Nonclassical dynamics induced by a quantum meter

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Conventionally, the effect of measurements on a quantum system is assumed to introduce decoherence, which renders the system classical-like. We consider here a microscopic meter, that is, an auxiliary essentially quantum system whose state is measured repeatedly, and show that it can be employed to induce transitions from classical states into inherently quantumlike states. The meter state is assumed to be lost in the environment and we derive a non-Markovian master equation for the dynamic system in the case of nondemolition coupling to the meter; this equation can be cast in the form of an (N_a)th-order differential equation in time, where N_a is the dimension of the meter basis. We apply the approach to a harmonic oscillator coupled to a spin-$\frac{1}{2}$ meter and demonstrate how it can be used to engineer effective Hamiltonian evolution, subject to decoherence induced by the projective meter measurements.

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I. INTRODUCTION

Arisal of the classical world from the underlying quantum domain is often attributed to decoherence, i.e., irrecoverable entanglement with an infinitely large environment. The mathematical description can be formulated in many, often somewhat metaphysical, ways that may include references to classical concepts, such as measurement devices, or meters.

The concept of measurement in quantum mechanics is still essential both for the interpretations and for the applications of the theory. Conventional wave-function quantum mechanics is limited to unitary evolution while measurements can be used to create nonpure quantum states conditional to the measurement result [1]; the conditional preparation of the initial quantum state can even be carried out after the final measurement on the prepared system [2]. Other applications of measurements also include quantum state preparation [3], quantum information processing [4], and coherence protection [5,6].

The measurement devices are themselves also quantum systems but often considered much larger than the dynamic system. Consequently, their interaction with the outer environment occurs on an exponentially shorter time scale than the interaction with the system [7], and the measurement act appears instantaneous. The fact that measurement devices often appear to be classical can be understood in terms of their pointer states [8], which correspond to classical states of a meter. It is the nature of the meter's coupling to the outer environment that determines the einselection, the observable that meter is able to measure if brought in contact with a dynamic quantum system [9]. We consider, in particular, nondemolition measurements, which do not transfer energy between the dynamic system and the measurement device [10–12].

In this paper we wish to demonstrate that measurements can bring in both classical-like and quantumlike evolution of a dynamic quantum system. We consider a harmonic oscillator coupled to a microscopic spin-$\frac{1}{2}$ measurement device that performs repeated, nondemolition measurements on the oscillator and loses the outcome into the environment. Note that the evolution of the dynamic system is not conditional to measurement outcomes but is affected unconditionally by the meter. In the limit of highly frequent measurements the precise effect is, however, unimportant for the discussion that follows.

We assume the meter only to detect the energy eigenstate of the dynamic system and not to transfer energy, and consider therefore nondemolition coupling between the two. The presence of the meter nonetheless affects the coherences of the reduced dynamic system, which modifies the dynamic system evolution. The nondemolition coupling also essentially simplifies the master equation since the matrix elements of the dynamic system decouple and the master equation may be written to each of them separately. The master equation is non-Markovian and can be written as an (N_a)th-order differential equation, where N_a is the dimension of the meter Hilbert space or as a Nakajima-Zwanzig equation with an explicit memory term.

II. EVOLUTION INDUCED BY A NONDEMOLISHING METER

The unitary evolution of the combined system (dynamical system and the meter) is described by

$$\frac{d}{dt}\hat{\varrho} = -\frac{i}{\hbar}[\hat{H}_1 + \hat{H}_{\text{int}} + \hat{H}_2, \hat{\varrho}], \tag{1}$$

where the nondemolition nature of the coupling implies $[\hat{H}_1, \hat{H}_{\text{int}}]=0$ and the subindices 1 and 2 refer to dynamic system and the meter (ancilla), respectively.

