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Efficient scheme for multipartite entanglement and quantum information processing with trapped ions

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In this paper, based on the recent experiment by Roos *et al.* [Science **304**, 1478 (2004)], a theoretical scheme is proposed to create the multipartite entanglement of many trapped ions and implement a two-qubit quantum phase gate between two ions in ion trap. In the scheme, the ion is illuminated by a single laser tuned to the first lower vibrational sideband. We also show that the scheme can be used to directly transfer information between two ions. The scheme has the advantage that it does not use the vibrational mode as the data bus and only requires a single resonant interaction. Thus the scheme is very simple and the quantum dynamics operation can be realized at a high speed. In view of the decoherence mechanism, the simplification for the entangled state preparation and experimental implementation of quantum logic operation may become crucial.

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I. INTRODUCTION

Entanglement is one of the most striking features of quantum physics. In particular entangled states for two or more particles serve to demonstrate fundamental quantum properties which are far beyond the conceptual framework defined by classical physics. Recently, two-particle entangled states have been realized in both cavity QED [1] and ion traps [2]. However, creating entanglement of many qubits (or particles) is still a great challenge for both theoretical and experimental physicist. Briegel and Raussendorf [3] introduced a special kind of multipartite entangled states, the so-called cluster states, which can be created via an Ising-type interaction. It has been shown that quantum computer can be realized via cluster states. In the same year Raussendorf and Briegel described the so-called one-way quantum computer [4], in which information is written onto the cluster and read out from the cluster by one-qubit measurements. However, an ideal Ising interaction is difficult to obtain experimentally. More recently, several interactions [5,6] have been proposed for the construction of those states, but so far no explicit physical system has been shown to serve as a realistic model.

On the other hand, quantum entanglement is useful in quantum information processing, such as quantum cryptography [7], computer [8], teleportation [9], and dense coding [10]. The main ingredient of quantum computer is the conditional quantum dynamics, in which one subsystem undergoes a coherent evolution depending on the state of another system. It has been shown that the building blocks of quantum computers are two-qubit logic gates. In some physics systems such as nuclear magnetic resonance [11–13], trapped ions [14,15], cavity QED [16–23], and optical systems [24,25] have been proposed for realizing quantum logic gates. In previous schemes for quantum information processing in ion trap, the scheme of Ref. [14], based on the resonant sideband excitation, uses the vibrational mode as the data bus. In this scheme, the information of an electronic system was stored in the vibrational mode and then transfers back to this electronic system after the conditional dynamics. Thus three resonant ion-laser interactions are required in order to achieve a quantum phase gate between two ions. The main obstacle for the implementation of quantum information is the difficulty to achieve the joint ground state of the ion motion and the heating of the ions. Recently, Sorensen and Molmer [26] have proposed a scheme for realizing quantum computation in ion traps via virtual vibrational excitations. The advantage of this scheme is that it does not use the motional mode as the data bus and is insensitive to the vibrational states. The idea of Ref. [26] can be used to create multiparticle entanglement and teleport the state of trapped ions [27]. More recently, Jonathan *et al.* [28] have proposed a scheme to implement relatively fast quantum gates for trapped ion in thermal motion with a single laser resonant with the ionic carrier frequency. In this scheme, in order for it to be insensitive to heating, one should invert the phase of the laser many times. All the above-mentioned schemes in ion trap put limitations on the intensity of the laser fields, which restricts the speed of the gates, making it difficult to perform a complex sequence of quantum operations withoutbeing significantly affected by decoherence.

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In this paper, we proposed a scheme to create the multipartite entanglement with many trapped ions and implement the quantum phase gate between two trapped ions. The scheme does not use the vibrational mode as the data bus and only requires a single resonant interaction. Thus the scheme is much simpler than the scheme of Ref. [14] and the required interaction time is shortened. The simplification of the procedure and decrease of operation time are important for the quantum dynamics operating at a high speed, which is important in view of decoherence. Finally we also show that the present scheme can be used for directly transferring quantum information between two ions. The paper is organized as follows. In Sec. II, we briefly review the dynamics of the ion-laser interaction in the ion trap. In Sec. III, we propose a procedure for creating multipartite entanglement of many trapped ions. In Sec. IV, we show how the scheme can be used to implement quantum phase gate between ions including the effect of atomic spontaneous emissions. And then we briefly discuss the directly transferring information between two ions. A summary appears in Sec. V.

