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Collective excitation of Bose-Einstein condensates in the transition region between three and one dimensions

M. Kottke,[1] T. Schulte,[1,*] L. Cacciapuoti,[2] D. Hellweg,[1] S. Drenkelforth,[1] W. Ertmer,[1] and J. J. Arlt[1]
[1]*Institut für Quantenoptik, Universität Hannover, Welfengarten 1, 30167 Hannover, Germany*
[2]*ESA Research and Scientific Support Department, ESTEC, Keplerlaan 1 - P.O. Box 299, 2200 AG Noordwijk ZH, The Netherlands*
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We measure the frequency of the low $m=0$ quadrupolar excitation mode of weakly interacting Bose-Einstein condensates in the transition region from the three-dimensional (3D) to the 1D mean-field regime. Various effects shifting the frequency of the mode are discussed. In particular we take the dynamic coupling of the condensate with the thermal component at finite temperature into account using a time-dependent Hartree-Fock-Bogoliubov treatment developed by Giorgini [Phys. Rev. A, **61**, 063615 (2000)]. We show that the frequency rises in the transition from 3D to 1D, in good agreement with the theoretical prediction of Menotti and Stringari [Phys. Rev. A **66**, 043610 (2002)].

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One-dimensional (1D) quantum degenerate Bose gases have recently attracted considerable theoretical and experimental interest [1]. On the one hand, this interest is due to their remarkable physical properties which are absent in three-dimensional systems. On the other hand, a rapid advance in trapping techniques for ultracold gases has put these systems within experimental reach. In particular optical lattices [2], optical dipole traps [3], and atom chips [4] have recently been used to realize such low-dimensional systems. For further experiments under these conditions a good understanding of the transition between the 3D and the 1D regime is therefore of crucial importance. In this paper this transition region is characterized experimentally by monitoring the oscillation of a quantum degenerate Bose gas.

From a fundamental point of view, one of the most striking features of 1D quantum degenerate Bose gases is the predominant role played by quantum fluctuations. For trapped 1D gases a rich variety of many particle states can exist, including true phase-coherent Bose-Einstein condensates (BECs) as well as phase-fluctuating BECs, the so-called quasicondensates [5]. The behavior of these 1D Bose gases is governed by the ratio of interaction and kinetic energy $\gamma = m\, g_{1D}/\hbar^2 n_{1D}$, where n_{1D} is the 1D atomic density, m the atomic mass, and g_{1D} the 1D coupling constant. Since this ratio scales as $1/n_{1D}$ these gases counter-intuitively become more non-ideal when the density is decreased. The fascinating features of these systems have led to continued theoretical interest over the past decades. For homogeneous 1D Bose gases with short-range interactions, the ground state [6], excitation spectrum [7] and thermodynamic properties [8] of the system can be determined with a Bethe ansatz for arbitrary values of γ. In the case of trapped systems the equation of state can be found by combining this approach with the local density approximation [9]. For high densities the system is in the weakly interacting regime, where it can be well described in the frame of mean-field theories. On the contrary, for low densities the system enters the strongly interacting or strongly correlated regime, where a description by mean-field theories fails. In the limit $\gamma \to \infty$ the system is equivalent to a 1D gas of impenetrable bosons, the so-called Tonks-Girardeau (TG) gas [10], where the bosonic particles effectively acquire fermionic properties.

To experimentally prepare a 1D gas in a harmonic trap with cylindrical symmetry one has to fulfill the condition

$$\mu, k_B T \ll \hbar\, \omega_\perp, \tag{1}$$

where $\omega_\perp$ denotes the radial trap frequency and μ is the chemical potential of the ensemble. If this condition is fulfilled, neither the thermal nor the interaction energy is sufficient to affect the radial shape of the ground-state wave function. The radial degree of freedom is then frozen, since the atoms are confined to the ground state of the radial trapping potential.

