Phase gate with a four-level inverted-Y system

Amitabh Joshi* and Min Xiao
Department of Physics, University of Arkansas, Fayetteville, Arkansas 72701, USA
(Received 3 June 2005; published 14 December 2005)

The four-level atomic system in an inverted-Y configuration is investigated for large Kerr nonlinearities. The cross-Kerr nonlinearity generated in such a system can produce a phase shift of order π and can be used for realizing polarization quantum phase gates.

DOI: 10.1103/PhysRevA.72.062319 PACS number(s): 03.67.Lx, 42.65.−k, 42.50.Gy

I. INTRODUCTION

An all-optical quantum computation has been proposed but a realization of two-qubit quantum gates, which are required for universal quantum computation, is not so straightforward due to lack of significant photon-photon interaction [1]. Many proposals have come up in recent years for efficiently implementing all-optical quantum computation. The linear optics quantum computation scheme is based on single-photon sources, passive linear optical devices, and detectors [2]. The scheme essentially uncovers the nonlinearities associated with the photodetection process and is basically a probabilistic scheme for implementing any quantum gate. Other schemes are based on the nonlinear responses of optical systems, i.e., making use of the optical nonlinearities to realize quantum phase gate operations. Under normal circumstances optical nonlinearities are too small to enhance the photon-photon interaction so the optical quantum gate operation cannot be efficiently implemented. The optical nonlinearity in atomic systems can be greatly enhanced in the presence of quantum interference in electromagnetically induced transparency (EIT) systems [3,4]. This enhancement is commonly observed in the weak probe laser beam in the presence of another strong coupling or driving laser beam(s) when these lasers are slightly off resonant from their respective transitions. The absorption, dispersion, and nonlinearity of the EIT systems are very sensitive to the probe, and coupling laser frequency detunings. This means that if exact two-photon resonance condition is disturbed in the typical three-level Λ-type system, the enhancement of nonlinearity could be observed [5]. Some other multilevel schemes have been investigated involving four and five levels in which enhancement of Kerr nonlinearities has been predicted, including the enhancement of self-phase modulation as well as cross-phase-modulation [6,7]. Schmidt and Imamoglu [6] used an EIT system of four levels in an N configuration and predicted dramatic improvement by several orders of magnitude in nonlinearities, as well as enhancement in cross-phase modulation in comparison with conventional three-level ladder system. The importance of cross-phase modulation is in implementing two-qubit all-optical quantum phase gates, where one qubit gets a phase shift dependent on the state of the other qubit. Cross-phase-modulation can occur in multi-level EIT systems when propagating pulses are allowed to interact for a considerably longer time by reducing their group velocities and also making these group velocities of the same order of magnitude, which was demonstrated in a tripod or adjacent Λ system [8]. Here we propose a phase-gating scheme using four-level atoms in an inverted-Y configuration [9]. Such a level scheme with a Λ system at its heart (or that can also be seen as an adjacent ladder system with a common middle level, as discussed in Sec. II), exhibiting double EIT (i.e., having double dark states [9]), is an alternate scheme to obtain enhanced nonlinearity and cross-phase-modulation by allowing the system to be tuned to its dark states, having EIT windows for each of the dark states to be narrow with a steep dispersion, enabling a significant reduction of group velocities as well as their matching due to symmetry of the system. In this way the EIT system in an inverted-Y configuration is different from the conventional three-level ladder scheme [6], and the latter does not give large nonlinearities and is limited by system parameters like detuning and the lifetime of the middle level. However, the EIT system in an *N* configuration of four levels [6] produces large nonlinearity and cross-phase modulation due to the simultaneous elimination of absorption loss. The inverted-Y configuration can easily be realized experimentally in rubidium atomic vapor and is very straightforward to implement. The two lower transitions are used for encoding the binary information in the polarization degree of freedom of the probe and signal pulses. The phase-gate mechanism operates on the cross-phase modulation effect between probe and signal fields. The electromagnetic fields are proposed as a possible environment for performing quantum logical operations. The atoms act as catalyzers of the logical operations. In the search of scalable quantum computing, the new methods of implementing QPG or CNOT gates are always desirable and has motivated us to do this work. Our purpose is first to explore the nonlinearities associated with such a kind of four-level system and their maneuverability using the atomic and laser parameters, and then finding the optimum conditions so that cross-phase-modulations of probe-signal fields become enhanced and can be used for a very efficient polarization phase gating for all-optical quantum computation purposes. The phase gate operation in multilevel systems in other configurations (adjacent Λ system, etc., which was different from the current work) has been demonstrated recently [7,8].

