# Single-particle entanglement

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I give a simple argument that demonstrates that the state $|0\rangle|1\rangle+|1\rangle|0\rangle$, with $|0\rangle$ denoting a state with 0 particles (or photons) and $|1\rangle$ a one-particle state, is entangled in spite of recent claims to the contrary. I also discuss viewpoints on the old controversy about whether the above state can be said to display single-particle or single-photon nonlocality.

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Every now and then I hear or read the claim that there is no entanglement in the state

$$|\psi\rangle_{A,B} = |0\rangle_A|1\rangle_B + |1\rangle_A|0\rangle_B, \tag{1}$$

where $|0\rangle_{A,B}$ and $|1\rangle_{A,B}$ denote states with zero and one particles, respectively, in modes $A$ and $B$. (See, for example, Ref. [1]; on the other hand, see also papers [2] that use or propose to use the same state for teleportation, quantum cryptography, or violating Bell inequalities, or that perform tomography on a similar state.) The reason for that claim is usually one of the following:

(i) One needs at least two particles for entanglement.

(ii) The state of Eq. (1) when written in second-quantized form as

$$|\psi\rangle_{A,B} = (a_A^\dagger + a_B^\dagger)|\text{vacuum}\rangle \tag{2}$$

clearly has no entanglement.

(iii) The entanglement is a property of a pathological representation of CCR/CAR algebras [1].

But here is a simple counterargument [3] that shows there is in fact entanglement in the state (1) provided modes $A$ and $B$ are spatially separated [5]. Just for the argument let us assume the particles are photons [6]. Also let us assume we place a cavity in each of the locations of the modes $A$ and $B$ and put an atom, initially in a ground state denoted by $|g\rangle$, inside each cavity. There are techniques [7] to make sure a photon in the proper mode will enter the cavity and excite the atom to a particular excited state, denoted by $|e\rangle$. In the ideal case, this process occurs with 100% efficiency. Starting with the two atoms both in state $|g\rangle$ we can then generate the joint atomic state (where the obvious assumption is made that the vacuum will not excite the atom)

$$|\Psi\rangle_{A,B} = |g\rangle_A|e\rangle_B + |e\rangle_A|g\rangle_B, \tag{3}$$

where $A$ and $B$ now refer to the locations of the atoms. The joint state of the two (photonic) modes is no longer relevant or entangled as both modes end up in the state $|0\rangle$.

In the state (3) there are two particles so that objection (i) from the above list does not apply. Furthermore, no one would insist on writing the state of the two atoms in separate cavities in a second-quantized form, so objection (ii) would not be raised. Moreover, the atoms used in the above-mentioned procedure do not have to be identical at all, so Eq. (2) would not apply in any case, and there are no problems arising from the role of quantum statistics of identical particles in the definition of entanglement. Finally, no one would complain about pathological representations of any sort of algebras when discussing (nonidentical) atoms. Thus I would say there is no doubt there is entanglement in the state (3). But since that state can be generated in principle, as just shown, from the state (1) by *local* operations, I would conclude that the state $|0\rangle|1\rangle+|1\rangle|0\rangle$ must have entanglement too. That concludes the simple argument.

Some further remarks are in order: First, in the famously [8] entangled two-mode squeezed state one has a Fock-state expansion $|0\rangle|0\rangle+|1\rangle|1\rangle+|2\rangle|2\rangle\cdots$ in the ideal (unrealistic) case of infinite squeezing, but in that case no one complains about the "vacuum term." Moreover, in a realistic finitely squeezed two-mode squeezed state the $|0\rangle|0\rangle$ term has, in fact, the largest amplitude. For example, for small amounts of squeezing the state is, approximately, $|0\rangle|0\rangle+r|1\rangle|1\rangle$, with $r\ll 1$, which has a small amount of entanglement on the order of $r^2\log_2(r)$, see Ref. [9] for further details.

Second, the entanglement in the state (1) is *not* between the photon and the vacuum, but between modes $A$ and $B$. This point has been made in more generality (for different physical systems, for different types of states and relative to sets of observables) in Ref. [5]. Similarly, in the case of the two-mode squeezed state with small squeezing, the entanglement is *not* between two photons and the vacuum: here the name of the state quite appropriately indicates what *is* entangled.