The first experimental realization of a weakly interacting 1D BEC was achieved in a magnetic trap with very high aspect ratio and detected by a change in the ballistic expansion [11,12]. The tight radial confinement required to fulfill Eq. (1) can also be provided by magnetic microtraps [13] or optical wave guides [14]. Such waveguides have allowed for the first realization of stable matter-wave bright solitons [15]. Two-dimensional optical lattices [16] offer the possibility to realize arrays of 1D gases which are coupled via tunneling between the lattice sites. In such a lattice gas a theoretically predicted reduction of the three-body recombination rate within the strongly correlated regime [17] has been experimentally observed [18]. Recently, seminal experiments have reached the TG regime in 2D optical lattices [19,20].

In this paper, we experimentally study collective excitations of weakly interacting BECs in the transition region between 3D and 1D, i.e., for values of the chemical potential $\mu \sim \hbar\, \omega_\perp$. The experimental investigation of collective excitations allows for quantitative tests of the underlying theoretical description. We focus on the observation of the low $m=0$ quadrupolar mode. Collective modes were investigated experimentally and theoretically in 3D BECs [21–27] and in arrays of 1D gases confined in a 2D optical lattice [28,29].

*Electronic address: schulte@iqo.uni-hannover.de

Our experimental investigation in the transition region reveals clear signatures of one-dimensional behavior and is complementary to experiments on the phase-coherence properties [30] of the system.

The dynamics of the low $m=0$ mode consists of an out-of-phase oscillation of the axial and radial diameters of the cloud. For Bose gases confined in cylindrically symmetric traps the hydrodynamic equations of superfluids [31] allow for the analytic determination of the mode frequency at $T=0$: For the case of a very elongated but 3D weakly interacting BEC the frequency is $\omega=\sqrt{5/2}\omega_z$ [32], where ω_z is the axial trap frequency. In the transition from the 3D mean-field regime to the 1D mean-field regime, the radial dynamics of the condensate freezes out, and accordingly the radial component of the low $m=0$ mode vanishes. In this process, the mode frequency rises to $\omega=\sqrt{3}\omega_z$ [32,33]. For the TG gas the frequency increases to $\omega=2\,\omega_z$ [34], identical to a noninteracting Fermi gas. The frequency in the intermediate regimes has been numerically determined by using a combination of hydrodynamic equations with a sum-rule approach [35] and later by exclusively relying on hydrodynamic equations [36]. Note that the analytic solutions given above depend on the axial trap frequency only, whereas the mode frequency in the intermediate regimes depends on the radial confinement as well [35].

Our experiments were performed with ^{87}Rb Bose-Einstein condensates in the $|F=1,m_F=-1\rangle$ hyperfine ground state confined in a strongly elongated magnetic trap. The axial trap frequency was 3.40 Hz and the radial trap frequency was varied between 265 and 385 Hz, resulting in aspect ratios as low as $\lambda=\omega_z/\omega_\perp\approx 1/113$. Further details of our experimental apparatus can be found elsewhere [37]. In such a trap with fixed geometry, the 1D criterion (1) reduces to a condition on the number of atoms in the sample. Neglecting the contribution of thermal atoms to the chemical potential, the gas is 1D if the number of condensed atoms N_0 is smaller than [11]

$$N_0^{1D}=\sqrt{\frac{32}{225}\frac{a_z^2}{a\,a_\perp}}, \tag{2}$$

where $a_\perp$ and a_z denote the harmonic oscillator length in the radial and axial directions and a is the s-wave scattering length. For our trap parameters $N_0^{1D}\approx 4000$. To approach this regime we reduce the number of atoms by adjusting the end of the forced evaporation ramp. We are restricted by our detection system to a minimum number of $N\approx 2000$ atoms. For this minimum number of atoms $\mu/\hbar\omega_\perp\approx 0.75$, thus we obtain ensembles on the border of one dimensionality.

