Entanglement from thermal blackbody radiation

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Two noninteracting quantum systems which couple to a common environment with many degrees of freedom initially in thermal equilibrium can become entangled due to the indirect interaction mediated through this heat bath. I examine here the dynamics of reservoir-induced entanglement for a heat bath consisting of a thermal electromagnetic radiation field, such as blackbody radiation or the cosmic microwave background, and show how the effect can be understood as result of an effective induced interaction.

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I. INTRODUCTION

About two decades of research in quantum information have led to the picture of quantum entanglement as a precious resource that plays a key role in processing information more securely and more efficiently than classically possible [1]. Entanglement is necessary for quantum teleportation [2] and the exponential acceleration of quantum algorithms [3], and it allows for secure quantum key distribution [4]. Recent experimental demonstrations of quantum teleportation [5–10] and small scale quantum computations [11] confirm this picture. Creating and manipulating entanglement in a controlled way remains a challenge, as environmentally induced decoherence tends to rapidly destroy entanglement. On the other hand, it has been shown that the coupling of two quantum systems to a common heat bath can also *create* substantial (mixed state) entanglement. This effect was originally demonstrated in the framework of an exactly solvable model [12], and has been confirmed by perturbative calculations in the Markovian regime [13,14]. Cirone *et al.* have unveiled a connection to the Casimir-Polder interaction [15], and Oh and Kim have shown by a renormalization group analysis how the effect can be understood through an effective-induced interaction between the quantum systems *via* the common heat bath [16].

Heat baths are ubiquitous, and one might therefore wonder, if "reservoir induced entanglement" (RIE) [17], i.e., the creation of the entanglement through coupling to a common heatbath is a common place and for over what distances and with what time dependence might arise. For example, all quantum systems containing charged particles couple to the ambient thermal electromagnetic radiation, i.e., the blackbody radiation (BBR) in a laboratory or cryostat at the corresponding temperatures, and even in free space there is still the cosmic microwave background (CMB), a basically perfect blackbody radiation at an absolute temperature $T=2.728\pm0.004$ K that fills the entire known universe [21]. Do these heat baths induce entanglement between remote quantum systems?

In this paper I show the BBR effectively constitutes two different heat baths which couple differently to the couple Alice-Bob, and whose effects largely cancel when it comes to entanglement creation. Entanglement is therefore created only very slowly, far behind the light cone, and the entanglement created oscillates as a function of time. The entanglement can be close to perfect, but the first maximum entanglement arrives only after a time t_1 which scales like the third power of the distance R between the two quantum systems,

$$t_1 \simeq \frac{\pi}{2\alpha_0}\frac{R^2}{d^2}\frac{R}{c_0}, \tag{1}$$

where $\alpha_0 \simeq \frac{1}{137}$ and c_0 are the fine structure constant and the speed of light in vacuum, respectively, and d denotes the dipole moment of the quantum system divided by the electron charge. The slow creation of entanglement limits the distance over which it can be created before competing decoherence processes set in.

I will elucidate the role of the high-frequency, far off-resonant modes of the heat bath, and discuss the temperature dependence of the phenomena.

II. THEORETICAL FRAMEWORK AND PHYSICAL SYSTEMS

Let us consider two identical quantum systems A and B which couple to the thermal electromagnetic radiation field in open space, as exemplified by the CMB, but which do not interact directly. We assume that A and B can be approximated as two-state systems with states $|0\rangle$ and $|1\rangle$. It turns out that the standard quantum optics approach of the rotating wave approximation and Markovian Master equations based on Fermi golden rule rates (i.e., second order perturbation theory in the atom-field coupling constants) is not fully adequate for describing RIE. First, the explicit dependence of the time scale on which RIE is produced on the cut-off frequency of the heat bath found in [12] hints to the importance of nonresonant modes, which invalidates the rotating wave approximation. One might argue that in [12] nonresonant modes came into play because degenerate energy levels were considered, but even for finite level spacing Δ the high-frequency modes should be relevant for times $t \ll \Delta$. Note that the rotating wave approximation was also avoided in [16,18].

Secondly, there is evidence that for the specific heat bath and quantum systems to be discussed here RIE is an effect that arises only at fourth order in the coupling constants [22].

Both issues combined call for a fourth order calculation without the rotating wave approximation, which makes the theory very heavy. On the other hand, the problem can be solved exactly and with less effort in the case of degenerate energy levels [12,23]. This approach will be followed here.

From an experimental point of view, two-state systems with exactly degenerate energy levels are hard to find. However, it should be kept in mind that a finite level spacing just introduces an upper limit to the time for which the present theoretical analysis is applicable: for any experiment terminated within a time $t \ll 1/\Delta$ the system Hamiltonian of the two-state systems A and B can be neglected [23]. Whether or not entanglement can still be produced beyond this time is an interesting experimental question.

