Quantum correlations of an atomic ensemble via an incoherent bath

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A rich variety of quantum features can be found in a collection of atoms driven only by an incoherent bath. To demonstrate this, we discuss a sample of three-level atoms in ladder configuration interacting via the surrounding bath, and show that the fluorescence light emitted by this system exhibits nonclassical properties. Realizations could be thermal baths for microwave transitions, or incoherent broadband fields for optical transitions. In a small sample of atoms, the emitted light can be switched from sub- to super-Poissonian and from antibunching to superbunching controlled by the mean number of atoms in the sample. Larger samples allow us to generate superbunched light over a wide range of bath parameters and thus fluorescence light intensities. We also identify parameter ranges where the fields emitted on the two transitions are strongly correlated or anticorrelated, such that the Cauchy-Schwarz inequality is violated. As in a moderately strong bath this violation occurs also for larger numbers of atoms, such samples exhibit macroscopic quantum effects.

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Initiated by Dicke [1], ensembles of few-level emitters interacting collectively with an environmental reservoir have been shown to be a source for many remarkable effects and applications [1–12]. In the recent past, this interest was renewed by the possible applications of such samples to quantum communications and logic devices. For example, the production of nonclassical light has been a subject of several recent experiments [13,14]. On the other hand, it was demonstrated that long-time entanglement between two arbitrary qubits can be generated if they interact with a common heat bath [15]. Thus quantum features can be induced through the interactions with an incoherent electromagnetic field, which may even be interpreted as a classical bath. This is not obvious, as, usually, it is believed that an interaction with a large environmental reservoir leads to decoherence. Thermal light may also produce effects like ghost imaging or subwavelength interference [16], which otherwise are known to occur for entangled light [17]. These results are of special interest, as nonclassical driving fields are often hard to produce experimentally with adequate intensities. Therefore schemes which allow us to extract quantum features from atoms in an otherwise incoherent external setup are highly desirable.

Here, we demonstrate that such a conversion scheme may be implemented with an ensemble of atomic few-level systems subject to an incoherent bath. For this, we discuss a three-level setup in ladder configuration, and show that the fluorescence light emitted spontaneously on the two transitions has nonclassical properties. The bath could, e.g., be a thermal bath for atoms with microwave transition frequencies, or incoherent broadband driving for the optical frequency region. In particular, we demonstrate that in a small sample of atoms, the mean number of atoms in the sample allows to switch the light emitted on one of the transitions from sub- to super-Poissonian statistics, and from anti- to superbunching. Here, both super- and antibunched light with super-Poissonian statistics can be produced. Larger samples, on the other hand, can be used to generate superbunched light over a wide range of bath parameters, thus enabling a control of the intensity of the strongly correlated light. We also identify parameter ranges where the fields emitted on the two transitions are correlated or anticorrelated. As an application for this, we show that the Cauchy-Schwarz inequalities are violated for a moderately strong reservoir and large samples, thus demonstrating a macroscopic quantum effect.

The basic element of our study is a sample of N identical nonoverlapping three-level radiators which interact with an incoherent reservoir; see Fig. 1. The emitters are located within a volume with linear dimensions smaller than the relevant emission wavelengths λ_{12}, λ_{23}, and densities low enough to avoid collisions (Dicke model) [1–11]. The excited atomic level $|1\rangle$ ($|2\rangle$) spontaneously decays to the state $|2\rangle$ ($|3\rangle$) with a decay rate $2\gamma_1$ ($2\gamma_2$). The only external driving is via the surrounding bath, which induces transitions

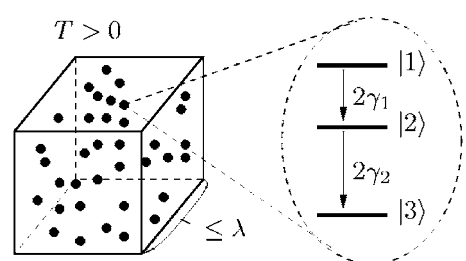

FIG. 1. The system setup. A sample of N atoms, each with three atomic states in ladder configuration, is confined to a region small as compared to the emission wavelengths λ. The whole setup is placed in an incoherent bath. The excited states decay spontaneously with rates $2\gamma_{1(2)}$, giving rise to fluorescence lights with nonclassical features.