*Electronic address: ajoshi@uark.edu

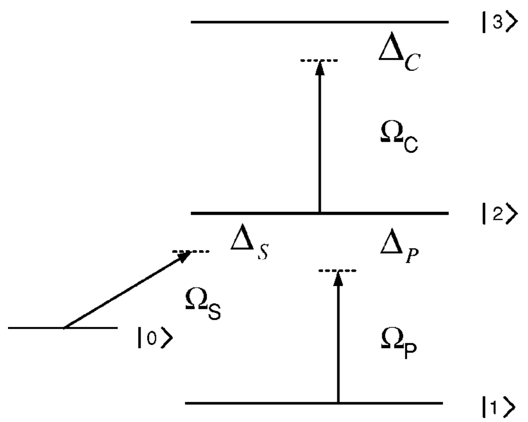

FIG. 1. Schematic diagram of a four-level atomic system in an inverted-Y configuration. Here, Ω_P, Ω_S, Ω_C are Rabi frequencies associated with the probe, signal, and control fields, respectively.

II. THE MODEL

The level scheme for the four-level atomic system in an inverted-Y configuration (with the lower three levels forming a Λ system) is shown in Fig. 1, which has been experimentally realized in Rb atoms. Under the condition that lower levels are equally populated and the decay rate of level $|0\rangle$ is quite small, then levels $|1\rangle$, $|2\rangle$ and $|3\rangle$ are in a three-level ladder-type configuration and level $|0\rangle$, together with levels $|2\rangle$ and $|3\rangle$, forms another three-level ladder-type configuration. So, this composite system consists of two subsystems; each one of them is a ladder-type system. The description of the optically allowed transitions in this system is as follows. The transition $|1\rangle$ to $|2\rangle$ (transition frequency ω_{12}) interacts with a weak probe field E_1 (frequency ω_1) having Rabi frequency Ω_P. A weak signal field E_0 (frequency ω_0) drives the transition $|0\rangle$ to $|2\rangle$ (with transition frequency ω_{02}) with the Rabi frequency of Ω_S while a coupling (control) field E_2 (frequency ω_2) is acting on the transition $|2\rangle$ and $|3\rangle$ (with transition frequency ω_{23}) and has a Rabi frequency of Ω_C. All the Rabi frequencies are assumed to be real. The radiative decay constants from levels $|3\rangle$ to $|2\rangle$, $|2\rangle$ to $|0\rangle$, and $|2\rangle$ to $|1\rangle$ are γ_3, γ_2, and γ_1, respectively. The corresponding atomic detunings for these transitions are $\Delta_C=\omega_2-\omega_{23}$, $\Delta_S=\omega_0-\omega_{02}$, and $\Delta_P=\omega_1-\omega_{12}$, respectively.

For this system the Hamiltonian under a dipole and rotating-wave approximation (RWA) can be written as

$$\hat{H}=\Delta_S|0\rangle\langle 0|+\Delta_P|1\rangle\langle 1|+\Delta_C|3\rangle\langle 3|+[\Omega_S|2\rangle\langle 0|+\Omega_P|2\rangle\langle 1| + \Omega_C|2\rangle\langle 3|+\text{H.c.}], \quad (1)$$

where $|i\rangle\langle j|$ $(i,j=0-3)$ are pseudospin atomic operators. The basic description of nonlinearity generation from such a system can be analyzed in terms of the dressed states created by the fields interacting with different energy levels. In the resonant condition of probe, signal and coupling field ($\Delta_P=\Delta_S=\Delta_C=0$), the eigenvalues of this Hamiltonian are 0, 0, $\pm\sqrt{\Omega_P^2+\Omega_S^2+\Omega_C^2}$, respectively. The corresponding eigenstates to the zero eigenvalues are linear combinations of the states $|1\rangle$, $|0\rangle$, and $|3\rangle$. Thus, we have two dark states or the degenerated dark states in this case. However, for different nonzero detuning values, the dark states are no longer degenerate. If the atoms are in any or both of the dark states, then EIT is exhibited. The interesting aspect of this EIT situation is its controllability by appropriately selecting the amplitude for the coupling field. In order to achieve a large cross-Kerr effect, both the probe and signal fields should be tuned to dark states such that transparency windows are very narrow for them. The variation of the refractive index should be steep so that there is significant group velocity reduction. The last crucial requirement is symmetry of transparency windows so that probe and signal velocities are equal. One can achieve these conditions by taking all the three detunings equal in magnitude. By slightly going away from the exact resonance condition, the strong cross-Kerr modulation and group velocity matching is readily obtained [8]. This situation helps in achieving the phase gate operation in this system.