The dynamic part is represented using the eigenstates $\hat{H}_1|n\rangle_1 = E_n|n\rangle_1$, for which $\hat{H}_{\text{int}}|n\rangle_1 = \hat{F}_n|n\rangle_1$; note that the quantities $\hat{\varrho}^{(m,n)} = {}_1\langle m|\hat{\varrho}|n\rangle_1$ and $\hat{F}_n$ are operators in the ancilla Hilbert space. Due to nondemolition coupling, matrix elements of the state operator decouple in the dynamical system space, and the unitary evolution over Δt is given componentwise by

$$\hat{\varrho}^{(m,n)}(t+\Delta t) = \hat{U}^{m}_{\Delta t}\hat{\varrho}^{(m,n)}(t)\hat{U}^{n\dagger}_{\Delta t}, \tag{2}$$

where $\hat{U}^{m}_{\Delta t}=e^{-(i/\hbar)(E_m+\hat{F}_m+\hat{H}_2)\Delta t}$.

We choose a measurement scheme where the microscopic meter measures the system state repeatedly with the period Δt and the outcome is lost in the environment of the meter; the measurement projection is therefore given by

$$\mathcal{M}\{\hat{\varrho}\} = \sum_k |\Psi_k\rangle_2\langle\Psi_k|\hat{\varrho}|\Psi_k\rangle_2\langle\Psi_k|. \tag{3}$$

We assume a stroboscopic picture where the quantum state is only observed after each measurement; consequently the off-diagonal elements (in the meter basis) are assumed to stay zero, $\hat{\varrho}^{(m,n)}_{kl}=0$ for $k\neq l$.

The nonunitary evolution over one measurement cycle is now determined by the discrete master equation

$$\varrho^{(m,n)}_{kk}(t+\Delta t) = \sum_l \mathcal{U}^{(m,n)}_{\Delta t,kl}\varrho^{(m,n)}_{ll}(t), \tag{4}$$

where the evolution is defined by the matrix

$$\mathcal{U}^{(m,n)}_{\Delta t,kl} = U^{m}_{\Delta t,kl}U^{n*}_{\Delta t,kl}. \tag{5}$$

For the purpose of deriving a (quasi-)continuous master equation, we define the evolution generator

$$\mathcal{L}^{(m,n)}_{\Delta t,kl} = \frac{1}{\Delta t}\ln(\mathcal{U}^{(m,n)}_{\Delta t,kl}) \tag{6}$$

that yields the master equation

$$\frac{d}{dt}\hat{\varrho}^{(m,n)}_{kk} = \sum_l \mathcal{L}^{(m,n)}_{\Delta t,kl}\hat{\varrho}^{(m,n)}_{ll}; \tag{7}$$

this equation interpolates the discrete evolution of Eq. (4) and also allows one to obtain the continuous measurement limit for $\Delta t\to 0$. The master equation only applies for the diagonal elements in the meter space since the projective measurements instantaneously reset the off-diagonal elements to zero, which process cannot be generated by any differential evolution.

III. TWO-STATE METER

We examine now the special case of a two-state meter although the approach is generalizable to arbitrary finite dimensions. In order to describe the time evolution of the dynamic system alone, we need to find an evolution equation for $\rho=\mathrm{Tr}_2[\hat{\varrho}]=\varrho_{00}(t)+\varrho_{11}(t)$ [we now omit the superscript (m,n); all matrix elements are separately evaluated for each pair (m,n)]. While ρ describes the matrix element of the system state, an auxiliary variable $\eta=\varrho_{00}(t)-\varrho_{11}(t)$ is included to cover the meter state and the entanglement between the system and the meter. In this basis, the equation of motion reads

$$\frac{d}{dt}\begin{bmatrix}\rho\\ \eta\end{bmatrix} = \mathbf{B}\begin{bmatrix}\rho\\ \eta\end{bmatrix}, \tag{8}$$

where

$$\mathbf{B} = \begin{bmatrix}B_{00} & B_{01}\\ B_{10} & B_{11}\end{bmatrix} = \begin{bmatrix}\dfrac{\mathcal{L}_{00}+\mathcal{L}_{10}+\mathcal{L}_{01}+\mathcal{L}_{11}}{2} & \dfrac{\mathcal{L}_{00}+\mathcal{L}_{10}-\mathcal{L}_{01}-\mathcal{L}_{11}}{2}\\ \dfrac{\mathcal{L}_{00}-\mathcal{L}_{10}+\mathcal{L}_{01}-\mathcal{L}_{11}}{2} & \dfrac{\mathcal{L}_{00}-\mathcal{L}_{10}-\mathcal{L}_{01}+\mathcal{L}_{11}}{2}\end{bmatrix} \tag{9}$$

is the time-evolution generator (i.e., Liouvillian) in the basis chosen above.