Double quantum dots (DQDs) seem to be a promising candidate, and the following analysis will be geared specifically towards these systems. Recently, the coherent manipulation of two states $|0\rangle$ and $|1\rangle$ located in the two wells of a single such device, as well as the state preparation and the state measurement were demonstrated; coherence times of the order 1–10 ns were achieved [24,25]. The energy barrier between the two wells as well as the energies $|0\rangle$ and $|1\rangle$ in a DQD can be tuned with the help of gate voltages and the barrier can be made very high after the preparation of a superposition, such that $|0\rangle$ and $|1\rangle$ become a good approximation of the degenerate eigenstates of the DQD Hamiltonians. The question of whether or not the two-state approximation still holds when we have to consider high-frequency modes might be addressed eventually experimentally by trying to shield each DQD with its own superconducting cavity. Modes with frequencies higher than the superconducting gap will be absorbed and one might thus envisage to control the cut-off frequency of the heat bath. Otherwise a cut-off of the order $\hbar\omega \simeq 1$ eV arises naturally due to the bandgap of the semiconductor material in which the DQDs are embedded; e.m. waves with higher frequencies get absorbed in the semiconductor. A disadvantage of the DQDs are the competing intrinsic decoherence mechanisms such as phonon scattering [26] and fluctuating electric fields [27] other than those from the BBR, which will be neglected in the following analysis.

As for the heat bath, I will consider specifically BBR with periodic boundary conditions. One might wonder if the mode structure (and thus the boundary conditions) make a difference, as each mode couples to two spatially separated quantum systems. For the geometrical situation considered below, and within the dipole approximation of the coupling, it turns out that a box-shaped cavity with perfectly conducting walls leads, in the limit of infinite volume and fixed distance R, to the same interaction Hamiltonian, and one can thus read in the following CMB with periodic boundary conditions or BBR in a box-shaped cavity interchangeably.

III. DOUBLE QUANTUM DOTS INTERACTING WITH BBR

Let us arrange the two DQDs such that the axes of the dots are aligned with the vector joining them, designated as the z axis in the following. The position operators of an electron in dots 1 and 2 have the matrix elements $\langle 0|z_{1,2}|0\rangle = -d/2 = -\langle 1|z_{1,2}|1\rangle$. All other matrix elements of $z_{1,2}$ vanish to a very good approximation due to the exponentially small overlap of the states $|0\rangle$ and $|1\rangle$, like the matrix elements of the other electron coordinates $x_{1,2}$ and $y_{1,2}$ for an assumed even parity of the ground state wave functions.

For describing the BBR I use a slightly unconventional representation of the electromagnetic field, which is very useful for the exact treatment for vanishing level spacing (or, in general, if one does not use the rotating wave approximation). In fact, the BBR can be considered as two independent heat baths (see AppendixA), one containing cos waves, the other sin waves, with an electric field operator

$$\mathbf{E}(\mathbf{r},t) = \sum_{\mathbf{k},\alpha}{}' \sqrt{\frac{2}{\epsilon_0 V}}\omega_k \boldsymbol{\epsilon}_{\mathbf{k}\alpha}(Q_{\mathbf{k}\alpha 1}\cos \mathbf{k}\cdot\mathbf{r} + Q_{\mathbf{k}\alpha 2}\sin \mathbf{k}\cdot\mathbf{r}), \tag{2}$$

and canonical coordinate operators $Q_{\mathbf{k}\alpha\nu}$ of the harmonic oscillators corresponding to the different electromagnetic field modes. The $\mathbf{k}$ are quantized wave vectors ($k_i = 2\pi n_i/L$, $n_i \in \mathcal{Z}$ for $i=x,y,z$, for quantization in a volume L^3 with periodic boundary conditions), $\alpha=1,2$ counts polarizations, ν distinguishes the cos waves ($\nu=1$) from the sin waves ($\nu=2$), and the prime on the sum means that the summation is restricted to $k_x > 0$. The free field Hamiltonian reads

$$H_{\mathrm{bath}} = \frac{1}{2}\sum_{\mathbf{k}}{}' \sum_{\alpha=1,2,} \sum_{\nu=1,2} (P_{\mathbf{k}\alpha\nu}^2 + \omega_k^2 Q_{\mathbf{k}\alpha\nu}^2), \tag{3}$$

with $\omega_k = c_0|\mathbf{k}| \equiv c_0 k$.

Placing the two DQDs at positions $\mathbf{R}/2$ and $-\mathbf{R}/2$ [$\mathbf{R} = (0,0,R)$] we obtain in dipole approximation the coupling Hamiltonian

$$H_{\mathrm{int}} = \sum_{\mathbf{k}}{}' \sum_{\alpha=1,2} [(\sigma_{z1} + \sigma_{z2})g_{\mathbf{k}\alpha 1}Q_{\mathbf{k}\alpha 1} + (\sigma_{z1} - \sigma_{z2})g_{\mathbf{k}\alpha 2}Q_{\mathbf{k}\alpha 2}], \tag{4}$$

written in terms of Pauli matrices σ_{z1}, σ_{z2} in the basis ($|0\rangle$, $|1\rangle$) of dots 1 and 2. The coupling coefficients $g_{\mathbf{k}\alpha\nu}$ are given by

$$g_{\mathbf{k}\alpha 1} = \frac{ed}{2}\sqrt{\frac{2}{V\epsilon_0}}\omega_k \boldsymbol{\epsilon}_{\mathbf{k}\alpha}\begin{pmatrix}0\\0\\1\end{pmatrix}\cos\left(\frac{\mathbf{k}\cdot\mathbf{R}}{2}\right), \tag{5}$$

and the same equation holds for $g_{\mathbf{k}\alpha 2}$ up to the change cos $\rightarrow$ sin.

