Muon transfer from muonic hydrogen to carbon

Arnaud Dupays
Laboratoire Collisions, Agrégats, Réactivité, IRSAMC, Université Paul Sabatier, 31062 Toulouse, France
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Exact three-dimensional quantum calculations of muon exchange between muonic hydrogen and carbon for collision energies in the range $10^{-3}-100$ eV, are presented. Muon transfer rates at thermal and epithermal energies are calculated including partial waves up to $J=7$. The relative populations of the final states are also given. The results show that above 1 eV, the relative population of $(\mu C)_{n=5}^{5+}$ can reach up to 30%.

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A significant advance has been achieved in recent years in the field of theoretical treatment of muon transfer processes from muonic hydrogen to elements with higher atomic charges X^{Z+} [1–4]:

$$p\mu + X^{Z+} \rightarrow p + (\mu X)^{(Z-1)+}. \quad (1)$$

In previous work, we have presented the first high-accuracy three-dimensional quantum calculations of muon exchange from muonic hydrogen to nitrogen, oxygen, and neon [2,3]. These calculations were performed using a time-dependent close-coupling method and hyperspherical elliptic coordinates [5] extended to nonzero total angular momentum [2]. Very good agreement with available measured muon transfer rates for the three systems has been obtained. Our calculations also reproduce perfectly the isotopic effects (when muonic protium is replaced by muonic deuterium) and confirm the reliability of our theoretical method to study muon transfer process. Moreover, in a recent work [4], Le and Lin, performing independent diabatic hyperspherical close-coupling calculations, have obtained very good agreement with our results for muon transfer from muonic hydrogen to oxygen and nitrogen.

Beyond the challenge to perform accurate calculations of reaction (1) with high value of Z, our study of muon transfer process has been motivated by a proposal to measure the hyperfine structure of the ground state of muonic hydrogen based on the collision energy dependence of the muon transfer to oxygen. The basic idea is the following: when muonic hydrogen in the $F=0$ hyperfine ground state is laser excited to the $F=1$ level, collisions with H_2 convert this excess energy into relative kinetic energy, giving an additional 0.12 eV translational energy to the muonic hydrogen. When the muon is transfered to an oxygen atom, it is captured predominantly in high $(n=5,6)$ Rydberg states, and the prompt de-excitation that follows is accompanied by the emission of characteristic x-rays. If the muon transfer rate to oxygen changes significantly from thermal (0.04 eV) to epithermal (0.16 eV) energies, the measurement of the x-ray emission intensity with and without laser excitation can be used for the determination of the hyperfine splitting in muonic hydrogen [6,7]. This dependence of the muon transfer rate on collision energy has been the subject of both experimental [8] and theoretical [2,9–12] studies. Up to now, theoretical results have not confirmed the expected increase of the transfer rate with energy for oxygen. However, our calculations on neon suggest to use this atom instead of oxygen to perform this measurement. Following this analysis, it appears necessary to study the energy dependence of the muon transfer process to other atoms in order to determine the most appropriate candidate for such an experiment.

In this paper, we present exact three-dimensional quantum calculations of muon exchange between muonic hydrogen and carbon $(Z=6)$ for collision energies in the range $10^{-3}-100$ eV. Muon transfer to carbon is not only important in the framework of the measurement of the hyperfine splitting in muonic hydrogen, but is of general interest because in standard experiments involving muonic atoms, walls of the experimental chamber are made of carbon. Thus, the muon transfer can also occur with the carbon of walls, complicating experimental data analysis. In particular, it appears important to evaluate the final state populations to make a prediction on the carbon x-ray emission spectrum. For example, this applies to the experiment on measuring the Lamb shift $2s$-$2p$ splitting in muonic hydrogen now in progress at the Paul Scherrer Institute (PSI, Switzerland) [13,14].

We have performed calculations of the reaction

$$(p\mu)_{n=1} + C^{6+} \rightarrow p + (\mu C)_{n,\ell}^{5+}. \quad (2)$$

The calculation were performed as follows. Hyperspherical elliptic coordinates [5] have been used. A piecewise diabatic basis set on the hyperspherical angles was used to expand the wave function. The resulting close-coupling time-independent Schrödinger equations in the hyper-radius were solved using a de Vogelaere algorithm, and the partial and total muon-transfer probabilities were determined by the standard S-matrix analysis at large distances. For each total angular momentum, we calculate the transition probabilities $P_J(E)$ and obtain the muon transfer cross section as

$$\sigma(E) = \frac{\pi\hbar^2}{2m_{C,p\mu}E}\sum_J (2J+1)P_J(E), \quad (3)$$

where $m_{C,p\mu}=m_C(m_p+m_\mu)/(m_C+m_p+m_\mu)$.

The muon transfer rate is then given by

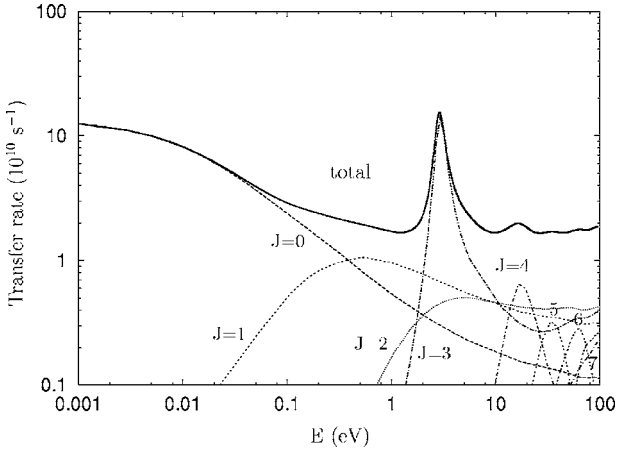

FIG. 1. Partial and total transfer rates versus collision energy for carbon.