The density-matrix equations of motion in the dipole and RWA for this system can be written as follows:

$$\dot{\rho}_{11}=2\gamma_1\rho_{22}+2\gamma_0\rho_{00}+i\Omega_P(\rho_{12}-\rho_{21}),$$

$$\dot{\rho}_{22}=-2(\gamma_1+\gamma_2)\rho_{22}+2\gamma_3\rho_{33}-i\Omega_P(\rho_{12}-\rho_{21}) - i\Omega_S(\rho_{02}-\rho_{20})+i\Omega_C(\rho_{23}-\rho_{32}),$$

$$\dot{\rho}_{33}=-2\gamma_3\rho_{33}-i\Omega_C(\rho_{23}-\rho_{32}),$$

$$\dot{\rho}_{00}=2\gamma_2\rho_{22}-2\gamma_0\rho_{00}+i\Omega_S(\rho_{02}-\rho_{20}),$$

$$\dot{\rho}_{12}=-(\gamma_1+\gamma_2-i\Delta_P)\rho_{12}+i\Omega_S\rho_{10}+i\Omega_C\rho_{13} + i\Omega_P(\rho_{11}-\rho_{22}),$$

$$\dot{\rho}_{13}=-[\gamma_3-i(\Delta_P+\Delta_C)]\rho_{13}+i\Omega_C\rho_{12}-i\Omega_P\rho_{23},$$

$$\dot{\rho}_{23}=-(\gamma_1+\gamma_2+\gamma_3-i\Delta_C)\rho_{23}-i\Omega_P\rho_{13}-i\Omega_S\rho_{03} + i\Omega_C(\rho_{22}-\rho_{33}),$$

$$\dot{\rho}_{10}=-(\gamma_0-i(\Delta_P-\Delta_S))\rho_{10}+i\Omega_S\rho_{12}-i\Omega_P\rho_{20},$$

$$\dot{\rho}_{02}=-(\gamma_1+\gamma_2-i\Delta_S)\rho_{02}+i\Omega_P\rho_{01}+i\Omega_C\rho_{03}+i\Omega_S(\rho_{00}-\rho_{22}),$$

$$\dot{\rho}_{03}=-[\gamma_0+\gamma_3-i(\Delta_S+\Delta_C)]\rho_{03}+i\Omega_C\rho_{02}-i\Omega_S\rho_{23}, \quad (2)$$

where γ_0 is related to the nonradiative relaxation rate of state $|0\rangle$ and the trace condition $\Sigma_i\rho_{ii}=1$. The probe and signal susceptibilities are defined by [8]

$$\chi_P=-\lim_{t\to\infty}\frac{N|\vec{d}_{12}|^2}{\hbar\varepsilon_0}\frac{\rho_{12}}{\Omega_P}, \quad (3)$$

$$\chi_S=-\lim_{t\to\infty}\frac{N|\vec{d}_{02}|^2}{\hbar\varepsilon_0}\frac{\rho_{02}}{\Omega_S}, \quad (4)$$

in which N is the number density of atoms and $\vec{d}_{12}$ ($\vec{d}_{02}$) is the electric-dipole matrix element for the probe (signal) transition.