In the regime of energy degenerate states $|0\rangle$ and $|1\rangle$ discussed previously; the time evolution of each dot to its own system Hamiltonian can be neglected, and the total Hamiltonian is thus simply $H = H_{\mathrm{int}} + H_{\mathrm{bath}}$ [12,23]. The interaction Hamiltonian represents a generalization of the situation considered in [23], in the sense that each mode of the BBR couples through a different "system coupling agent" $S_{\mathbf{k}\alpha\nu} = g_{\mathbf{k}\alpha\nu}[\sigma_{z1} - (-1)^\nu \sigma_{z2}]$. Note, however, that all coupling agents commute with each other. This allows us to generalize the time evolution derived in [12,23] of the reduced density matrix describing the two DQDs alone to

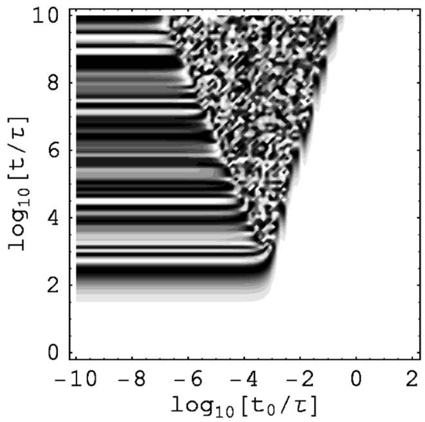

FIG. 1. Entanglement of formation E for two initially not entangled DQDs with $d=10$ nm coupled to the CMB at $T=2.73$ K as a function of $\log_{10}(t_0/\tau)$ and $\log_{10}(t/\tau)$, $\tau=\beta\hbar$. Black means perfect entanglement, $E=1$, white no entanglement $E=0$. Entanglement is created only for $t/\tau \gtrsim 10^{12}(t_0/\tau)^3$.

$$\langle s|\rho(t,t_0)|s'\rangle = \exp\left\{-\sum_{\mathbf{k},\alpha,\nu}{}' [(\lambda_{s,\mathbf{k}\alpha\nu}-\lambda_{s',\mathbf{k}\alpha\nu})^2 f(k) - \mathrm{i}(\lambda^2_{s,\mathbf{k}\alpha\nu}-\lambda^2_{s',\mathbf{k}\alpha\nu})\varphi(k)]\right\}\langle s|\rho(0,t_0)|s'\rangle, \tag{6}$$

where $|s\rangle$ is one of the four states $|00\rangle$, $|01\rangle$, $|10\rangle$, or $|11\rangle$ and $\lambda_{s,\mathbf{k}\alpha\nu}$ are the corresponding eigenvalues of $S_{\mathbf{k}\alpha\nu}$. It was assumed that the bath is initially in thermal equilibrium, and that the total initial density matrix factorizes into a system part and a bath part. The dependence on the time of travel of a light signal between the two DQDs, $t_0=R/c_0$ with $R=|\mathbf{R}|$, will appear below.

For the arrangement of the DQDs described above, only one polarization direction ($\alpha=1$) contributes, with $\boldsymbol{\epsilon}_{\mathbf{k}1}=\boldsymbol{\epsilon}_\theta$ in polar coordinates for $\mathbf{k}$. With the abbreviation $G_k = -ed/2\sqrt{(2/V\epsilon_0)}\omega_k \sin\theta$ the relevant eigenvalues are $\lambda_{00,\mathbf{k}11}=2\cos(kR\cos\theta/2)G_k=-\lambda_{11,\mathbf{k}11}$ and $\lambda_{01,\mathbf{k}12}=2\sin(kR\cos\theta/2)G_k=-\lambda_{10,\mathbf{k}12}$. All other eigenvalues vanish. The functions $f(k)$ and $\varphi(k)$ depend on $k=|\mathbf{k}|$ as $[\beta=1/(k_\mathrm{B}T)]$

$$f(k)=\coth\left(\frac{\beta\hbar\omega_k}{2}\right)\frac{1-\cos\omega_k t}{2\hbar\omega_k^3}, \quad \varphi(k)=\frac{1}{2\hbar\omega_k^2}\left(t-\frac{\sin\omega_k t}{\omega_k}\right). \tag{7}$$