We present here two different methods for finding an equation of motion for ρ alone. Elimination of η from Eq. (8) yields a second-order differential equation in time that can be written in the form

$$\mathrm{Det}\left(\mathbf{B}-\mathbf{I}\frac{d}{dt}\right)\rho = 0, \tag{10}$$

where $\mathbf{I}$ denotes the identity matrix. Note that the order of the differential equation is directly determined by the dimension of the meter. For a two-level meter the equation for ρ reads

$$\frac{d^2}{dt^2}\rho = (B_{00}+B_{11})\frac{d}{dt}\rho + (B_{01}B_{10}-B_{11}B_{00})\rho. \tag{11}$$

The initial condition still depends on the initial ancilla state since

$$\left.\frac{d}{dt}\rho(t)\right|_{t=0} = B_{00}\rho(0) + B_{01}\eta(0); \tag{12}$$

hence the time evolution has a memory.

An equivalent formulation can be given in the form of a Nakajima-Zwanzig equation with memory, given by

$$\frac{d}{dt}\rho(t) = B_{00}\rho(t) + \int_0^t K(t-s)\rho(s)ds + B_{01}e^{B_{11}t}\eta(0), \tag{13}$$

where the memory kernel is given by

$$K(t-s) = B_{01}e^{B_{11}(t-s)}B_{10}; \tag{14}$$

see, e.g., Ref. [20]. The influence of the initial spin state is carried out by the last term but it decays exponentially, since the real part of B_{11} is expected to be negative.

This equation can also be used for obtaining the master equation in the Markovian limit. In that case, the exponential decays faster than the state ρ evolves; $\rho(s)$ may therefore be replaced with $\rho(t)$ and taken out of the integral. Assuming still $\mathrm{Re}(B_{11})<0$ the integral may be performed explicitly and the master equation is

$$\frac{d}{dt}\rho(t) = \left(B_{00} - B_{01}\frac{1}{B_{11}}B_{10}\right)\rho(t). \tag{15}$$

HARMONIC OSCILLATOR MEASURED BY A SPIN-1/2 METER

We consider a special case in which the meter-induced evolution can transform the classical-like coherent state into quantumlike superposition (Schrödinger "cat") state [13] via generation of an artificial Kerr interaction. The possibility of using Kerr effect to create nonclassical states has been considered theoretically in optical fibers and electrical circuits [14–16]. Since the Kerr coefficient is typically many orders of magnitude smaller than unity, an accumulation over large optical path lengths is required and decoherence has so far prevented the experimental observation of a Schrödinger cat-like state in an optical fiber. It has however been successfully prepared and its decoherence observed for a mesoscopic cavity field [17–19].

We now apply the above general scheme to a harmonic oscillator coupled to a spin-$\frac{1}{2}$ meter by means of a time-independent interaction. In the absence of measurements, we assume the Hamiltonian

$$\hat{H} = \hbar\omega\left(\hat{n} + \tfrac{1}{2}\right) + \hbar\beta\hat{\sigma}_z + \alpha m\omega\hat{x}^2\hat{\sigma}_{\mathbf{q}}, \tag{16}$$

where $\hat{n}=\hat{a}^\dagger\hat{a}$ and $\hat{x}=\sqrt{\hbar/(2m\omega)}(\hat{a}+\hat{a}^\dagger)$ are the number and the position operator for the oscillator, whereas $\hat{\sigma}_z$ and $\hat{\sigma}_{\mathbf{q}}=\mathbf{q}_x\hat{\sigma}_x+\mathbf{q}_y\hat{\sigma}_y+\mathbf{q}_z\hat{\sigma}_z$ are the respective Pauli (spin) matrices. z is the quantization axis of the meter and $\mathbf{q}$ is called the interaction axis. This model applies, for example, to a spin-$\frac{1}{2}$ particle confined to move in a one-dimensional harmonic potential, subject to a spatially varying magnetic field or a two-level atomic system interacting with a single mode of a cavity field. It is of interest here as example of an interaction between a discrete- and a continuous-variable system.