For obtaining these susceptibilities, we need to solve the density-matrix equations (2) under the steady-state condition. Under the assumption that the coupling field (E_2) is much stronger than the probe field (E_1) and signal field (E_0), and the detunings are of the same orders of magnitude, the steady-state population will be equally distributed in the ground states $|1\rangle$ and $|0\rangle$. The expressions of ρ_{12} and ρ_{02} under perturbation expansion can be written as

$$\rho_{12} \cong \frac{i\Omega_P(\rho_{11}-\rho_{22})}{\left(\gamma_1+\gamma_2-i\Delta_P+\dfrac{\Omega_S^2}{\gamma_0-i(\Delta_P-\Delta_S)}+\dfrac{\Omega_C^2}{\gamma_3-i(\Delta_P+\Delta_C)}\right)} - \frac{i\Omega_P\Omega_S^2(\rho_{00}-\rho_{22})}{(\gamma_0-i(\Delta_P-\Delta_S))\left(\gamma_1+\gamma_2+i\Delta_S+\dfrac{\Omega_C^2}{\gamma_3+\gamma_0+i(\Delta_S+\Delta_C)}\right)\left(\gamma_1+\gamma_2-i\Delta_P+\dfrac{\Omega_C^2}{\gamma_3-i(\Delta_P+\Delta_C)}\right)}, \tag{5}$$

$$\rho_{02} \cong \frac{i\Omega_S(\rho_{00}-\rho_{22})}{\left(\gamma_1+\gamma_2-i\Delta_S+\dfrac{\Omega_P^2}{\gamma_0+i(\Delta_P-\Delta_S)}+\dfrac{\Omega_C^2}{\gamma_3+\gamma_0-i(\Delta_S+\Delta_C)}\right)} - \frac{i\Omega_S\Omega_P^2(\rho_{11}-\rho_{22})}{(\gamma_0+i(\Delta_P-\Delta_S))\left(\gamma_1+\gamma_2+i\Delta_P+\dfrac{\Omega_C^2}{\gamma_3+i(\Delta_P+\Delta_C)}\right)\left(\gamma_1+\gamma_2-i\Delta_S+\dfrac{\Omega_C^2}{\gamma_0+\gamma_3-i(\Delta_S+\Delta_C)}\right)}. \tag{6}$$

In the previous expressions we have kept two lowest-order contributions in probe and signal susceptibilities, which include the cross-phase-modulation between probe and signal fields, but higher-order contributions are neglected. The susceptibilities can be expressed as

$$\chi_P = \chi_P^{(1)} + \chi_P^{(3)}|E_0|^2, \tag{7}$$

$$\chi_S = \chi_S^{(1)} + \chi_S^{(3)}|E_1|^2, \tag{8}$$

and the expressions for $\chi_l^{(1)}, \chi_l^{(3)}$ $(l=P,S)$ are symmetric with the simultaneous exchange of $0 \longleftrightarrow 1$ (in ρ_{ij}) and $P \longleftrightarrow S$ when γ_0 is of the same order of magnitude as γ_3 or smaller.

The system under consideration can be viewed as two adjacent ladder systems: one involving the probe field while other involving the signal field sharing the coupling field. Hence both fields exhibit EIT, which is manifested as different transparency windows corresponding to two dark states. The two windows of EIT coincide when the dark states become degenerate. This occurs when the detunings are zero or of equal magnitudes. The equal detunings give rise to almost similar dispersive properties for both probe and the signal fields, and thus the same group velocity [8]. The group velocity of light pulse is defined by $v_g = c/(1+n_g)$, in which c is the speed of light in vacuum and the group index is given by

$$n_g = (1/2)\mathrm{Re}[\chi] + (\omega_L/2)\left.\frac{\partial\,\mathrm{Re}[\chi]}{\partial\omega}\right|_{\omega=\omega_L}. \tag{9}$$

The group index is basically dependent on the linear susceptibility as the contribution from higher-order susceptibilities are much smaller and can be neglected. In the case of equal detunings it is possible to write down the expression for the group velocity at the center of the transparency window for each field where $\mathrm{Re}[\chi]$ vanishes and the group velocity reduction takes place as a result of large dispersion gradient. The expressions for the group velocities of probe and signal fields are [8–11]

$$v_g(\text{probe}) = \frac{4\hbar c\varepsilon_0}{\omega_P N|d_{12}|^2}(\Omega_C^2+\Omega_S^2), \tag{10}$$

$$v_g(\text{signal}) = \frac{4\hbar c\varepsilon_0}{\omega_S N|d_{02}|^2}(\Omega_C^2+\Omega_P^2). \tag{11}$$

The group velocity matching can be easily achieved because the orders of dipole matrix elements under consideration are nearly equal and so are the signal and probe Rabi frequencies. Under the resonance condition or the condition of equal detunings, the nonlinear susceptibility vanishes and so will the cross-phase-modulation. In order to have a nonlinear refractive index, we need to disturb the EIT conditions by taking slightly different detuning such that the common transparency window is not disturbed and absorption is still smaller. By doing so, the group velocity matching can also be maintained and cross-phase-modulation is also obtained, which is the requirement for the phase-gate operation [8].