We performed the following experimental procedure: Laser-cooled atoms were loaded into a moderately elongated magnetic trap with $\lambda\approx 1/25$. Radio-frequency evaporative cooling was performed to obtain temperatures just above the transition temperature. Then the trap was axially decompressed to reach the desired aspect ratio, and the final evaporation ramp to obtain BEC was performed in the strongly elongated trap. The low $m=0$ mode was excited by modulating the current of the magnetic trap for five periods at a frequency close to the expected value of the mode frequency ω. The condensate was allowed to oscillate in the magnetic trap for a hold time τ. Subsequently, the trapping potential was switched off and the atomic density distribution was detected by resonant absorption imaging after 30 ms time of flight. A bimodal fit to the density distribution was used to extract the aspect ratio, the total and condensed particle numbers and the temperature of the ensemble. By varying the hold time τ in the magnetic trap the oscillation was stroboscopically monitored as shown in Fig. 1. The axial component of the oscillation is clearly visible in these images. However, the amplitude of the radial oscillation is very small in the dimensional transition region. In addition these images show axial density modulations, which are related to the presence of phase fluctuations in the condensate. These phase fluctuations are caused by the finite temperature and reduced dimensionality of the system [30]. However, for the parameter regime under investigation, density modulations in the magnetic trap are suppressed by interactions [38] and hence the phase fluctuations do not directly affect the frequency of oscillation.

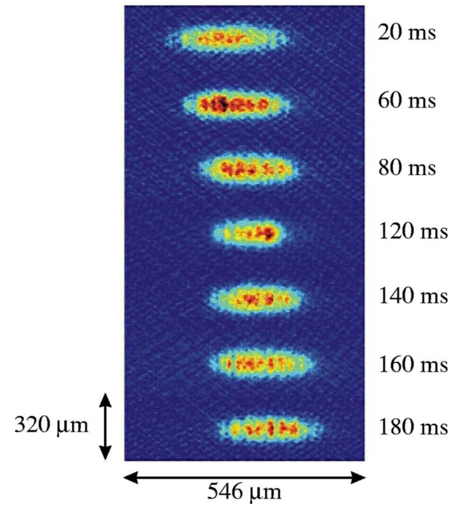

FIG. 1. (Color online) Absorption images after 30 ms time of flight for various hold times τ in the magnetic trap (indicated on the right). The axial component of the oscillation is clearly visible. The axial displacement of the cloud is due to the simultaneous excitation of the dipole mode. Axial density modulations observable in the absorption images are related to the presence of phase fluctuations in the condensate and develop during time of flight [30]. We have vertically compressed the images for graphical representation.

Since the mode frequency depends on the dimensionality of the system, the data was sorted according to the parameter

$$P=N_0\lambda a/a_\perp, \tag{3}$$

which appropriately describes the degree of dimensionality for our purposes [35]. For $P\ll 1$ the cloud enters the $1D$ regime, whereas large values $P\gg 1$ correspond to a $3D$ cloud. Figure 2 shows the aspect ratio of clouds within an interval of the parameter P versus the hold time τ. To extract the frequency we fit this data with a damped sinusoidal function.

The frequency rise which accompanies the transition from the 3D to the 1D mean-field regime constitutes a shift of only 10%. Due to the small size of this effect other influences on

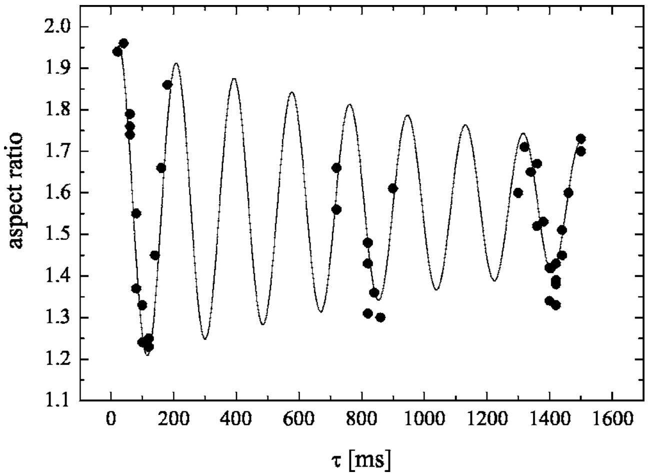

FIG. 2. Aspect ratio of the clouds after time of flight as a function of the hold time in the magnetic trap. The fit to the data gives a mode frequency of $\omega = 5.41 \pm 0.02$ Hz. The data points originate from clouds with an average dimensionality parameter $P = 13.3$ and reduced temperature $T/T_c = 0.34$.