II. THE TIME EVOLUTION OF AN ION IN A LINEAR POTENTIAL

We consider a two-level ion confined in a one-dimensional harmonic potential and interacting with a traveling wave laser field, tuned to the first lower vibrational sideband. In the rotating wave approximation, the Hamiltonian for such a system is given by ($\hbar=1$)

$$H = \nu a^\dagger a + \omega_0 \sigma_z + [\lambda E^+(x,t)\sigma^+ + \text{H.c.}], \qquad (1)$$

where a and $a^\dagger$ are the annihilation and creating operators for the motion of the trapped ion, $\sigma_z=\frac{1}{2}(|e\rangle\langle e|-|g\rangle\langle g|)$, $\sigma^+=|e\rangle\langle g|$, $\sigma^-=|g\rangle\langle e|$ are raising, lowering and inversion operators for the two-level ion with $|e\rangle$ and $|g\rangle$ being the excited state and ground state, ν, ω_0 and λ are the trap frequency, the transition frequency and dipole matrix element characterizing the transition in the two-level ion, and $E^+(x,t)$ is the positive frequency part of the classical field,

$$E^+(x,t) = E_0 e^{-i[(\omega_0-\nu)t-kx+\phi]}, \qquad (2)$$

where E_0, ϕ, and k are the amplitude, phase, and wave vector for the driving laser field. The operator of the center-of-mass position x can be reexpressed as $x=(1/\sqrt{2\nu M})(a+a^\dagger)$ with M being the mass of the ion.

In the resolve sideband limit, the ion-laser interaction can be described by the nonlinear Jaynes-Cummings model [29–32]. Then the Hamiltonian of Eq. (1), in the interaction picture, can be simplified to

$$H_i = \Omega e^{-i\phi} e^{-\eta^2/2} \sigma^+ \sum_{j=0}^{\infty} \frac{(i\eta)^{2j+1}}{j\,!\,(j+1)!} (a^\dagger)^j a^{j+1} + \text{H.c.}, \qquad (3)$$

where $\Omega=\lambda E_0$ is the Rabi frequency of the laser field and $\eta=k/\sqrt{2\nu M}$ is the Lamb-Dicke parameter. We consider the behavior of the ion in the Lamb-Dicke regime, i.e., $\eta \ll 1$. The Hamiltonian can be approximated by the expansion to the first order in η,

$$H_i = i\eta\Omega e^{-i\phi} a\sigma^+ + \text{H.c.} \qquad (4)$$

III. GENERATION OF MULTIPARTITE ENTANGLEMENT FOR MANY TRAPPED IONS

In order to prepare the cluster states we use a third ionic state $|f\rangle$. In the experiment of Ref. [33], $S_{1/2}(m_j=-1/2)$ and $D_{5/2}(m_j=-1/2)$ of $^{40}Ca^+$ ions can be used for the states $|g\rangle$ and $|e\rangle$, respectively. In the case $S_{1/2}(m_j=1/2)$ can be used for the state $|f\rangle$. The transition frequency between the states $|f\rangle$ and $|g\rangle$ is highly detuned and thus the state $|g\rangle$ is not affected during the laser-ion interaction. In the case that $\phi=2\pi$, according to the Hamiltonian (4), the evolution of the system will be described by the unitary operator

$$U(t) = \exp[\eta\Omega t(a\sigma^+ - \text{H.c.})]. \qquad (5)$$

It is easy to prove that the following transformation keeps the state $|f\rangle|0\rangle_m$ unaltered under the application of the unitary operator (5), whereas

$$U(t)|f\rangle|1\rangle_m = \cos(\eta\Omega t)|f\rangle|1\rangle_m - \sin(\eta\Omega t)|e\rangle|0\rangle_m, \qquad (6)$$

$$U(t)|e\rangle|0\rangle_m = \cos(\eta\Omega t)|e\rangle|0\rangle_m + \sin(\eta\Omega t)|f\rangle|1\rangle_m, \qquad (7)$$

where $|0\rangle_m$ ($|1\rangle_m$) denotes a state of the vibrational mode with no (one) phonon.