Transforming the sums over modes $\mathbf{k}$ into integrals for the limit of large L, we find

$$\langle s_1|\rho(t,t_0)|s_2\rangle = \exp(-A\{f_1(t,t_0)C_{s_1,s_2}+f_2(t,t_0)S_{s_1,s_2} - \mathrm{i}[\varphi_1(t,t_0)\tilde{C}_{s_1,s_2}+\varphi_2(t,t_0)\tilde{S}_{s_1,s_2}]\}) \times\langle s_1|\rho(0,t_0)|s_2\rangle \tag{8}$$

with

$$S=\begin{pmatrix}0&1&1&0\\1&0&4&1\\1&4&0&1\\0&1&1&0\end{pmatrix},\quad C=\begin{pmatrix}0&1&1&4\\1&0&0&1\\1&0&0&1\\4&1&1&0\end{pmatrix},$$

$$\tilde{S}=\begin{pmatrix}0&-1&-1&0\\1&0&0&1\\1&0&0&1\\0&-1&-1&0\end{pmatrix},\quad \tilde{C}=-\tilde{S}, \tag{9}$$

in the basis $|00\rangle$, $|01\rangle$, $|10\rangle$, $|11\rangle$, and $A=\alpha_0 d^2/\pi c_0^2\tau^2$, where $\alpha_0=e^2/4\pi\epsilon_0\hbar c_0 \simeq \frac{1}{137}$ is the fine-structure constant. In the following, both t and t_0 will be expressed in units of the thermal time $\tau=\beta\hbar$ for all finite temperatures. The functions $f_{1,2}(t,t_0)$ and $\varphi_{1,2}(t,t_0)$ are then given by

$$f_\nu(t,t_0)=\int_0^{y_{max}} dy\, y\coth(y/2)[1-\cos(yt)] \times\left[\frac{1}{3}+(-1)^\nu\left(\frac{\cos(yt_0)}{(yt_0)^2}-\frac{\sin(yt_0)}{(yt_0)^3}\right)\right], \tag{10}$$

$$\varphi_\nu(t,t_0)=\int_0^{y_{max}} dy\, y[yt-\sin(yt)] \times\left[\frac{1}{3}+(-1)^\nu\left(\frac{\cos(yt_0)}{(yt_0)^2}-\frac{\sin(yt_0)}{(yt_0)^3}\right)\right]. \tag{11}$$

The integrals extend, in principle, from zero to infinity, but a UV cut-off $y_{max}=\omega_{max}\tau$ is needed to regularize them.

Due to $\tilde{C}=-\tilde{S}$, the final density matrix depends only on the phase difference $\varphi_-(t)=\varphi_1(t)-\varphi_2(t)$, which can be written in closed form as

$$\varphi_-(t,t_0)=\frac{t}{t_0^3}\{-2\sin(y_{max}t_0)+\mathrm{Si}[y_{max}(t-t_0)]+2\,\mathrm{Si}(y_{max}t_0) - \mathrm{Si}[y_{max}(t+t_0)]\}+\frac{2}{y_{max}t_0^3}\sin(y_{max}t)\sin(y_{max}t_0). \tag{12}$$

Note that $\varphi_-(t,t_0)$ remains finite for $y_{max}\to\infty$. For large t and t_0, $\varphi_-(t,t_0)$ increases proportional to t and decays as $1/t_0^3$. The functions $f_\nu(t,t_0)$, on the other hand, scale like y_{max}^2 with the cut-off, which gives a physical significance to the high-frequency modes. The influence of the form of the cut-off function will be examined below. For the moment we assume the simplest form, a sharp cut-off at $\omega=\omega_{max}$.

IV. RESERVOIR-INDUCED ENTANGLEMENT

I have evaluated the entanglement of formation $E[\rho(t,t_0)]$ for an initial state in the form of a pure product state, $(|0\rangle+|1\rangle)\otimes(|0\rangle+|1\rangle)/2$, i.e., $E[\rho(t,t_0)]=0$ using Wootter's formula [28] and by numerically integrating the remaining expressions for $f_{1,2}(t,t_0)$. Alternatively, one may approximate $\coth\simeq 1$ for $y_{max}\gg 1$, which allows for an analytical solution also for f_ν. The result for $T=2.73$ K and $d=10$ nm is shown in Fig. 1. Clearly, there is no entanglement for spacelike separated points, $t<t_0$. However, almost perfect mixed state entanglement arises for $t>t_0$, starting at around $t\simeq 100\tau$. The entanglement oscillates rapidly as function of t, and vanishes for $t/\tau<c(t_0/\tau)^3$, where c is a constant of order 10^{12} in agreement with Eq. (1). This behavior can be understood by considering the long time limit $t\gg t_0\gg\tau$, which leads to $f_{1,2}\simeq y_{max}^2/6$ for $y_{max}\gg 1$. In this case the final density matrix depends only on two variables, $v=d\omega_{max}/c_0$ and $\phi_-=A\varphi_-=\phi_-(t,t_0)$, with

$$\langle s|\rho(v,\phi_-)|s'\rangle \simeq \exp\left(-\frac{\alpha_0}{12\pi}v^2(S_{s,s'}+C_{s,s'})+\mathrm{i}\phi_-\widetilde{C}_{s,s'}\right)\times\langle s|\rho[v,\phi_-(0,t_0)]|s'\rangle. \tag{13}$$

Figure 2 shows the corresponding entanglement. The function is π-periodic in ϕ_- with a first maximum at $\phi_-=\pi/2$, which leads to Eq. (1) for the first maximum entanglement. The scaling of t_1 with R^3 makes the entanglement production extremely slow: during the lifetime of the universe, it would have reached a distance of only about 8.4 km for $d=1$ μm. For the same dipole moment, a distance of about 52 μm should be reached during a coherence time of the order of 100 ns. The maximum amount of entanglement as well as the time to first finite entanglement creation is controlled by v, which has to be smaller than about 50. One should, therefore, try to have a large d for a large maximum distance and a small ω_{max} to get large maximum entanglement, whereas the temperature becomes irrelevant in this regime ($\hbar\omega_{max}\gg k_\mathrm{B}T$). Note that τ cancels in the expression for ϕ_- for large t,t_0, such that also t_1 becomes independent of temperature.