III. IMPLEMENTATION OF TWO OPTICAL QUBIT PHASE GATE

In order to implement the quantum phase gate between two optical qubits cross-phase-modulation is a very essential requirement. This could be realized very easily for a system exhibiting the cross-Kerr effect, where one optical field acquires a phase shift conditioned to the state of another optical field, as the inverted-Y system discussed above. The gate transformation is defined by the input-output relationship [1,12]

$$|i\rangle_1|j\rangle_2 \rightarrow \exp(i\theta_{ij})|i\rangle_1|j\rangle_2, \quad (i,j=0,1), \tag{12}$$

in which, $|i\rangle$, $|j\rangle$ denote the qubit basis. By this operation the two initially factorized qubits get entangled when the conditional phase shift $\theta=\theta_{11}+\theta_{00}-\theta_{10}-\theta_{01}$ becomes different from zero and thus a universal two-qubit gate can be realized [1,12,13]. We briefly outline the phase gate operation following [8], just for the sake of completeness. For optical beams, polarization degrees of freedom can form two logical basis states $|0\rangle$ and $|1\rangle$ of the gate transformation under consideration that corresponds to two orthogonal light polarizations [7,8]. We can have two different polarizations each for probe and signal beams. For the universal Quantum-Phase-Gate (QPG) operation one needs nontrivial cross-phase-modulation between probe and signal fields for one of the four input probe and signal field polarization configurations. We fix the polarization of the probe light to be σ^+ and the signal light to be σ^- for this to occur. If probe has σ^- polarization then the trivial vacuum phase shift $\theta_0^P=k^PL$ (L is the length of the medium) will occur as it is not able to interact with any level, but the signal field with σ^- polarization will acquire a linear phase-shift $\theta_{linear}^s=k^SL(1+2\pi\chi_S^{(1)})$. The cross-phase-modulation between these two σ^- polarized pulses will not occur for narrow laser linewidths and nearly equal detuning conditions. Similarly, if both probe and signal pulses are of σ^+ polarizations then the phase shifts acquired by them will reverse their roles, i.e., $\theta_{linear}^P=k^PL(1+2\pi\chi_P^{(1)})$ and $\theta_0^S=k^SL$. When polarizations of the probe and signal fields are σ^- and σ^+, and there is no nearby levels to which these fields can couple, then both of the fields just acquire the vacuum phase shift $\theta_0^{P(S)}=k^{P(S)}L$. The qubit formed by superposition of two circularly polarized states of field can be written in the single photon wave packet as [7,8]

$$|q_i\rangle=\beta_i^+|\sigma^+\rangle_i+\beta_i^-|\sigma^-\rangle_i, \quad i=P,S, \tag{13}$$

where $\beta_i^\pm$ are constants. The photon field operators $b^\pm(\omega)$ corresponding to $\sigma^\pm$ polarizations undergo a transformation while propagating through the atomic medium of length L,

$$b^\pm(\omega)\rightarrow b^\pm(\omega)\exp\left((i\omega/c)\int_0^L dz\,\eta_\pm(\omega,z)\right). \tag{14}$$

Under the assumption that Re[$\eta_\pm$] (which is the real part of the refractive index) is slowly varying over the bandwidth of the wave packet centered at ω_i, $\eta_\pm(\omega,z)=\eta_\pm(\omega_i,z)$, a phase shift in circularly polarized state is obtained to be $|\sigma^\pm\rangle_i\rightarrow e^{-\theta_\pm^i}|\sigma^\pm\rangle_i$, with $\theta_\pm^i=(\omega/c)\int_0^L dz\;\eta_\pm(\omega_i,z)$.