If the vibrational modes are initially in the entangled state with other subsystems $(1/\sqrt{2})(|0\rangle_m|\phi_1\rangle+|1\rangle_m|\phi_2\rangle)$, and $|\phi_1\rangle$ and $|\phi_2\rangle$ are arbitrary normalized wave functions of the subsystems. We assume the first ion prepared in the superposition state $(1/\sqrt{2})(|g_1\rangle+|f_1\rangle)$. Choosing the parameter to satisfy $\eta\Omega t=\pi$, the state of the system becomes

$$|\psi\rangle = \tfrac{1}{2}[|0\rangle_m(|g_1\rangle + |f_1\rangle)|\phi_1\rangle + |1\rangle_m(|g_1\rangle - |f_1\rangle)|\phi_2\rangle]. \qquad (8)$$

Then let us consider the following step process, the first ion is subjected to one classical pulse by choosing the amplitudes and phases appropriately so that this ion undergoes the transition $|f_1\rangle \to (1/\sqrt{2})(|g_1\rangle-|f_1\rangle)$ and $|g_1\rangle \to (1/\sqrt{2})\times(|g_1\rangle+|f_1\rangle)$. The state (8) becomes

$$|\psi\rangle = \frac{1}{\sqrt{2}}(|0\rangle_m|g_1\rangle|\phi_1\rangle + |1\rangle_m|f_1\rangle|\phi_2\rangle) \qquad (9)$$

Now we consider one identical auxiliary ion in the state $|f_2\rangle$. After the interaction time $\pi/2\eta\Omega$, the state evolution of the system is given by

$$|\psi\rangle = U(t)\tfrac{1}{\sqrt{2}}(|0\rangle_m|g_1\rangle|\phi_1\rangle + |1\rangle_m|f_1\rangle|\phi_2\rangle)|f_2\rangle$$
$$= \tfrac{1}{\sqrt{2}}|0\rangle_m(|f_2\rangle|g_1\rangle|\phi_1\rangle + |e_2\rangle|f_1\rangle|\phi_2\rangle). \qquad (10)$$

Then a short strong classical pulse is applied to the auxiliary ion, which induces the transition $|f_2\rangle \to (1/\sqrt{2})(|e_2\rangle+|f_2\rangle)$ and $|e_2\rangle \to (1/\sqrt{2})(|f_2\rangle-|e_2\rangle)$ (assuming the pulse is strong enough that the contributions of the interaction of the ion with laser field can be neglected). Thus the state of the system becomes

$$|\psi\rangle = \tfrac{1}{2}[|f_2\rangle(|g_1\rangle|\phi_1\rangle + |f_1\rangle|\phi_2\rangle) + |e_2\rangle(|g_1\rangle|\phi_1\rangle - |f_1\rangle|\phi_2\rangle)]$$
$$\times|0\rangle_m. \qquad (11)$$

Now we consider the interaction of the auxiliary ion again interacts with the laser field for the time $\pi/2\eta\Omega$. We can obtain

$$|\psi\rangle=\tfrac{1}{2}[|0\rangle_m(|g_1\rangle|\phi_1\rangle+|f_1\rangle|\phi_2\rangle)+|1\rangle_m(|g_1\rangle|\phi_1\rangle-|f_1\rangle|\phi_2\rangle)], \tag{12}$$

which can be rewritten as follows:

$$|\psi\rangle=\tfrac{1}{2}(|0\rangle_m+|1\rangle_m\sigma_1)(|g_1\rangle|\phi_1\rangle+|f_1\rangle|\phi_2\rangle) \tag{13}$$

with $\sigma_i=|g_i\rangle\langle g_i|-|f_i\rangle\langle f_i|$, $(i=1,2,\ldots)$. And then we consider $2(N-1)$ ions; which is initially prepared in the superposition state $(1/\sqrt{2})(|0\rangle_m+|1\rangle_m)$. Assume the $(2i-1)$th ions perform the same operation as the first ion, and the $(2i)$th ions perform the same operation as the auxiliary ion. We can obtain the state of the system

$$|\psi\rangle=\frac{1}{\sqrt{2^N}}(|0\rangle_m+|1\rangle_m\sigma_{2N-3})(|g_{2N-3}\rangle+|f_{2N-3}\rangle\sigma_{2N-5})\otimes\cdots\otimes(|g_3\rangle+|f_3\rangle\sigma_1)(|g_1\rangle+|f_1\rangle). \tag{14}$$