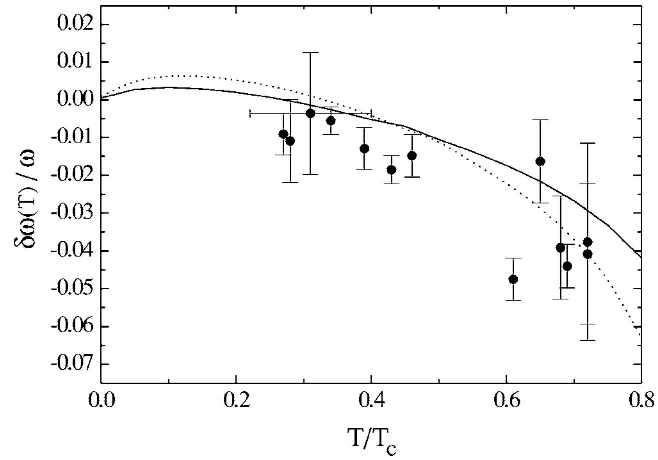

FIG. 3. Normalized frequency difference between our measurements and the $T=0$ prediction in Ref. [35] as a function of the reduced temperature. The values of P for the data points shown lie in the dimensional transition region, see Fig. 4. The lines indicate the theoretical frequency corrections for an interaction parameter of $\eta = 0.33$ (solid) and $\eta = 0.41$ (dotted) according to Ref. [44].

the oscillation frequency have to be considered carefully. In principle, frequency shifts can be caused by effects beyond the mean-field approximation, by large oscillation amplitudes and by the finite temperature of the system.

The frequency shift due to corrections to the mean-field approximation can be analyzed in the frame of the hydrodynamic theory of superfluids. Using the first quantum correction to the Bogoliubov equation of state, this shift of the lowest quadrupolar mode at $T=0$ is [39]

$$\frac{\delta\omega}{\omega} = \frac{63\sqrt{\pi}}{128}\sqrt{a^3 n(0)} f(\lambda), \tag{4}$$

where $f(\lambda)$ is a function that depends on the geometry of the trap [see Eq. (17) in Ref. [39]]. Since the gas parameter $a^3 n(0)$ is very small for our experimental conditions, this frequency shift is below the 0.1% level and thus negligibly small.

The hydrodynamic equations are also well suited to investigate the effect of large oscillation amplitudes at $T=0$. Large amplitudes cause nonlinear coupling between the normal modes of the system and shifts of the oscillation frequency. The frequency shift for the low $m=0$ quadrupolar mode is [40]

$$\frac{\delta\omega(A_z)}{\omega} = \frac{\delta(\lambda) A_z^2}{16}, \tag{5}$$

where A_z is the relative oscillation amplitude of the condensate length in the trap and $\delta(\lambda)$ is a factor depending on the trap geometry [see Eq. (22) in Ref. [40]]. The amplitude A_z can be extracted from the oscillation amplitude in the time of flight images A_z^{TOF} by using scaling theory [41]. For our parameters we obtain $A_z \approx A_z^{\mathrm{TOF}}/\sqrt{2}$ and the oscillation amplitudes in the trap are $A_z \leqslant 20\%$ for our measurements. Hence the corresponding frequency shift is limited to $\delta\omega(A_z)/\omega \leqslant 0.5\%$ and is thus clearly smaller than the expected shift due to dimensional effects.

Let us now turn to the most important correction to the mode frequency, which is caused by the finite temperature of the system. Early experiments on collective excitations in 3D systems investigated effects due to the finite temperature of the sample, including frequency shifts with respect to the zero temperature predictions [42,43]. On the theory side various approaches have been proposed to include finite temperature into the theory of weakly interacting trapped gases. To describe the observed features, such as the damping of excitations or the dynamic coupling between the condensed and thermal components, time-dependent mean-field schemes are appropriate. In this paper, we rely on the linearized time-dependent Hartree-Fock-Bogoliubov approach proposed in [44]. Within this approach the coupled equations for the dynamics of the condensate and the thermal component are not solved self-consistently, but perturbatively up to second order in the coupling constant g. Thus the nonphysical gap in the self-consistent static quasiparticle excitation spectrum is replaced by a well-behaved gapless Bogoliubov type of spectrum. Moreover at $T=0$ this approach recovers the first order quantum correction (4), and therefore includes the frequency shift due to corrections to the mean-field approximation.