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among the atomic levels. In the microwave region, the bath could be a thermal bath, where the rates are proportional to the mean thermal photon number at the corresponding transition frequencies. In the optical frequency region, thermal excitations are negligible, and the bath can be realized by a pseudothermal bath, i.e., broadband incoherent driving fields perpendicular to the observation direction. Then, the rates depend on the field strength of the incoherent fields. Possible atomic systems for a realization are, e.g., Rb or Cs Rydberg states in the microwave region, or low-lying atomic states in Rb at optical frequencies [5]. In the usual mean-field, Born-Markov, and rotating-wave approximations, the system is described by the following master equation [6,8]:

$$\dot{\rho}(t) = -\gamma_1(1+\bar{n}_1)[S_{12},S_{21}\rho] - \gamma_2(1+\bar{n}_2)[S_{23},S_{32}\rho] - \gamma_1\bar{n}_1[S_{21},S_{12}\rho] - \gamma_2\bar{n}_2[S_{32},S_{23}\rho] + \mathrm{H.c.} \tag{1}$$

Here, an overdot denotes differentiation with respect to time. For thermal baths,

$$\bar{n}_i = \frac{1}{\exp(\beta\hbar\,\omega_{i,i+1}) - 1} \tag{2}$$

is the mean thermal photon number at transition frequency $\omega_{i,i+1}=\omega_i-\omega_{i+1}$ and for temperature T, where $\beta=(k_BT)^{-1}$ with k_B as the Boltzmann constant. For pseudothermal baths,

$$\bar{n}_i = \frac{R_{i,i+1}d_{i,i+1}^2}{\gamma_i\hbar^2}, \tag{3}$$

where $R_{i,i+1}$ describes the strength of the incoherent pumping [18]. It is important to note that Eq. (1) contains collective atomic operators $S_{ij}=\Sigma_{k=1}^N|i\rangle_{kk}\langle j|$, which describe populations for $i=j$, transitions for $i\neq j$, and which obey the commutation relation $(i,j\in\{1,2,3\})$ [8]

$$[S_{ij},S_{i'j'}] = \delta_{ji'}S_{ij'} - \delta_{j'i}S_{i'j}. \tag{4}$$

The steady-state limit of Eq. (1) can conveniently be evaluated with the help of coherent atomic states $|N,n,m\rangle$ for the su(3) algebra, which denote a symmetric collective state of N atoms with n atoms in bare state $|1\rangle$, $m-n$ in bare state $|2\rangle$, and $N-m$ atoms in bare state $|3\rangle$ with $0\leqslant n\leqslant N$, $n\leqslant m\leqslant N$ [6–9]. The collective operators S_{ij} act on these states as follows:

$$S_{31}|N,n,m\rangle = \sqrt{n(N-m+1)}|N,n-1,m-1\rangle, \tag{5a}$$

$$S_{32}|N,n,m\rangle = \sqrt{(m-n)(N-m+1)}|N,n,m-1\rangle, \tag{5b}$$

$$S_{21}|N,n,m\rangle = \sqrt{n(m-n+1)}|N,n-1,m\rangle, \tag{5c}$$

$$S_{11}|N,n,m\rangle = n|N,n,m\rangle, \tag{5d}$$

$$S_{22}|N,n,m\rangle = (m-n)|N,n,m\rangle. \tag{5e}$$

The diagonal elements $P_{nm}=\langle N,n,m|\rho_{ss}|N,n,m\rangle$ of the steady-state density operator ρ_{ss} evaluate to

$$P_{nm} = (1-\eta_2)\eta_1^n\eta_2^m\left[\frac{1-(\eta_1\eta_2)^{N+1}}{1-\eta_1\eta_2} - \eta_2^{N+1}\frac{1-\eta_1^{N+1}}{1-\eta_1}\right]^{-1}, \tag{6}$$

with $\eta_i=\bar{n}_i/(1+\bar{n}_i)$, $(i\in\{1,2\})$. From Eq. (6), the atomic expectation values can easily be evaluated.

As a reference, we first consider the single atom case $N=1$, where $P_{nm}=\eta_1^n\eta_2^m/[1+\eta_2+\eta_1\eta_2]$, and the mean steady-state populations are given by

$$\langle S_{11}\rangle_s = \frac{\eta_1\eta_2}{1+\eta_2+\eta_1\eta_2}, \tag{7a}$$

$$\langle S_{22}\rangle_s = \frac{\eta_2}{1+\eta_2+\eta_1\eta_2}, \tag{7b}$$

$$\langle S_{33}\rangle_s = \frac{1}{1+\eta_2+\eta_1\eta_2}. \tag{7c}$$

It is easy to see that the ratios of these populations obey the equilibrium Boltzmann distribution $(i,j=1,2,3)$

$$\frac{\langle S_{ii}\rangle_s}{\langle S_{jj}\rangle_s} = e^{-\beta\hbar\omega_{ij}}. \tag{8}$$