Now consider the $(2N-1)$th ion-laser interaction with the time $\pi/2\eta\Omega$. With encode $|g_i\rangle\to|0\rangle$ and $|f_i\rangle\to|1\rangle$, the state of the system becomes

$$|\psi\rangle=\frac{1}{\sqrt{2^N}}(|0\rangle+|1\rangle\sigma_1)(|0\rangle+|1\rangle)\otimes\cdots\otimes(|0\rangle+|1\rangle\sigma_{2N-3})(|0\rangle+|1\rangle\sigma_{2N-5}). \tag{15}$$

By this way, we obtain a multipartite entangled state for many trapped ions, i.e., the so-called cluster state [3–6].

We now give a brief discussion on the experimental feasibility of the proposed scheme. The presented scheme is based on the resonant sideband excitation (the first lower sideband), and it requires negligible loss and atomic spontaneous decay during the whole preparation process, detection of ions in given states and controlling interaction time between ion and laser pulse. An experimental realization of the scheme can be straightforward modification of a previous scheme for control and measurement of entangled state [33] in ion trap. In order to realize these conditions, the ion-laser interaction times are at the order of 10^{-5} s. At this time scale, the times needed for the classical field pulse are negligible. We have taken the metastable $D_{5/2}(m_j=-1/2)$ state of $^{40}Ca^+$ ion to be the excited state $|e\rangle$, the lifetime of the excited state $|e\rangle$ is about $1s$, much longer than the required interaction times needed to implement the present scheme 10^{-5} s. Thus the losses due to the spontaneous emission can be neglected. In addition, it is necessary to give a brief discussion on the experimental feasibility of the ion subjected to a classical pulse, which can be easily realized with current experimental technology of Ref. [33]. The difference between our scheme and experiment is that the ion is subjected to a different classical pulse in the Ramsey zone.

IV. IMPLEMENTATION OF TWO-ION PHASE GATE INCLUDING DECAY

In this section the present scheme will be used for the implementation of two-ion phase gate, and then the effect of the atomic spontaneous emission is considered. Finally, we also briefly discuss the idea can be used for the directly transferring information between two ions.

We now consider two two-level ions confined a linear trap. Assume that the first ion is excited by a laser and the other ion is excited by another laser. Both lasers are tuned to the first lower vibrational sideband. According to the Hamiltonian (4), with the choice $\phi_1=\phi_2=\pi/2$ the evolution of the system will be described by the unitary operator

$$U(t)=\exp[-i\eta\Omega_1 t(a^+\sigma_1+a\sigma_1^+)]\exp[-i\eta\Omega_2 t(a^+\sigma_2+a\sigma_2^+)]. \tag{16}$$

In order to realize the two-qubit quantum gate we use a third atomic state $|f\rangle$, which is not affected during the ion-laser interaction [33]. Assume that the quantum information of the control qubit is encoded on the states $|e_1\rangle$ and $|g_1\rangle$, while the quantum information of the controlled qubit is encoded on the states $|g_2\rangle$ and $|f_2\rangle$. If the two ions are initially in the state $|e_1\rangle|f_2\rangle$ and the vibrational mode is initially in the state $|0\rangle_m$, after the time interval,

$$U(t)|e_1\rangle|f_2\rangle|0\rangle_m\to\cos(\eta\Omega_1 t)|e_1\rangle|0\rangle_m|f_2\rangle-i\sin(\eta\Omega_1 t)|g_1\rangle\times|1\rangle_m|f_2\rangle. \tag{17}$$

By choosing $\eta\Omega_1 t=\pi$, we can obtain

$$|e_1\rangle|f_2\rangle|0\rangle_m\to-|e_1\rangle|f_2\rangle|0\rangle_m. \tag{18}$$

Thus the system returns to the initial state with an additional phase shift π.

