\rightarrow$QWP1, cavity A(or M1)$\rightarrow$PBS1$\rightarrow$HWP1$\rightarrow C\rightarrow$PBS2$\rightarrow$QWP2, cavity B(or M2)$\rightarrow$PBS2$\rightarrow C\rightarrow$HWP2$\rightarrow$PBS3. When the detector D1 is triggered, one can find that a MCPF gate for $N+M$ atoms described by $U_{N+M}^{CPF}=e^{i\pi|N+M,0\rangle_a\langle N+M,0|}$ is performed. But when the detector D2 is triggered, we must let single-photon pulse with R polarization enter cavity A again and be reflected (as shown in Fig. 1). After the detector D triggered, the same unitary operation U_{N+M}^{CPF} is performed. In particular, if cavities A and B both trap only one atom, through the

above process the state of two atoms is prepared into

$$|\Psi\rangle = -\alpha_1\alpha_2|00\rangle + \alpha_1\beta_2|01\rangle + \beta_1\alpha_2|10\rangle + \beta_1\beta_2|11\rangle. \tag{34}$$

Namely we have implemented two-qubit phase gate $U_2^{CPF} = e^{i\pi|2,0\rangle_a\langle 2,0|}$. If $\alpha_i = \beta_i = 1/\sqrt{2}(i=1,2)$, the state (34) can be written as

$$|\Psi\rangle = \frac{1}{\sqrt{2}}(|0\rangle|-\rangle + |1\rangle|+\rangle) \tag{35}$$

where $|+\rangle = 1/\sqrt{2}(|1\rangle + |0\rangle)$, $|-\rangle = 1/\sqrt{2}(|1\rangle - |0\rangle)$, i.e., we have prepared the maximally entangled state between distant atoms, which is one of key problems of quantum computation and quantum communication.

V. CONCLUSION

In conclusion, we have made a protocol to realize not only the CPF gate but also the MCPF gate by one step. The direct implementation of multiqubit controlled-phase-flip gate would be more efficient than implementation built from a series of one- and two-qubit rotations, and this efficiency would become even more significant with an increasing number of the qubits. Obviously, a smaller number of gate steps keeps the scheme easier to implement from the experimental point of view. The numerical simulations show that the scheme has the following significant advantages: (i) It is so insensitive to variation of the atom-photon coupling rate that it has high fidelity even if the atoms are not localized in the Lamb-Dicke regime. (ii) The scheme is inherently robust to atomic spontaneous emission, which simply decreases the success probability but exerts no influence on the fidelity of the gate. In addition, the protocol may be adopted to achieve other implement multiqubit controlled unitary gates, and implement MCPF gate on distant atoms for large-scale quantum computation.

Note added.—After submission of this work, we become aware that similar ideas have been recently proposed by Duan, Wang, and Kimble [23] independently.

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