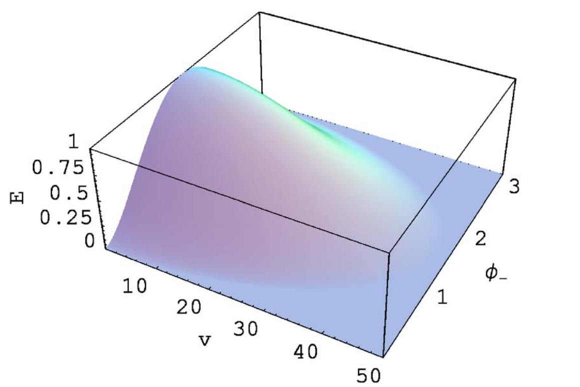

FIG. 2. (Color online) Entanglement of formation for $t\gg t_0\gg\tau$ as function of $v=\omega_{max}d/c_0$ and the phase φ_-. The latter increases linearly with time t leading to entanglement oscillations, and decays as $t_0^{-3}=(R/c_0)^{-3}$, which makes the entanglement creation very slow for large distances R.

V. THE EFFECTIVE INTERACTION

The physical origin of the entanglement generation can be understood as arising from an effective interaction H_eff mediated through the coupling to the common bath [16]. This idea can be made quantitative by observing that a Hamiltonian of the type

$$H=\sum_k\hbar\omega_k a_k^\dagger a_k+\hbar\sum_k g_k(S_k a_k+S_k^\dagger a_k^\dagger), \tag{14}$$

where S_k is an operator satisfying $[S_k,S_k^\dagger]=0$ acting on the Hilbert space of the system (i.e., here the qubits of Alice and Bob), leads to the time evolution operator [29]

$$\mathrm{e}^{-\mathrm{i}/\hbar Ht}=D\left(-\sum_k\frac{g_k}{\omega_k}S_k^\dagger\right)\mathrm{e}^{-\mathrm{i}/\hbar(H_\mathrm{bath}+H_\mathrm{eff})t}D\left(\sum_k\frac{g_k}{\omega_k}S_k^\dagger\right), \tag{15}$$

with the shift operator $D(\Sigma_k g_k/\omega_k S_k^\dagger)=\exp[\Sigma_k(g_k/\omega_k)(S_K^\dagger a_k^\dagger-S_k a_k)]$, bath Hamiltonian $H_\mathrm{bath}=\Sigma_k\hbar\omega_k a_k^\dagger a_k$, and the effective interaction

$$H_\mathrm{eff}=-\hbar\sum_k\frac{g_k^2}{\omega_k}S_kS_k^\dagger. \tag{16}$$

It is very instructive to calculate this interaction explicitly for the example at hand. To that end we revert momentarily to the standard expression of the electric field, Eq. (A1), and allow for arbitrary orientation of the DQDs. If we denote the orientations by unit vectors $\hat{u}_i$ and observe that the index k in Eq. (14) stands for wave vector and polarization $(\mathbf{k},\alpha)$, we have $S_{\mathbf{k}\alpha}=\Sigma_{i=1,2}g_{\mathbf{k}\alpha}^{(i)}\sigma_{zi}\mathrm{e}^{\mathrm{i}\mathbf{k}\mathbf{r}_i}$ with coupling constants

$$g_{\mathbf{k}\alpha}^{(i)}=\mathbf{d}_i\cdot\boldsymbol{\epsilon}_{\mathbf{k}\alpha}\sqrt{\frac{c_0|\mathbf{k}|}{2\epsilon_0\hbar V}}, \tag{17}$$

with $\mathbf{d}_i=(ed/2)\hat{u}_i$. The evaluation of the sum over all modes in Eq. (16) leads, in the limit of continuous $\mathbf{k}$ and infinite cut-off frequency, to

$$H_\mathrm{eff}=\frac{\mathbf{d}_1\cdot\mathbf{d}_2-3(\mathbf{d}_1\cdot\hat{r})(\mathbf{d}_2\cdot\hat{r})}{4\pi\epsilon_0 r^3}\sigma_{z1}\sigma_{z2}, \tag{18}$$