This Hamiltonian can be transformed into an effective nondemolition Hamiltonian provided that the system is far from resonance. For the case in which the interaction axis is orthogonal to quantization axis, second-order perturbation yields the effective Hamiltonian

$$\hat{H}_{\text{eff}} = \hbar[a\hat{n} + b\hat{\sigma}_z + c\hat{n}(\hat{n}+1)\hat{\sigma}_z], \tag{17}$$

where a, b, and c are real constants, see Ref. [21]. The unitary evolution between measurements is represented with

$$U^m_{\Delta t,kl} = e^{-iam\Delta t}(\delta_{kl}\cos\zeta_m - i\gamma_{kl}\sin\zeta_m), \tag{18}$$

where $\zeta_m=[b+cm(m+1)]\Delta t$ and $\gamma_{kl}={}_2\langle\Psi_k|\hat{\sigma}_z|\Psi_l\rangle_2$ are the matrix elements of $\hat{\sigma}_z$ in the meter basis,

$$\gamma = \begin{bmatrix} \cos\theta & -\sin\theta \\ -\sin\theta & -\cos\theta \end{bmatrix}. \tag{19}$$

Here, θ is the polar angle of the measurement axis relative to the quantization axis.

The dynamics simplifies essentially if the measurement axis is either parallel with or perpendicular to the quantization axis. For the parallel case $\gamma_{kl}=\pm\delta_{kl}$, and the different k and l states decouple. If, on the other hand, the meter axis is orthogonal to the quantization axis, $\gamma_{kk}=0$. Therefore, the measurement axis should be chosen neither parallel with nor orthogonal to the quantization axis in order to obtain a nontrivial contribution to the dynamic system.

We choose system parameters such that $b=0$ in the Hamiltonian (17) and assume the parameter c to be sufficiently large such that the measurement axis can be chosen to satisfy $\gamma_{00}=\cos\theta=-c^{-1}a$. Then, in the limit of continuous measurement, $\Delta t\to 0$, with the meter state $|\Psi_0\rangle$, the reduced dynamic system evolves according to the Hamiltonian $\langle\Psi_0|\hat{H}_{\text{eff}}|\Psi_0\rangle=-\hbar a\hat{n}^2$. This is similar to Kerr nonlinearity and can be used to generate Schrödinger cat superposition states, in line with proposals based on an optical single-mode Kerr nonlinearity [14–16].

We assume a coherent initial state $|\beta\rangle$ and evaluate it only at discrete times $t_m=\pi/(2|c\cos\theta|)m$. Under the Hamiltonian quadratic in $\hat{n}$, the evolution is cyclic (via complicated intermediate states) according to

$$|\beta\rangle \to \frac{1+i}{2}|\beta\rangle + \frac{1-i}{2}|-\beta\rangle \to |-\beta\rangle \to \frac{1-i}{2}|\beta\rangle + \frac{1+i}{2}|-\beta\rangle \to |\beta\rangle, \tag{20}$$

from now on we choose $m=4k+1$ and hence consider the first superposition state of each cycle. The evolution period is given by $T=2\pi/|c\cos\theta|$.

With finite measurement intervals the evolution deviates from that presented above, and is subject to decoherence. Consequently, the state departs from the superposition as k increases. We divide the evolution period into N measurement cycles, such that $\Delta t=T/N$. The eventual steady state is obtained directly from Eq. (4). For $m\neq n$ (and the measurement angle $\theta\neq 0$) the eigenvalues of the time-step operator are smaller than one but not equal to zero, and the steady state value is given by $\varrho^{(m,n)}_{k,l}(\infty)=0$. The diagonal elements of the oscillator state stay constant whereas the coherences tend to zero, and the initial coherent state $|\beta\rangle$ evolves into a Poissonian mixture of Fock states. We note that the rate of coherence loss increases exponentially in the coherent amplitude.