The polarization QPG truth table for the inverted-Y configuration goes as

$$|\sigma^-\rangle_P|\sigma^-\rangle_S=e^{-i(\theta_0^P+\theta_{linear}^S)}|\sigma^-\rangle_P|\sigma^-\rangle_S,$$

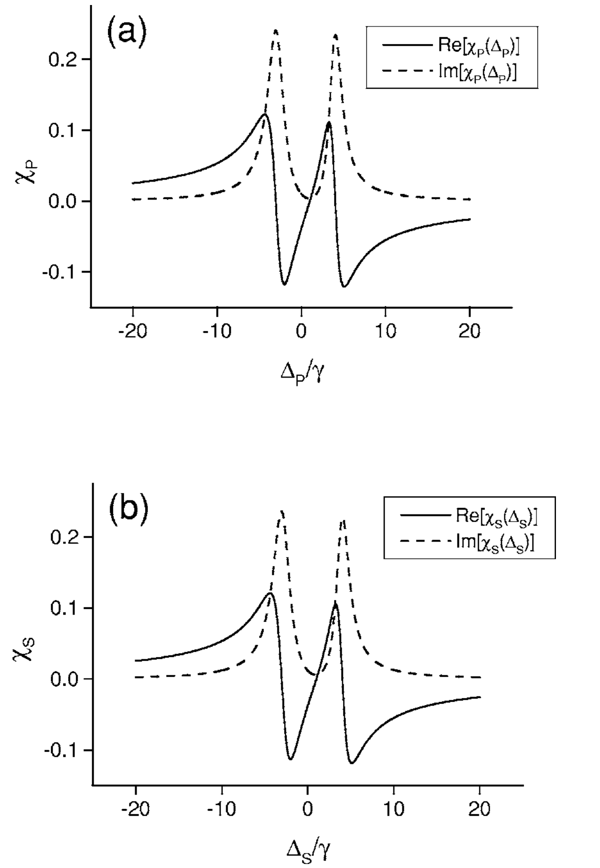

FIG. 2. Probe susceptibility χ_P (a) as a function of probe detuning Δ_P/γ for $\Delta_C/\gamma=-1.0$ and $\Delta_S/\gamma=1.04$. Signal susceptibility χ_S (b) as a function of signal detuning Δ_S/γ for $\Delta_C/\gamma=-1.0$ and $\Delta_P/\gamma=1.02$. The other parameters for both the cases are $\Omega_P=\Omega_S=0.1\gamma$, $\Omega_C=2.5\gamma$. The dispersion and absorption are represented by solid and dashed curves, respectively.

$$|\sigma^-\rangle_P|\sigma^+\rangle_S=e^{-i(\theta_0^P+\theta_0^S)}|\sigma^-\rangle_P|\sigma^+\rangle_S,$$

$$|\sigma^+\rangle_P|\sigma^+\rangle_S=e^{-i(\theta_{linear}^P+\theta_0^S)}|\sigma^+\rangle_P|\sigma^+\rangle_S,$$

$$|\sigma^+\rangle_P|\sigma^-\rangle_S=e^{-i(\theta_+^P+\theta_-^S)}||\sigma^+\rangle_P|\sigma^-\rangle_S, \tag{15}$$

in which, $\theta_+^P=\theta_{linear}^P+\theta_{non}^P$, and $\theta_-^S=\theta_{linear}^S+\theta_{non}^S$ and the conditional phase shift is defined through $\theta=\theta_+^P+\theta_-^S-\theta_{linear}^P-\theta_{linear}^S$, which implies that only nonlinear phases contribute to θ. The nonlinear phase shift θ_{non}^P (θ_{non}^S) related to the real part of the nonlinear susceptibility $\chi_P^{(3)}$ ($\chi_S^{(3)}$) [as defined in Eqs. (3)–(8)] is given by [7,8]

$$\theta_{non}^l=k_lL\frac{\pi^{3/2}\hbar^2\Omega_m^2}{4|d_n|^2}\;\mathrm{Re}[\chi_l^{(3)}]\frac{\mathrm{erf}(\xi_l)}{\xi_l},$$

$$\xi_l=\sqrt{2}L(1-v_g^l/v_g^m)/(T_mv_g^l), \quad (l\equiv P,S;\; m\equiv S,P;\; n\equiv 02,12); \tag{16}$$