Third, a different reason altogether for not attributing entanglement to the state (1) under certain conditions is given in Ref. [10]. That paper refers to the situation where the relative phase between the two states $|0\rangle_A|1\rangle_B$ and $|1\rangle_A|0\rangle_B$ is not well defined. This occurs when, e.g., the two parties located at $A$ and $B$ do not share a reference that defines that phase (for instance, a clock or a spatial reference frame). More precisely, suppose Alice and Bob, to use modern parlance, share a state

$$|\psi\rangle_{A,B} = |0\rangle_A|1\rangle_B + \exp(i\phi)|1\rangle_A|0\rangle_B, \tag{4}$$

where $\phi$ is defined relative to a (possibly fictituous) third-party reference frame; the states $|0\rangle$ and $|1\rangle$ may refer now to any types of orthogonal quantum states, be it polarization states of single photons, states of Josephson junctions with different charges, or spin "up" and "down" states of electrons. Alice and Bob may have their own local reference frames but the difference between their local phases or their relative orientation is not known to them. Hence Alice and Bob do not know the phase $\phi$ and so they would in fact not assign the state (4) but rather a mixture over the unknown phase $\phi$ to a single copy [11]. The description (4) is used by anyone with access to the third-party reference frame. In this situation Alice and Bob cannot make use of a single copy [11] of the state (4) for teleportation [12] or violating Bell inequalities. In that sense, according to Alice and Bob, there is no entanglement between Alice and Bob's systems $A$ and $B$ when they do not share a reference frame. Of course, when they *do* share a reference frame (and in experiments this is always explicitly or implicitly assumed), there *is* entanglement (see also Ref. [4]). Note, for example, that the abovementioned operation involving atoms in cavities requires a phase reference, too. For completeness let us note, in contrast, that even in the absence of a shared reference frame, one can still perform quantum communication protocols and violate Bell inequalities, not by using a state of the form (4) but by using reference-frame invariant encoding, as discussed in Ref. [13].

Fourth, in the 1990s a related but different discussion arose as to whether nonlocality can arise from a single-particle or single-photon state [14]. The issue then was not whether there is entanglement in the state $|0\rangle|1\rangle+|1\rangle|0\rangle$ (apparently, there was agreement there is entanglement), but whether an experiment using that state can demonstrate nonlocality with just one particle or photon. The idea is simply that all proposed (and in the meantime performed) optics experiments with the state (1) detect, at least sometimes, more than a single photon. In that case, it was argued, nonlocality arises from multiparticle entanglement. We can add some interesting insights to that discussion by relating it to the role reference frames play in quantum-communication protocols

In certain types of experiments the shared reference frame is such a trivial resource that no one cares to mention it. This applies, for instance, to experiments using a spatial reference frame (the earth or the fixed stars). On the other hand, the role of a clock (another example of a reference frame) in optics experiments is inevitably, conveniently, and quite visibly, played by lasers (e.g., Alice and Bob both having a laser, phase-locked to one another) [15]. The confusing aspect is that in optics experiments on Bell inequalities photons are detected that may originate both from the entangled state (1) and from the phase reference laser beam. In contrast, in experiments with a spin-entangled electron pair or a polarization-entangled photon pair the particles making up the spatial reference frame are *not* detected by the same detector that detects the electrons or photons. Hence it may seem that indeed only two electrons or two photons have to be detected. However, this apparent distinction is not so clear: One could argue, on the one hand, that the reference frame particles *are* detected, not by a detector but by the experimenter. On the other hand, one could argue that in optics experiments a different sort of clock could be used, at least in principle, that requires no photons (say, based purely on electronics). In that case, Bell inequalities could be violated using just the single-photon state (1) without more than one photodetector clicking.

In fact, this is a good example of the difference between "internal" and "external" reference frames [16]: In optics experiment one is more inclined to treat the laser field as an internal reference frame that must be quantized too, whereas the earth or the fixed stars are typically treated as a classical, external, reference frame. However, as a matter of principle, there is *no* difference between those two cases, and in both cases one has a choice whether to internalize or externalize the reference frame.

In short, if one has the point of view that a singlet state of two spin-entangled electrons or two polarization-entangled photons can display "two-particle nonlocality," then it is just as valid to claim that the state (1) can display "single-particle nonlocality." In particular, $|0\rangle|1\rangle+|1\rangle|0\rangle$ is entangled.