i.e., a dipole-dipole interaction. This explains the $1/r^3$ dependence of the phase $\varphi_-(t,t_0)$ and its proportionality to t for large times. Of course, the electric field cannot mediate an instantaneous interaction, as Eq. (18) might suggest. The full time dependence must take into account also the free bath Hamiltonian, and $H_\mathrm{bath}+H_\mathrm{eff}$ together lead indeed to the correct retardation behavior. This is seen from Eq. (12) when we take the limit $\omega_{max}\to\infty$. The first oscillating term $\sin(y_{max}t_0)$ arises from the sharp cut-off and will average to zero for a smoother cut-off. The last term vanishes for $y_{max}\to\infty$, and the remaining sin-integral functions conspire to a Heaviside θ function on the light cone, $\pi\theta(t/t_0-1)$, with the consequence that no entanglement can be created faster than the speed of light with this effect. The matrix $\widetilde{C}$ in Eq. (8) is seen to arise from the difference of eigenvalues of $\sigma_{z1}\sigma_{z2}$ defined by the indices of the density matrix.

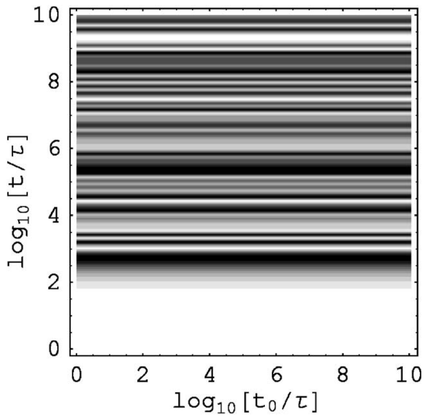

FIG. 3. Entanglement of formation E for a hypothetical coupling to the cos waves only. Entanglement production is basically independent of R in the interval shown, and thus possible quasiinstantaneously ($t < t_0$). Same parameters and gray-scale code as in Fig. 1.

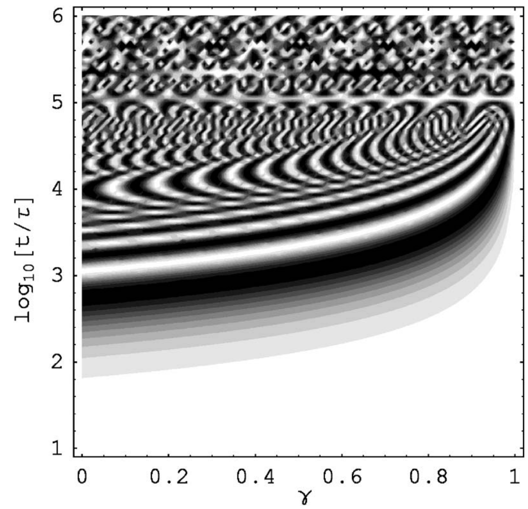

FIG. 4. Entanglement of formation for $t_0 = 10^6 \tau$ as function of γ and $\log_{10}(t/\tau)$. The parameter γ measures the coupling strength to the sin waves (0 no coupling, 1 full coupling). Already values of γ only slightly smaller than 1 would lead to the entanglement creation before the light cone ($t < t_0$). Same parameters and gray-scale code as in Fig. 1.

One might be tempted to think at this point that the entanglement generation is trivial in the sense that the Hamiltonian for the two DQDs and the heat bath just amounts to a fancy reformulation of the dipole-dipole interaction between the DQDs initially assumed noninteracting. This is, however, not the case: First of all, as can be seen from Eq. (15) the effective interaction is not the only term that determines the dynamics of the entanglement generation. Rather it is supplemented by two shifts in the harmonic oscillators, which depend on the state of the two DQDs. The reduced overlaps of the harmonic oscillators are responsible for the decoherence quantified by the functions $f_1(t,t_0)$ and $f_2(t,t_0)$. Thus, there needs to be a balance between the effective interaction induced and the decoherence due to state dependent coupling to the heat bath, and this balance is made quantitative by the functions $f_1(t,t_0)$, $f_2(t,t_0)$ and $\varphi_-(t,t_0)$. The effect of decoherence (and possibly the retardation) would be overlooked, if one started directly with a Hamiltonian containing the dipole-dipole interaction.

Secondly, the heat bath is initially in thermal equilibrium, and the thermal noise reduces the amount of entanglement additionally compared to the $T=0$ case. So the nature of the environment as a heat bath is important, and not just the fact that it induces a well-known interaction.

One might wonder whether one can speed up the entanglement generation by reservoir engineering. One obvious attempt would be to selectively supress the coupling to one of the two heat baths, say the sin waves. In fact, the calculation shows that for a complete supression of the coupling to the sin waves the entanglement becomes basically independent of R and entanglement would be created quasiinstantaneously, Fig. 3. The difference arises from the fact that $\varphi_- = \varphi_1 - \varphi_2$, a phase difference accumulated from the couplings to the cos and sin waves, is replaced by a single phase from the cos waves alone. The latter contains a large distance independent term which normally cancels a corresponding term from the sin waves. Without the sin waves ($\varphi_2 = 0$) this term leads to a very rapid, distance independent growth of φ_- and therefore to the quasiinstantaneous entanglement creation. We conclude that the two independent heat baths counteract each other when it comes to entanglement creation, and this leads to a much slower entanglement creation based on the small remaining, distance dependent phase accumulation.