Figure 3 shows a comparison between the measured frequency shifts with respect to the $T=0$ calculation in Ref. [35] and the theoretical prediction for the shifts due to finite temperature according to Ref. [44]. The vertical error bars correspond to the uncertainty of the frequency determination as shown in Fig. 2. The typical horizontal error bar shown reflects the temperature distribution of the clouds contributing to one frequency measurement. The calculated frequency shift depends on temperature, the aspect ratio of the trap, and on the parameter $\eta = \mu/(k_B T_c)$ which describes the interactions in a 3D Bose gas [44]. Here T_c denotes the critical temperature for a non-interacting gas. Due to the variation of the atom number in the experiments, the corresponding values for η range from 0.33 to 0.41. The theoretical prediction for these cases is shown in Fig. 3. Despite the low atom numbers used in some of the measurements this theory, developed for 3D gases in the thermodynamic limit, shows good agreement with the measured frequency shifts in the dimensional transition region. Since the finite-temperature shifts are of the order of a few percent, they constitute an important contribution to the mode frequency. For a subsequent correction of the mode frequency, the parameter η and the resulting frequency shift was calculated for each data point.

To compare our finite-temperature measurements with the zero-temperature prediction in Ref. [35], we correct our data for the finite-temperature shift and the smaller shift due to the oscillation amplitude. Figure 4 shows the measured mode frequencies, including these corrections. The variation in the number of particles allowed for measurements covering almost the entire transition region between the 3D and 1D mean-field regimes. Within the experimental error, our data shows good agreement with the predicted frequency dependence at zero temperature. The error increases towards small values of the dimensionality parameter P due to the small particle numbers involved in these measurements. A small discrepancy between the theoretical prediction and the measurement might be due to the loss of atoms during the oscillation or systematic uncertainties in the determination of the number of atoms.

In conclusion, we have measured the increase of the oscillation frequency of the low $m=0$ quadrupolar mode in the crossover region between the 3D and 1D regimes. To compare our measurements with the zero-temperature prediction, various effects that can lead to shifts of this frequency were carefully evaluated. We have identified finite-temperature effects as an important contribution to the frequency shift. Including frequency corrections due to the finite temperature and the oscillation amplitude, our data shows good agreement with the theoretical prediction. The observed frequency increase constitutes a clear signature of the onset of one dimensionality. These results confirm that mode frequency measurements provide a sensitive probe of the dimensionality of quantum degenerate Bose gases.

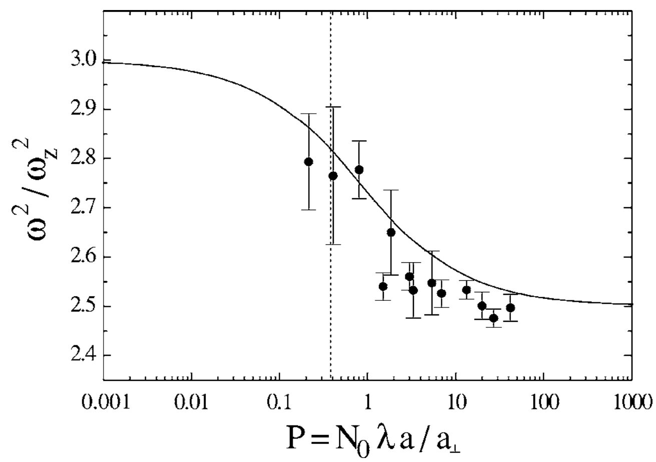

FIG. 4. Measurement of the mode frequency ω for the transition between the 3D and the 1D mean-field regimes. The frequency shifts due to finite temperature and large oscillation amplitudes are included. The solid line shows the theoretical prediction [35]. The dashed vertical line indicates the transition point between the two regimes according to (2).

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