Things are different for an ensemble of N atoms. Then the collective interaction between the atoms drives the system into a steady-state different to a Boltzmann distribution. For instance, if $\eta_{1,2}<1$ and $N\gg 1$ such that $(\eta_1\eta_2)^N\to 0$ and $\eta_{1(2)}^N\to 0$, then one finds $P_{nm}=(1-\eta_2)(1-\eta_1\eta_2)\eta_1^n\eta_2^m$. The populations are

$$\langle S_{11}\rangle_s = \frac{\eta_1\eta_2}{1-\eta_1\eta_2}, \tag{9a}$$

$$\langle S_{22}\rangle_s = \frac{\eta_2}{1-\eta_2}, \tag{9b}$$

$$\langle S_{33}\rangle_s = N - \langle S_{11}\rangle_s - \langle S_{22}\rangle_s, \tag{9c}$$

thus violating Eq. (8). A very strong thermal bath $(\eta_i\to 1)$ leads to an equal distribution of the atoms on the energy levels with $P_{nm}=1/[(N+1)(N/2+1)]$. Finally, setting $\eta_1=0$ in Eq. (6), one recovers the thermal atomic distribution for two-level atoms $(|2\rangle\leftrightarrow|3\rangle)$, $P_m=(1-\eta_2)\eta_2^m/(1-\eta_2^{N+1})$ $(m=0,1,\ldots,N)$ [3]. Thus for $\eta_2<1$ and $N\gg 1$ such that $\eta_2^N\to 0$, the emitters obey the Bose-Einstein statistics [4].

We now turn to our main interests in this study, the coherence properties of the collective fluorescent light generated on the transitions $|1\rangle\to|2\rangle$ and $|2\rangle\to|3\rangle$. The photons emitted on the two transitions are distinguishable by their polarizations and frequencies, and can be detected, e.g., by a pair of single-photon detectors (optical frequency region) or by atomic state detection (microwave transitions) [4,5]. The second-order coherence function is defined as [19]

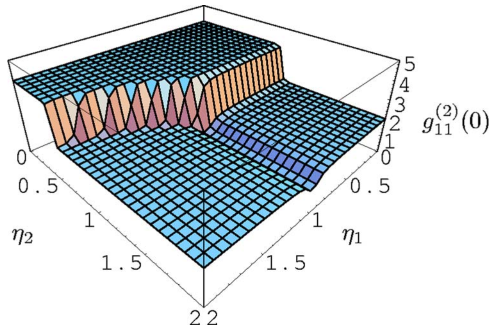

FIG. 2. (Color online) Plot of the second-order correlation function $g_{11}^{(2)}(0)$ against bath parameters η_1, η_2. The number of atoms in the sample is $N=150$.

$$g_{ij}^{(2)}(\tau)=\frac{\langle J_i^{+}(t)J_j^{+}(t+\tau)J_j(t+\tau)J_i(t)\rangle}{\langle J_i^{+}(t)J_i(t)\rangle\langle J_j^{+}(t)J_j(t)\rangle}, \tag{10}$$

where $i,j\in\{1,2\}$ with $J_1=S_{21}$ and $J_2=S_{32}$. The quantity $g_{ij}^{(2)}(\tau)$ can be interpreted as a measure for the probability for detecting one photon emitted on transition i and another photon emitted on transition j with time delay τ. $g_{ij}^{(2)}(0)<1$ characterizes sub-Poissonian, $g_{ij}^{(2)}(0)>1$ super-Poissonian, and $g_{ij}^{(2)}(0)=1$ Poissonian photon statistics. $g_{ij}^{(2)}(\tau)>g_{ij}^{(2)}(0)$ is the condition for photon antibunching, whereas $g_{ij}^{(2)}(\tau)<g_{ij}^{(2)}(0)$ means bunching. We further define superbunching as bunching with $g_{ij}^{(2)}(0)>2$ [20,21]. More specific, correlation functions with $i=j$ describe the photon statistics of the fluorescence light emitted on a single atomic transition, and $g_{i\neq j}^{(2)}(0)$ the cross correlations between the photon emission on two different transitions. We also need to consider the fluorescence intensities $G_i^{(1)}(0)=\langle J_i^{+}J_i\rangle_s$ of the two transitions. For example, applications may require particularly strong or weak nonclassical fields. On the other hands, in the microwave region, the signal from the sample competes with noise from the surrounding heat bath proportional to $\bar{n}_i$, which especially for small samples of atoms with low signal can render experimental verifications difficult. For bath parameters $0<\eta_1$, $\eta_2<1$ and larger samples $N\gg1$ such that $\eta_{1(2)}^{N}\to0$, one has