While a complete suppression of the sin waves (or the coupling to them) might be illusive, even a very small suppression (as one might imagine in a cavity by using a thin, unchanged wire) would lead quickly to the entanglement creation faster than the speed of light. Figure 4 shows the entanglement for $t_0 = 10^6 \tau$ as a function of γ and $\log_{10}(t/\tau)$, where $0 \leq \gamma \leq 1$ measures the relative coupling strength to the sin waves $[f_2(t,t_0) \to \gamma f_2(t,t_0)$ and $\varphi_2(t,t_0) \to \gamma \varphi_2(t,t_0)]$. The smaller γ, the faster the entanglement arises, but up to γ very close to unity, almost perfect entanglement is created for $t < t_0$.

However, causality leads to strong restrictions of what should be possible in this respect: also in a classical field theory the retardation of the dipole-dipole interaction would be modified, and an electromagnetic interaction spreading faster than the speed of light is certainly not possible, regardless of whatever arrangement of conductors one might come up with in order to supress certain modes.

In [18,19] it was proposed that two atoms can get entangled by coupling them for a time $t < t_0$ to the vacuum of a massless scalar relativistic field (thus explicitly avoiding the effects of any effective induced interaction), and an experiment was proposed in [20] to demonstrate the corresponding effect in ion traps, namely, the entangling of the internal degrees of freedom of two ions in an ion-string faster than the speed of sound in the ion string. The vacuum case can be retrieved from the calculation presented here by setting $T = 0$, and replacing the coth function in Eq. (10) by unity. Since τ diverges then, it is more reasonable to use directly t_0

as time scale. This is achieved formally by substituting $y = z\tau/t_0$ in Eqs. (10) and (11) whereupon the functions depend only on t/t_0 up to an additional prefactor τ^2/t_0^2. The latter combines with the prefactor A and reads then $\alpha_0 d^2/(\pi c_0^2 t_0^2)$. The cut-off y_{max} is replaced by $z_{max}=\omega_{max}t_0$. However, nothing changes about the fact that the function φ_- is proportional to a Heaviside θ function centered on the light cone. Thus, no phase accumulation is possible for $t<t_0$, unless one uses a finite cut-off. In this case the Heaviside θ function gets "softened," which is physically plausible as now the heat bath does not contain sufficiently small wavelengths anymore to precisely locate Bob's DQD. But even if in such a case one was to operate slightly before the light cone, $t \lesssim t_0$, one would still have to overcome the smallness of the prefactor $\alpha_0 d^2/(\pi c_0^2 t_0^2)$. This seems to be excluded for atoms, as the distance between Alice and Bob would have to become smaller than the length attached to the atomic dipole moment d, in which case the whole dipole approximation breaks down. In DQDs one might hope to create states $|0\rangle$ and $|1\rangle$ with large amounts of charge, thus increasing artificially the dipole moments, but even for ten excess electrons in one well the DQDs would have to be so close that the dipole approximation becomes doubtful. Thus, within the above theoretical framework of fixed DQDs with two degenerate energy levels dipole-coupled to BBR, there seems to be not much room for substantially entangling the atoms faster than the speed of light by coupling them for a time $t<t_0$ to the BBR, even at $T=0$.

VI. CUT-OFF DEPENDENCE

As the function φ_- remains finite for $\omega_{max}\to\infty$, a different cut-off function has little influence on φ_- (besides the eventual removal of the oscillating terms mentioned earlier) and, therefore, on the speed of entanglement generation. However, since $f_{1,2}$ diverge for $y_{max}\to\infty$, the cut-off function can change the maximum amount of entanglement. Let us suppose that the cut-off function to be used in Eq. (10) is equal to unity for $y<y_{max}$ and equal to a function $C(y)$ for $y \geqslant y_{max}$. For $y_{max}\gg 1$, the $\coth(y/2)$ can be replaced by unity without impacting the following scaling arguments. Also, the strongest diverging t_0-dependent part is given by

$$\int_{y_{max}}^{\infty} dy\, C(y)\frac{\cos(yt_0)}{yt_0^2}, \tag{19}$$

which even for $C(y)=1$ remains finite. The only remaining question is then, if a different cut-off function changes significantly the t-dependent behavior of $f_{1,2}$ and, in particular, if for $t\to\infty$ $f_{1,2}$ always remain finite. To answer this question observe that for $0\leqslant C(y)\leqslant 1$,

$$0 \leqslant \frac{1}{3}\int_{y_{max}}^{\infty} dy\, C(y)y[1-\cos(yt)] \leqslant \frac{2}{3}\int_{y_{max}}^{\infty} dy\, C(y)y. \tag{20}$$

Thus, if the cut-off function decays fast enough to make the integral finite, it will be finite for all times. More specifically, if $C(y)\propto 1/y^p$, p needs to be larger than 2, and the upper bound in (20) is then of the order y_{max}^{2-p}, which is completely negligible compared to the dominating y_{max}^2 behavior from $y<y_{max}$. Therefore, the results obtained, in particular, the entanglement for large t, are robust against a change of the cut-off function.