In Fig. 1 we illustrate the evolution of initial state $|\beta=3\rangle$ both over one unitary evolution cycle with Kerr-type Hamiltonian, and simulated nonunitary evolution subject to meter with measurement period $\Delta t=10^{-6}T$. In the ideal case of continuous measurements, one evolution cycle follows Eq. (20), illustrated in the upper row. For a finite measurement interval Δt, decoherence appears and the decohering evolution of the Schrödinger cat state is shown over several cycles until the final steady state is nearly reached. Note again that this is stroboscopic picture and the actual evolution within each cycle roughly follows that shown in the upper row.

V. CONCLUSIONS

In this paper we have considered the time evolution of a dynamic quantum system coupled via nondemolition interaction to a microscopic meter, represented with a repeatedly measured ancilla. Given the time lapse Δt between two subsequent measurements, the combined system may be described using a difference master equation whereas, in the

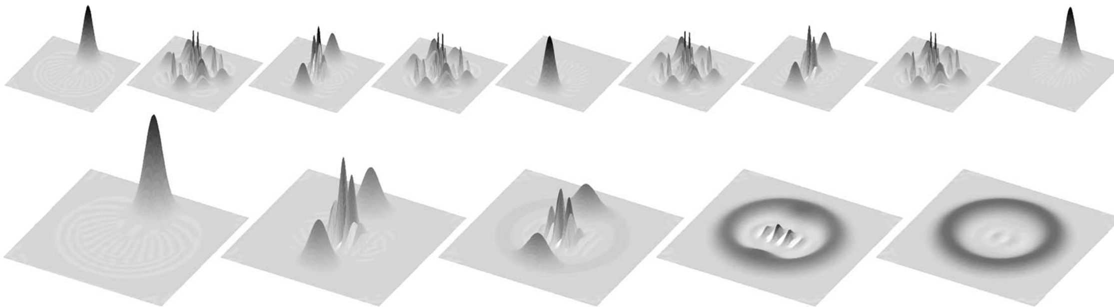

FIG. 1. Upper row, unitary evolution cycle of an initial coherent state under Kerr-type Hamiltonian, cf. Eq. (20). Lower row, generation and decoherence of a Schrödinger cat state for finite measurement intervals. Duration of one evolution cycle is $T=2\pi$ and the meter state has been measured with the time step $\Delta t=10^{-6}T$. Pictures correspond to the initial state and to the Schrödinger cat states after 1, 10, 10^2, and 10^3 evolution cycles. The initial coherent amplitude is $\beta=3$ and the other parameters are $\theta=37°$, $a=1$, $b=0$, and $c=-a/\cos\theta$. The states are represented with the help of the Wigner functions.

continuous measurement limit $\Delta t\rightarrow 0$, the evolution of the dynamic system is unitary and defined by the state of the ancilla. For an arbitrary Δt, we also formulated a master equation that interpolates smoothly the exact evolution given by the difference equation.

The master equation of the dynamic system can be represented independently for each element of the density operator and each obeys a non-Markovian master equation. We formulate these equations as (N_a)th-order differential equations in time, where N_a is the dimension of the ancilla Hilbert space or, equivalently, as Zwanzig equations with an explicit memory over the system evolution.

The meter may represent the internal degrees of freedom of a particle [22], which themselves can be measured by the environment. For a rising number of meter levels the quantumlike evolution of the dynamic system is increasingly difficult to maintain until eventually for a very large meter the classical behavior of the dynamic system is observed.

The above approach has been applied to a harmonic oscillator coupled to a two-level system, that serves as the meter. In the limit of continuous measurements, suitably chosen physical parameters lead to an effective meter-induced Kerr interaction, but finite measurement intervals are shown to eventually destroy the coherence. This is demonstrated by a periodic generation of a Schrödinger cat state. The decoherence process is highly nonlinear in the initial state amplitude and the decoherence time decreases rapidly for increasing amplitude.

We conclude that the presence of a microscopic quantum meter can introduce evolution that transforms classical-like states into quantumlike states and vice versa. Although the projective measurements performed on the meter eventually lead to loss of quantum coherence, close to the limit of continuous measurements the effect is clearly quantum and not classical.

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