$$G_1^{(1)}(0)=\frac{\eta_1\eta_2}{(1-\eta_2)(1-\eta_1\eta_2)}=\frac{\bar{n}_1\bar{n}_2}{1+\bar{n}_1/(1+\bar{n}_2)}, \tag{11a}$$

$$G_2^{(1)}(0)=\frac{\eta_2}{1-\eta_2}\left(N-\frac{2\eta_2}{1-\eta_2}-\frac{\eta_1\eta_2}{1-\eta_1\eta_2}\right)\approx\bar{n}_2N. \tag{11b}$$

Thus $G_1^{(1)}(0)$ does not depend on N explicitly, while $G_2^{(1)}(0)$ increases linearly with N. In the strong-field limit ($\eta_1,\eta_2\to1$), one has $G_{1(2)}^{(1)}(0)=N(3+N)/12\sim N^2$.

The first results are shown in Fig. 2, where the correlation function $g_{11}^{(2)}(0)$ of the light emitted on transition $|1\rangle\to|2\rangle$ is plotted against η_1 and η_2 for a sample of $N=150$ atoms. It can be seen that photons with super-Poissonian statistics are generated on this transition for moderate baths. In this region, the emitted light is superbunched except for small values of η_2, where the light is antibunched. This range of η_2 for antibunching depends on the number of atoms N and η_1. In Fig. 2, for $\eta_1<0.6$, antibunching requires $\eta_2<0.01$. For smaller samples, however, the range for antibunching increases. If η_2 is small, then almost all atoms are in the ground state $|3\rangle$ due to collective effects, and on average at most one atom gets excited to $|1\rangle$ to emit light contributing to $g_{11}^{(2)}(0)$. Thus the light is antibunched. For higher η_2, more atoms can be excited to $|1\rangle$ simultaneously, and the light is bunched. As the superbunched photons are produced over a wide range of values for η_i, the intensity of the generated light can be controlled via the bath parameters $\bar{n}_1$, $\bar{n}_2$ from very weak up to intense flux. In other words, by modifying the reservoir characteristics, we can obtain a low or an intense flux of strongly correlated photons. This setup is particularly suitable for microwave transitions, as then thermal baths with high values for η_i can be achieved, and larger samples of atoms allow for a good signal-to-noise ratio (SNR). An optical realization, however, is also possible. We now turn to the "channel" around $\eta_1=1$, $\eta_2\geq1$ in Fig. 2. For $\eta_1=\eta_2=1$, one finds $g_{11}^{(2)}(0)=8(N-1)(N+4)/[5N(N+3)]$, which for $N\to\infty$ goes to $8/5$. The corresponding limit for $\eta_1=1$, $\eta_2>1$ yields $6/5$. This "channel" can be understood by noting that for these parameters the sample acts collectively, i.e., $G_i^{(1)}(0)\propto N^2$ ($i\in\{1,2\}$), resulting in a close to coherent photon statistics that corresponds to a superfluorescent atomic sample [1–3]. It should be emphasized here that while a thermal reservoir or a direct incoherent pumping of the transitions only admits for values $0<\eta_1$, $\eta_2\leq1$, we have also used larger values for these parameters in Figs. 2 and 4. Such situations may occur if additional driving fields are applied to the sample of atoms, e.g., an incoherent repumping from the lower to the upper atomic states [8,22]. We stress, however, that our main results are obtained without such driving.

In the following, we discuss the special case of a small collection of atoms. This is of particular interest for an experimental verification in the optical region, whereas in the microwave region the small number of atoms makes it hard to obtain a decent SNR against the thermal background. In Fig. 3(a), we show $g_{11}^{(2)}(0)$ against the number of atoms in the sample for different values of $\eta_1=\eta_2$. Consider, for example, an experimental setup with an atomic beam passing through a low quality cavity. Then, depending on the mean number of atoms which are simultaneously inside the cavity and on the bath parameters, switching between sub- and super-Poissonian statistics or antibunching and superbunching of the emitted light can be observed [14]. In the figure, squares (triangles) denote antibunching (bunching) of the emitted photons. Note that together with a super-Poissonian statistics, both bunching and antibunching can be observed. Only collective states $|N,1,m\rangle$ with a single atom in state $|1\rangle$ may lead to antibunching. All other states $|N,n>1,m\rangle$ contribute to bunching. The total system behavior depends on the ratio of these two contributions. With increasing bath strength and increasing number of atoms, more bunching states $|N,n>1,m\rangle$ are available and populated, such that the switching from antibunching to superbunching occurs. Some examples