Under the application of the unitary operator (16), it is easy to prove that the following transformation keeps the state $|g_1\rangle|g_2\rangle|0\rangle_m$ and $|g_1\rangle|f_2\rangle|0\rangle_m$, the system does not undergo any transition since $U(t)|g_1\rangle|g_2\rangle|0\rangle_m=U(t)|g_1\rangle|f_2\rangle|0\rangle_m=0$. Therefore we have

$$|g_1\rangle|g_2\rangle|0\rangle_m\to|g_1\rangle|g_2\rangle|0\rangle_m, \tag{19}$$

$$|g_1\rangle|f_2\rangle|0\rangle_m\to|g_1\rangle|f_2\rangle|0\rangle_m. \tag{20}$$

If the two ions are initially in the state $|e_1\rangle|g_2\rangle$, after the time interval,

$$\begin{aligned}U(t)|e_1\rangle|g_2\rangle|0\rangle_m&\\ \to\frac{\Omega_1}{\sqrt{\Omega_1^2+\Omega_2^2}}&\left[\frac{1}{\sqrt{\Omega_1^2+\Omega_2^2}}\times\left(\Omega_1\cos(\eta\sqrt{\Omega_1^2+\Omega_2^2}t)+\frac{\Omega_2^2}{\Omega_1}\right)|e_1\rangle|g_2\rangle|0\rangle_m\right]\\ &+\frac{\Omega_2}{\sqrt{\Omega_1^2+\Omega_2^2}}[\cos(\eta\sqrt{\Omega_1^2+\Omega_2^2}t)-1]|g_1\rangle|g_2\rangle|0\rangle_m\\ &-i\sin(\eta\sqrt{\Omega_1^2+\Omega_2^2}t)|g_1\rangle|g_2\rangle|0\rangle_m.\end{aligned} \tag{21}$$

By choosing $\eta\sqrt{\Omega_1^2+\Omega_2^2}t=2\pi$, we can obtain

$$|e_1\rangle|g_2\rangle|0\rangle_m\to|e_1\rangle|g_2\rangle|0\rangle_m. \tag{22}$$

Here we have assumed the ratio between the two Rabi frequencies and interaction time appropriately so that $\Omega_2=\sqrt{3}\Omega_1$ and $t=\pi/\eta\Omega_1$ to satisfy $\eta\sqrt{\Omega_1^2+\Omega_2^2}t=2\pi$ and $\eta\Omega_1 t$

$=\pi$. Combining Eqs. (18)–(20) and Eq. (22), we obtain a quantum phase gate for two ions with the vibrational mode left in the state $|0\rangle_m$.

So far we have considered a system-evolution described by unitary operator (16), where only the center-of-mass motion is present in the ion trap, while the coupling of this mode to the surroundings and the atomic spontaneous emission induced by surroundings are negligible. We now consider the effect of the atomic spontaneous emission. Under the condition that no phonon is detected by the spontaneous emission, the unitary operator (16) describing the evolution of the system can be rewritten as

$$U'(t) = U(t)\exp\left(-\frac{\Gamma t}{2}(|e_1\rangle\langle e_1| + |e_2\rangle\langle e_2|)\right), \qquad (23)$$

where Γ characterized the spontaneous emission rate. Based on the unitary operator (23), after the interaction time $t=\pi/\eta\Omega_1$, we can obtain the evolution as follows (see Appendix for the derivation of the following expression):

$$|g_1\rangle|g_2\rangle|0\rangle_m \to |g_1\rangle|g_2\rangle|0\rangle_m,$$

$$|g_1\rangle|f_2\rangle|0\rangle_m \to |g_1\rangle|f_2\rangle|0\rangle_m,$$

$$|e_1\rangle|f_2\rangle|0\rangle_m \to -e^{-\pi/40}|e_1\rangle|f_2\rangle|0\rangle_m,$$

$$|e_1\rangle|g_2\rangle|0\rangle_m \to e^{-\pi/40}|e_1\rangle|g_2\rangle|0\rangle_m. \qquad (24)$$