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[1] M. Pawlowski and M. Czachor, quant-ph/0507151, and references therein.

[2] A. I. Lvovsky *et al.*, Phys. Rev. Lett. **87**, 050402 (2001); G. Björk, P. Jonsson, and L. L. Sanchez Soto, Phys. Rev. A **64**, 042106 (2001); E. Lombardi *et al.*, Phys. Rev. Lett. **88**, 070402 (2002); J. W. Lee *et al.*, Phys. Rev. A **68**, 012324 (2003); B. Hessmo *et al.*, Phys. Rev. Lett. **92**, 180401 (2004); S. A. Babichev *et al.*, *ibid.* **92**, 193601 (2004).

[3] This argument is not new: it appears in C. C. Gerry, Phys. Rev. A **53**, 4583 (1996). Also, M. S. Kim has used this argument in the same way [M. S. Kim (private communication)]. See also Ref. [4].

[4] Y. Aharonov and L. Vaidman, Phys. Rev. A **61**, 052108 (2000).

[5] The qualification that the modes be spatially separated is important, and in certain cases absolutely crucial. This should be clear from the argument presented here, but also from S. J. van Enk, Phys. Rev. A **67**, 022303 (2003); see also P. Zanardi, *ibid.* **65**, 042101 (2002); Y. Shi, *ibid.* **67**, 024301 (2003); and for a generalized and more technical notion of what is important for entanglement, see H. Barnum *et al.*, quant-ph/0305023; quant-ph/0506099.

[6] Actually, the argument would work just as well in principle with electrons instead of photons, as electrons, too, can be used to excite atoms.

[7] J. I. Cirac *et al.*, Phys. Rev. Lett. **78**, 3221 (1997). Actually, how one does this precisely, or whether one can achieve this perfectly experimentally at the present moment is not relevant.

[8] Famously, because the original EPR [A. Einstein, B. Podolski, and N. Rosen, Phys. Rev. **47**, 777 (1935)] state is equivalent to an infinitely squeezed two-mode squeezed state.

[9] The two-mode squeezed state for general (finite) squeezing parameter $r$ is [D. Walls and G. Milburn, *Quantum Optics* (Springer-Verlag, Berlin, 1994)]

$$\frac{1}{\cosh r}\sum_{n=0}^{\infty}(\tanh r)^n|n\rangle|n\rangle.$$

The entanglement in this state is [S. J. van Enk, Phys. Rev. A **60**, 5095 (1999)]

$$E=\cosh^2 r\log_2(\cosh^2 r)-\sinh^2 r\log_2(\sinh^2 r).$$

[10] H. M. Wiseman and J. A. Vaccaro, Phys. Rev. Lett. **91**, 097902 (2003).

[11] As shown in Ref. [10] there is entanglement in two (or more) copies of the same state (4); see also S. J. van Enk, Phys. Rev. A **71**, 032339 (2005). This demonstrates the asymmetry in the roles the state (4) and the reference frame play: although both seem to be needed to demonstrate entanglement, the entanglement can be said to arise from the state (4), not from the reference frame.

[12] S. J. van Enk, J. Mod. Opt. **48**, 2049 (2001) pointed out the importance of a shared reference frame for teleportation, but see Ref. [13] for an important exception.

[13] S. D. Bartlett, T. Rudolph, and R. W. Spekkens, Phys. Rev. Lett. **91**, 027901 (2003).

[14] S. M. Tan, D. F. Walls, and M. J. Collett, Phys. Rev. Lett. **66**, 252 (1991); L. Hardy, *ibid.* **73**, 2279 (1994); A. Peres, *ibid.* **74**, 4571 (1995); L. Vaidman, *ibid.* **75**, 2063 (1995); D. M. Greenberger, M. A. Horne, and A. Zeilinger, *ibid.* **75**, 2064 (1995); L. Hardy, *ibid.* **75**, 2065 (1995).

[15] H. M. Wiseman, J. Opt. B: Quantum Semiclassical Opt. **6**, S849 (2004).

[16] For a detailed and amusing account, see S. D. Bartlett, T. Rudolph, and R. W. Spekkens, quant-ph/0507114.