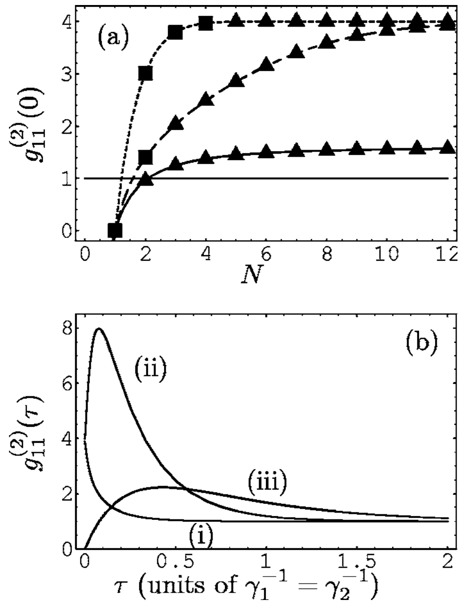

FIG. 3. (a) The second-order correlation function $g_{11}^{(2)}(0)$ for small numbers of atoms N in the sample. Solid, long dashed, and short dashed curves are for $\eta_1=\eta_2=1$, 0.5, and 0.1, respectively. Noninteger values for N are possible if N is a mean number of atoms, e.g., for atoms crossing a cavity field. Triangles (squares) denote bunching (antibunching). (b) $g_{11}^{(2)}(\tau)$ vs delay time τ. (i) $N=15$, $\eta_1=\eta_2=0.6$; (ii) $N=6$, $\eta_1=0.8$, $\eta_2=0.05$; (iii) $N=1$, $\eta_1=\eta_2=0.2$.

for $g_{11}^{(2)}(\tau)$ versus delay time τ are shown in Fig. 3(b). Curve (i) shows superbunching for $N=15$ and $\eta_1=\eta_2=0.6$. Starting from an initial value close to 4, the correlation function drops rapidly to unity with increasing τ. Example (ii) shows antibunching with super-Poissonian photon statistics and large intermediate values of $g_{11}^{(2)}(\tau)$. The maximum value of the correlation function can be further increased, however, at the cost of intensity. As a reference, (iii) shows an evolution for the single-atom case:

$$g_{11}^{(2)}(\tau) = 1 + \frac{2\gamma_1\bar{n}_1}{(\lambda_+ - \lambda_-)\langle S_{11}\rangle_s}\left[e^{-\lambda_-\tau}\left\{1 - \frac{2\gamma_2\bar{n}_2}{\lambda_-}\right\} - e^{-\lambda_+\tau}\left\{1 - \frac{2\gamma_2\bar{n}_2}{\lambda_+}\right\}\right], \tag{12}$$

where $\lambda_\pm=\gamma_1(1+2\bar{n}_1)+\gamma_2(1+2\bar{n}_2)\pm\{[\gamma_1(1+2\bar{n}_1)-\gamma_2(1+2\bar{n}_2)]^2+4\gamma_1\gamma_2\bar{n}_1(1+\bar{n}_2)\}^{1/2}$.

The cross correlations $g_{i\neq j}^{(2)}(0)$ also show nonclassical behavior. For an atomic sample in a weak bath, $g_{12}^{(2)}(0)$ is much larger than unity as shown in Fig. 4, indicating super-Poissonian light statistics, which is accompanied by strong correlation between the fluorescence light radiated on both atomic transitions, i.e., cross superbunching. The reason is that then atoms which decay from $|1\rangle$ to $|2\rangle$ also decay further to $|3\rangle$ with a high probability. For stronger baths, however, larger samples exhibit bunched sub-Poissonian light. Then $\lim_{\{\eta_1,\eta_2\}\to 1} g_{12}^{(2)}(0)=4(N+2)(N+4)/[5N(N+3)]$, with limit $4/5<1$ for $N\to\infty$. In this case, atoms decaying from $|1\rangle$ to $|2\rangle$ are repumped by the bath rather than decaying further to $|3\rangle$.