If the two ions are initially in the state $(1/\sqrt{2})(|g_1\rangle+|e_1\rangle)(|g_1\rangle+|f_2\rangle)$, according to Eqs. (18)–(20) and Eq. (22), we can obtain the entangled state $|\varphi_1\rangle=\frac{1}{2}[|g_1\rangle(|g_2\rangle+|f_2\rangle)+|e_1\rangle(|g_2\rangle-|f_2\rangle)]$, however, considering the atomic spontaneous emission, we can obtain $|\varphi_2\rangle=[1/\sqrt{2(1+e^{-\pi/20})}][|g_1\rangle\times(|g_2\rangle+|f_2\rangle)+e^{-\pi/40}|e_1\rangle(|g_2\rangle-|f_2\rangle)]$. The difference between the two states can be characterized in terms of fidelity $|\langle\varphi_1||\varphi_2\rangle|^2$, in this case the approximate value is 0.99. Therefore, the present scheme is slightly affected by the atomic spontaneous emission. In addition, we can calculate the probability of success as $P=\frac{1}{2}(1+e^{-\pi/20})\simeq 0.921$.

Now we turn to use the idea for the directly transferring quantum information between two ions. If the two ions are initially in the state $|g_2\rangle(\varepsilon|g_1\rangle+|e_2\rangle)$ (ε is an arbitrary parameter) and the vibrational mode is initially in the vacuum state $|0\rangle_m$. According to Eqs. (19) and (21), with different parameters $\Omega_2=\Omega_1$ and $t=\pi/(\eta\sqrt{\Omega_1^2+\Omega_2^2})$ the evolution of the system is given by

$$U(t)[\varepsilon|g_1\rangle|g_2\rangle + |e_1\rangle|g_2\rangle]|0\rangle_m \to [\varepsilon|g_1\rangle|g_2\rangle - |g_1\rangle|e_2\rangle]|0\rangle_m. \qquad (25)$$

Then rotating a single-qubit state $|e_2\rangle$ to $-|e_2\rangle$, we can obtain

$$[\varepsilon|g_1\rangle|g_2\rangle + |e_1\rangle|g_2\rangle]|0\rangle_m \to |g_1\rangle[\varepsilon|g_2\rangle + |e_2\rangle]|0\rangle_m. \qquad (26)$$

By this way the quantum information of the first ion can be transferred to the second ion.

We briefly discuss the experimental feasibility of the proposed scheme. The scheme requires that the two lasers have different Rabi frequencies ($\Omega_2=\sqrt{3}\Omega_1$). The recent experiment of trapped $^{40}Ca^+$ ions [33] have realized the precise different couplings for the two ions in the Lamb-Dicke regime. In addition, unlike the Cirac-Zoller scheme of Ref. [14], which also based on the resonant sideband excitation, uses the vibrational mode as the data bus and requires three interactions, the present scheme does not use the vibrational mode as the data bus and only requires a single resonant interaction. Thus the scheme is very simple and the quantum dynamics operation can be realized at a high speed. In view of the decoherence mechanism, the simplification for the state preparation and experimental implementation of quantum logic operation may become crucial.

A number of schemes [28,34,35] have also been proposed for fast quantum computations in ion trap. The schemes of Refs. [34,35] require a number of laser-ion interactions, while the present scheme only requires a single interaction. The scheme of Ref. [35] requires two distinct harmonic wells and uses internal-state-selective and time dependent pushing force, which is difficult in view of experiment. The scheme of Ref. [28] also requires a single laser resonant interaction, but the scheme uses light shift induced virtual transitions and works in the regime with a comparatively high Rabi frequency (on the order of the vibrational frequency) and a comparatively small Lamb-Dicke parameter (on the order of 10^{-2}), which restricts the speed of the gates and makes it difficult to perform a complex sequence of quantum operations without being significantly affected by decoherence. The techniques required by the present scheme are within the scope of what can be obtained in the ion trap setup of Ref. [33].