where the duration of the probe (signal) pulse is given by $T_{P(S)}$ and group velocity $v_g^{P(S)}$. The advantage of the system discussed above is that it can easily be realized in ^{87}Rb atoms confined in a vapor cell or MOT (magneto-optical trap) where the cold atoms are transferred in the $5S_{1/2}$ state with $F=1$, $m=(-1,1)$ of ^{87}Rb, which corresponds to the lower two states of the inverted-Y system and the state $|2\rangle$ corresponds to $5P_{3/2}$, $F=0$ level while state $|3\rangle$ corresponds to the $5D_{5/2}$, $F=1$ level. In this configuration of rubidium atoms in a vapor cell, the conditional phase-shift of π radians can be obtained for $\Omega_P=\Omega_S=0.1\gamma$, $\Omega_C=2.5\gamma$, $\Delta_P=1.02\gamma$, $\Delta_S=1.04\gamma$, $\Delta_C=-1.0\gamma$, $(\gamma_1=\gamma_2=1, \gamma_3=0.1\gamma, \gamma_0\cong 0.05\gamma)$, $N=6\times 10^{12}$ cm^{-3}, $L\cong 1$ cm. Under these conditions the group velocity matching is also achieved. We have to slightly go away from the exact EIT condition by suitably adjusting the detunings so that nonlinearity becomes optimum and the group velocity matching condition is also fulfilled. For these parameters the probe and signal susceptibilities are depicted in Fig. 2. We do not find a very appreciable amount of absorption for this set of parameters, so there will not be much fidelity contamination for phase gate. Some other advantages of our four-level scheme is as follows. Since there is no optical pumping issue involved in the inverted-Y system, i.e., as the signal and probe field intensities increase, the population will still be distributed among the two ground states $|0\rangle$ and $|1\rangle$. When an atomic cell is used for the experiment, the ladder systems (the inverted-Y system is an adjacent ladder system) require counterpropagating beams, so signals have less of a background from the strong pumping (control) beam. By choosing lower-energy levels suitably or using appropriate buffer gas pressure one can have γ_0 as minimum as possible to enhance the cross-phase-modulation, which is another advantage of this system.

IV. SUMMARY

To summarize, in this work we studied nonlinear properties of a four-level system in an inverted-Y configuration interacting with two weak fields and a strong field. This system is different from the previously studied four-level systems for manipulating nonlinearity and can be seen as composite or two adjacent ladder systems. This system can provide large cross-Kerr effect between the probe and the signal fields (by appropriately selecting or manipulating the decay constant of coherence between the lower two levels) which is useful in implementing a conditional phase gate. Also, the practical realization of such a four-level system is easily achievable in a rubidium atomic system in a gas cell where the probe and signal fields copropagating and control field counterpropagating to cancel the first-order Doppler effect.

ACKNOWLEDGMENT

We acknowledge the funding support from the National Science Foundation.

[1] M. A. Nielsen and I. L. Chuang, *Quantum Computation and Quantum Information* (Cambridge University Press, Cambridge, England, 2000).

[2] E. Knill, R. Laflamme, and G. J. Milburn, Nature **409**, 46 (2001).

[3] K. J. Boller, A. Imamoglu, and S. E. Harris, Phys. Rev. Lett. **66**, 2593 (1991).

[4] S. E. Harris, Phys. Today **50**, 36 (1997).

[5] H. Wang, D. Goorskey, and M. Xiao, Phys. Rev. Lett. **87**, 073601 (2001).

[6] H. Schmidt and A. Imamoglu, Opt. Lett. **21**, 1936 (1996).

[7] C. Ottaviani, D. Vitali, M. Artoni, F. Cataliotti, and P. Tombesi, Phys. Rev. Lett. **90**, 197902 (2003).

[8] S. Rebic, D. Vitali, C. Ottaviani, P. Tombesi, M. Artoni, F. Cataliotti, and R. Corbalan, Phys. Rev. A **70**, 032317 (2004).

[9] A. Joshi and M. Xiao, Phys. Lett. A **317**, 370 (2003).

[10] M. Fleischhauer and M. D. Lukin, Phys. Rev. Lett. **84**, 5094 (2000); Phys. Rev. A **65**, 022314 (2002).

[11] A. Joshi and M. Xiao, Phys. Rev. A **71**, 041801(R) (2005).

[12] Q. A. Turchette, C. J. Hood, W. Lange, H. Mabuchi, and H. J. Kimble, Phys. Rev. Lett. **75**, 4710 (1995).

[13] S. Lloyd, Phys. Rev. Lett. **75**, 346 (1995).