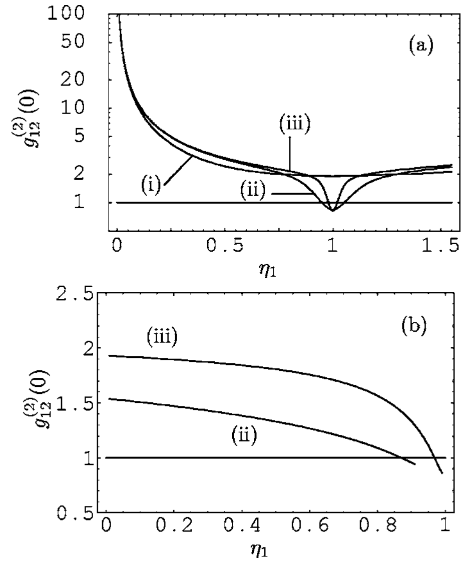

FIG. 4. The cross-correlation function $g_{12}^{(2)}(0)$ against bath parameters $\eta_1=\eta_2$. The number of atoms in the sample is (i) $N=2$, (ii) $N=50$, and (iii) $N=150$. (a) $\eta_1=\eta_2$, (b) η_2 chosen such that the SNR on both transitions is above 10.

The other cross correlation, $g_{21}^{(2)}(0)$, is below unity for a weak bath, as a transition $|2\rangle$ to $|3\rangle$ cannot directly be followed by a transition $|1\rangle$ to $|2\rangle$ without an extra excitation. If both transitions of an intermediate size sample ($N\lesssim 100$) are driven strongly, one finds cross antibunching with $G_i^{(1)}(0)\propto N^2$. Further, $\lim_{\{\eta_1,\eta_2\}\to 1} g_{21}^{(2)}(0)=4(N^2-1)/[5N(N+3)]$, which tends to $4/5$ as $N\to\infty$. For larger samples and smaller η_1, the light is antibunched for small η_2, but switches to bunched light with increasing η_2.

As an application for the nonclassical features, we now show that the light emitted from the sample of atoms violates the Cauchy-Schwarz inequalities (CSI) [23]. The CSI are violated if

$$\chi_{1(2)} = \frac{g_{11}^{(2)}(0)g_{22}^{(2)}(0)}{[g_{12(21)}^{(2)}(0)]^2} < 1, \tag{13}$$

i.e., if the cross correlations between photons emitted on two different transitions are larger than the correlation between

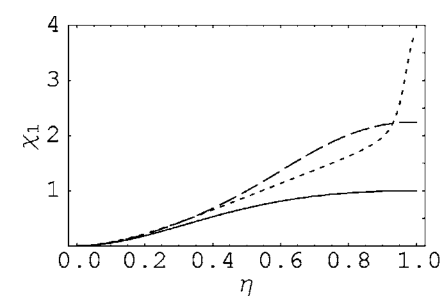

FIG. 5. The Cauchy-Schwarz parameters χ_1 shown vs bath parameters $\eta_1=\eta_2\equiv\eta$. The solid, long-dashed, and short-dashed curves are plotted for $N=4$, 10, and 150.

photons emitted from the individual levels. Figure 5 shows the violation of the CSI function for moderately strong baths. Within the Dicke model, this violation is present for any number of atoms in the sample, thus demonstrating a macroscopic quantum effect. In addition, χ_1 is always smaller than unity for $N \leqslant 3$ and the entire range of $0 \leqslant \eta_1, \eta_2 \leqslant 1$ ($\lim_{\{\eta_1,\eta_2\}\to 1}\chi_1 = 4[(N-1)/(N+2)]^2$), while χ_2 is larger than unity for any number of atoms and for any values of η_1, η_2. The CSI violation can best be observed in the optical region, as low values of η_i are favorable, and as it is more difficult to obtain a decent SNR in the microwave region. Large cross correlations together with a violation of the CSI indicate quantum entanglement among the photons emitted by incoherently driven atoms, which currently is studied intensively as an intriguing aspect of noise in quantum information processing [15,17].

In summary, we have demonstrated quantum features in the fluorescence light of a sample of atoms driven only by a surrounding incoherent bath. We discussed both thermal baths with microwave atomic transition frequencies and pseudothermal baths for optical transition frequencies as realizations of the bath. For small samples, a change of the mean number of atoms in the sample induces sensitive switching between sub- or super-Poissonian statistics and antibunching or superbunching of the light emitted on one of the transitions. For appropriate bath parameters, even macroscopic samples exhibit antibunching. As an application, we have shown that the Cauchy-Schwarz inequalities are violated in our system over a wide range of parameters.

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