V. SUMMARY

In conclusion, in contrast with other systems, the linear ion trap system is more suited for the experimental implementation of the quantum logic operation and quantum state engineering, because the prominent property of this system is that the vibrational mode damping is extremely weak. we have proposed a scheme for creating the so-called cluster state with many trapped ions, which can be used to test quantum nonlocality [36–38] and constitute a universal resource for quantum computation assisted by local measurement only. We also show the scheme can be used to implement the quantum phase gate between two ions (including the atomic spontaneous). The scheme is based on the excitations of the ions by a classical laser field. During each excitation, the laser field is tuned to the respective first lower vibrational sideband. The present scheme has the advantage that it does not use the vibrational mode as the data bus and only requires a single resonant interaction. Thus the scheme is very simple and the quantum dynamics operation can be realized at a high speed. Unlike other schemes [28,34,35] for fast quantum computation, the present scheme only requires a single interaction. The simplification of the procedure and decrease of operation time may be important for suppressing decoherence [39,40]. Finally, we show that the present scheme can be used for directly transferring information between two ions. Based on the currently available techniques [33], the present scheme might be realized soon.

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APPENDIX

In this appendix, we present the detailed derivation of Eq. (22) in the main text. If the two ions are initially in the state $|g_1\rangle|g_2\rangle$ and $|g_1\rangle|f_2\rangle$, the system does not undergo any transition since $H_i|g_1\rangle|g_2\rangle=H_i|g_1\rangle|f_2\rangle=0$. Therefore we have

$$|g_1\rangle|g_2\rangle|0\rangle_m \to |g_1\rangle|g_2\rangle|0\rangle_m, \tag{A1}$$

$$|g_1\rangle|f_2\rangle|0\rangle_m \to |g_1\rangle|f_2\rangle|0\rangle_m. \tag{A2}$$

If the system is initially in the state $|\psi(0)\rangle=|e_1\rangle|f_2\rangle|0\rangle_m$, based on the interaction Hamiltonian (21), we can obtain the evolution as

$$|\psi(t)\rangle = U'(t)|\psi(0)\rangle = e^{-\eta\Omega_1 t/40}\Big\{\Big[\cos(\eta\Omega_1 t) - \tfrac{1}{40}\sin(\eta\Omega_1 t)\Big] \times |e_1\rangle|0\rangle_m|f_2\rangle - i\sin(\eta\Omega_1 t)|g_1\rangle|0\rangle_m|f_2\rangle\Big\}. \tag{A3}$$

As the same, if the system is initially in the state $|\psi(0)\rangle=|e_1\rangle|g_2\rangle|0\rangle_m$, we can obtain

$$\begin{aligned}|\psi(t)\rangle = U'(t)|\psi(0)\rangle = {}& \frac{\Omega_1^2}{\Omega_1^2+\Omega_2^2}e^{-\eta\Omega_1 t/40}\Bigg[\Bigg(\cos(\eta\sqrt{\Omega_1^2+\Omega_2^2}t) \\ & - \frac{\Omega_1}{4\sqrt{\Omega_1^2+\Omega_2^2}}\sin(\eta\sqrt{\Omega_1^2+\Omega_2^2}t) + \Omega_2^2|e_1\rangle|g_2\rangle|0\rangle_m \\ & + \frac{\Omega_1\Omega_2}{\Omega_1^2+\Omega_2^2}\Bigg(e^{-\eta\Omega_1 t/40}\cos(\eta\sqrt{\Omega_1^2+\Omega_2^2}t) \\ & - \frac{\Omega_1}{4\sqrt{\Omega_1^2+\Omega_2^2}}\sin(\eta\sqrt{\Omega_1^2+\Omega_2^2}t) - e^{-\eta\Omega_1 t/40}\Bigg)|g_1\rangle|g_2\rangle \\ & \times|0\rangle_m - i\frac{\Omega_1}{\sqrt{\Omega_1^2+\Omega_2^2}}e^{-\eta\Omega_1 t/40}\sin(\eta\sqrt{\Omega_1^2+\Omega_2^2}t)|g_1\rangle|g_2\rangle \\ & \times|1\rangle_m.\end{aligned} \tag{A4}$$

Here we choose $\Gamma=0.1\eta\Omega_1$, with the choice $\Omega_2=\sqrt{3}\Omega_1$ and $t=\pi/\eta\Omega_1$, we can obtain

$$|e_1\rangle|f_2\rangle|0\rangle_m \to -e^{-\pi/40}|e_1\rangle|f_2\rangle|0\rangle_m,$$

$$|e_1\rangle|g_2\rangle|0\rangle_m \to e^{-\pi/40}|e_1\rangle|g_2\rangle|0\rangle_m. \tag{A5}$$

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