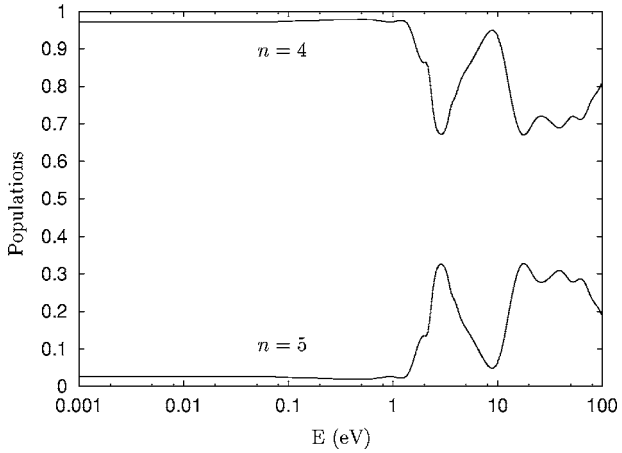

FIG. 2. Relative populations of $(\mu C)^{5+}_{n=4}$ and $(\mu C)^{5+}_{n=5}$ final states versus collision energy for carbon.

$$\lambda(E) = N\left(\frac{2E}{m_{C,p\mu}}\right)^{1/2}\sigma(E), \tag{4}$$

N being the number density of liquid hydrogen (4.25×10^{22} cm^{-3}). We can also define a temperature-dependent muon transfer rate as

$$\lambda(T) = \int_0^{+\infty} \lambda(E)n(E;T)dE, \tag{5}$$

where $n(E;T)$ is the kinetic energy distribution of the $(p\mu)_{1s}$ at temperature T. This rate can be directly compared with experimental results obtained with thermalized muonic hydrogen.

Figure 1 presents partial and total transfer rates for collision energies in the range $10^{-3}-100$ eV. For the energy range considered here, we used $N_{ch}=94$ coupled diabatic channels $[(p\mu)_{n=1-2}+C$ and $p+(\mu C)_{n'=1-13}]$. By comparison with a larger computation obtained with a larger number of channels ($N_{ch}=209$), we estimate that we achieve a convergence of the transfer probabilities better than 0.1%. The partial waves up to $J=7$ have to be included to converge the transfer rate up to 100 eV. The broad maximum near 5 eV is due to an intense resonance associated to $J=3$ partial wave. Using Eq. (5) we find a muon transfer rate at thermal energy $\lambda_{th}=5.5\times10^{10}$ s^{-1}. The agreement with measured rates is not clear because of the conflicting available experimental results: $(5.1\pm1.0)\times10^{10}$ s^{-1} [15] and $(9.5\pm0.5)\times10^{10}$ s^{-1} [16]. At epithermal energy, we obtain $\lambda_e=2.1\times10^{10}$ s^{-1}. Thus, the transfer rate decreases by a factor of 2.6 from thermal to epithermal energies. This result must be compared with the previous variations obtained with others atoms: a decrease by a factor of 1.4 (2) for nitrogen (oxygen) and an increase by a factor of 5 for neon. These results confirm that neon remains the most appropriate element to provide the greatest change of muon transfer rate with energy from thermal to epithermal energy.

We have also determined the relative population of final states versus collision energy. For each final state $(\mu C)^{5+}_n$ we can define a partial cross section as

$$\sigma_n(E) = \frac{\pi\hbar^2}{2m_{C,p\mu}E}\sum_J (2J+1)P^J_n(E). \tag{6}$$

The relative population of the final state is thus given by

$$p_n = \frac{\sigma_n(E)}{\sigma(E)}. \tag{7}$$

Figure 2 shows the relative populations of $(\mu C)^{5+}_{n=4}$ and $(\mu C)^{5+}_{n=5}$ final state. Below 1 eV, only $(\mu C)^{5+}_{n=4}$ is populated, but above this energy the relative population of $(\mu C)^{5+}_{n=5}$ can reach up to 30%. This result suggests that x-ray emission from de-excitation of $(\mu C)^{5+}_{n=5}$ must be considered when carbon walls are used in an experiment.

In conclusion, we have performed accuracy calculations of the muon transfer rate from muonic hydrogen to carbon using a hyperspherical elliptic close-coupling method. Concerning the energy dependence of muon transfer rate from thermal to epithermal energy, comparisons with previous results obtained with nitrogen, oxygen, and neon confirm that neon remains the most appropriate element, at this time, to perform a measurement of the hyperfine structure of muonic hydrogen. Moreover, we have shown that above 1 eV, the relative population of $(\mu C)^{5+}_{n=5}$ final state can reach up to 30%, and, therefore, de-excitation lines from $(\mu C)^{5+}_{n=5}$ should be present in x-ray spectra in standard muonic atoms experiments using carbon walls.

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