VII. CONCLUSIONS

The CMB or the BBR in a box-shaped cavity can be regarded as two independent heat baths, which couple differently to two spatially separated DQDs. Two spatially separated, noninteracting quantum systems can get entangled by interacting with the CMB or the same BBR. However, the effects of both heat baths cancel to a large extent. As a result, the entanglement is created only very slowly, with a first maximum entanglement arriving at a time t_1 that scales like the third power of the distance R between the quantum systems. The effect can be understood as originating from an effectively induced dipole-dipole interaction, and consequently is retarded by the propagation of a light signal.

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APPENDIX: THE ELECTROMAGNETIC FIELD

I derive here the expression for the electric field operator, Eq. (2). The starting point is the standard representation of the quantum operator of the electric field for periodic boundary conditions [30–34],

$$\mathbf{E}(\mathbf{r},t)=\sum_{\mathbf{k},\alpha}\boldsymbol{\epsilon}_{\mathbf{k}\alpha}\mathcal{E}_k(a_{\mathbf{k}\alpha}\mathrm{e}^{-\mathrm{i}(\omega_k t-\mathbf{kr})}+a_{\mathbf{k}\alpha}^{\dagger}\mathrm{e}^{\mathrm{i}(\omega_k t-\mathbf{kr})}), \tag{A1}$$

with $\mathcal{E}_k=\sqrt{\hbar\omega_k/(2\epsilon_0 V)}$, unit polarization vectors $\boldsymbol{\epsilon}_{\mathbf{k}\alpha}(\alpha=1,2)$, and the creation and annihilation operators $a_{\mathbf{k}\alpha}^{\dagger}$ and $a_{\mathbf{k}\alpha}$, respectively. We express these operators in terms of canonical coordinate and momentum operators $q_{\mathbf{k}\alpha}$ and $p_{\mathbf{k}\alpha}$, respectively, to obtain

$$\mathbf{E}(\mathbf{r},t)=\sum_{\mathbf{k},\alpha}\boldsymbol{\epsilon}_{\mathbf{k}\alpha}\frac{1}{\sqrt{\epsilon_0 V}}[\omega_k q_{\mathbf{k}\alpha}\cos(\mathbf{kr}-\omega_k t) - p_{\mathbf{k}\alpha}\sin(\mathbf{kr}-\omega_k t)]. \tag{A2}$$

We then split the set of modes into two sets, one with $k_x>0$, the other with $k_x<0$, and introduce $Q_{\mathbf{k}\alpha}^{\pm}=(q_{\mathbf{k}\alpha}\pm q_{-\mathbf{k}\alpha})/\sqrt{2}$, $P_{\mathbf{k}\alpha}^{\pm}=(p_{\mathbf{k}\alpha}\pm p_{-\mathbf{k}\alpha})/\sqrt{2}$. Changing the summation variable from $\mathbf{k}\to-\mathbf{k}$ in the $k_x<0$ part, and with $\boldsymbol{\epsilon}_{\mathbf{k}1}=-\boldsymbol{\epsilon}_{-\mathbf{k}1}$, $\boldsymbol{\epsilon}_{\mathbf{k}2}=\boldsymbol{\epsilon}_{-\mathbf{k}2}$, $s_1=-$, $s_2=+$, Eq. (A1) can be rewritten

$$\mathbf{E}(\mathbf{r},t)=\sum_{\mathbf{k},\alpha}{}'\sqrt{\frac{2}{\epsilon_0 V}}\boldsymbol{\epsilon}_{\mathbf{k}\alpha}[(\omega_k Q_{\mathbf{k}\alpha}^{s_\alpha}\cos\mathbf{kr}-P_{\mathbf{k}\alpha}^{s_{3-\alpha}}\sin\mathbf{kr})\cos\omega_k t + (\omega_k Q_{\mathbf{k}\alpha}^{s_{3-\alpha}}\sin\mathbf{kr}+P_{\mathbf{k}\alpha}^{s_\alpha}\cos\mathbf{kr})\sin\omega_k t],$$

where the prime at the sum denotes summation over modes with $k_x > 0$ only. Note that hereby the total number of modes is kept unchanged. We finally perform a time dependent canonical transformation,

$$Q_{\mathbf{k}11}(t) = Q_{\mathbf{k}1}^{-} \cos \omega_k t + \frac{1}{\omega_k} P_{\mathbf{k}1}^{-} \sin \omega_k t, \tag{A3}$$

$$Q_{\mathbf{k}12}(t) = Q_{\mathbf{k}1}^{+} \sin \omega_k t - \frac{1}{\omega_k} P_{\mathbf{k}1}^{+} \cos \omega_k t, \tag{A4}$$

and the same set of equations, but with $Q_{\mathbf{k}1}^{\pm} \to Q_{\mathbf{k}2}^{\mp}$, $P_{\mathbf{k}1}^{\pm} \to P_{\mathbf{k}2}^{\mp}$ for the second polarization direction, $Q_{\mathbf{k}21}$, $Q_{\mathbf{k}22}$. This leads to Eq. (2). The advantage of this representation is that all modes are coupled via their canonical position to the DQDs.

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