Bose-Einstein condensates in optical quasicrystal lattices

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We analyze the physics of Bose-Einstein condensates confined in two dimensional (2D) quasiperiodic optical lattices, which offer an intermediate situation between ordered and disordered systems. First, we analyze the time-of-flight interference pattern that reveals quasiperiodic long-range order. Second, we demonstrate localization effects associated with quasidisorder as well as quasiperiodic Bloch oscillations associated with the extended nature of the wave function of a Bose-Einstein condensate in an optical quasicrystal. In addition, we discuss in detail the crossover between diffusive and localized regimes when the quasiperiodic potential is switched on, as well as the effects of interactions.

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The last few years have witnessed a fastly growing interest on ultracold atomic gases in laser-generated periodic potentials [optical lattices (OLs)]. These present neither defects nor phonons, offering a powerful tool for investigating the quantum behavior of periodic systems under unique control possibilities. Thus, ultracold atoms trapped in OLs show fascinating resemblances with solid-state physics, which range from Bloch oscillations [1,2] and Wannier-Stark ladders [3], to Josephson arrays of Bose-Einstein condensates (BECs) [4], or to the superfluid to a Mott-insulator transition [5].

These remarkable experiments have been performed in regular cubic OLs. However, these lattices do not exhaust the rich possibilities offered by optical potentials. More sophisticated lattice geometries have been proposed, as honeycomb [6] or Kagomé and triangular [7] lattices. Beyond, controlled defects may be introduced to generate random or pseudorandom potentials [8], allowing for the realization of Kondo-like physics [9], Anderson localization, and Bose-Glass phases [10]. Exploiting this possibility, laser speckle fields have been employed very recently to produce BECs in random potentials [11–13] opening very exciting experimental possibilities.

Bridging between ordered and disordered structures, quasicrystals (QC) have attracted a wide interest since their discovery in 1984 [14]. QCs are long-range ordered materials but without translational invariance, and consequently they share properties with both ordered crystals and amorphous solids [15]. In particular, QCs show intriguing structure [16] as well as electronic conduction properties [17] at the border between ordered and disordered systems.

Surprisingly, up to now, few works have been devoted to optical analogues of QCs, despite of the fact that OLs offer dramatic possibilities for designing a wide range of geometries [18]. Optical QCs have been first studied in laser cooling experiments [19], in which the atomic gas, far from quantum degeneracy, was confined in a dissipative OL where quantum coherence was lost due to spontaneous emission. In these systems, the temperature and spatial diffusion were found to behave similarly as in periodic OLs. The physics of one-dimensional (1D) quasiperiodic OLs has also been subject of recent research in the context of cold atomic gases, including a proposal for the atom-optical realization of the Harper model [20], and the analysis of Fibonacci potentials [21].

In this paper, we study the dynamics of a BEC in a two-dimensional (2D) optical QC. First, we show that the BEC wave function displays quasiperiodic long-range order, a property that may be easily probed via matter-wave interferometry. Second, we show that macroscopic quantum coherence dramatically modifies the transport on the lattice compared with the dissipative case. On the one hand, due to quasidisorder in optical QC, spatial localization occurs, in contrast to ballistic expansion in periodic lattices. The crossover between ballistic expansion and localization is analyzed when the quasiperiodicity of the lattices is continuously increased. On the other hand, we show that due to the extended character of the BEC wave function, Bloch oscillations take place. These oscillations are however quasiperiodic rather than periodic. Additionally, we briefly discuss the effects of the interatomic interactions in the BEC diffusion.

In the following we consider a dilute Bose gas trapped in the combination of a smooth harmonic potential $V_{\mathrm{ho}}(\vec{r})=(M/2)(\omega_\perp^2\vec{r}_\perp^2+\omega_z^2z^2)$ plus an OL $V_{\mathrm{latt}}(\vec{r}_\perp)$. In the previous expression, M is the atomic mass, ω_j are the harmonic trap frequencies, and $\vec{r}_\perp=(x,y)$ is the position vector on the lattice plane. We assume ω_z to be large enough to keep a 2D physics on the xy plane. We consider a laser configuration [19] consisting on N_{b} laser beams arranged on the xy plane with N_{b}-fold symmetry rotation (Fig. 1). The polarization $\vec{\epsilon}_j$ of laser j with wave vector $\vec{k}_j$ is linear and makes an angle α_j with the xy plane. The optical potential is thus [22]

$$V_{\mathrm{latt}}(\vec{r}_\perp)=\frac{V_0}{\left|\sum_j \varepsilon_j\right|^2}\left|\sum_{j=0}^{N_{\mathrm{b}}}\varepsilon_j\vec{\epsilon}_j e^{-i(\vec{k}_j\cdot\vec{r}_\perp+\varphi_j)}\right|^2, \qquad (1)$$

where $0\leqslant\varepsilon_j\leqslant 1$ stand for eventually different laser intensities and φ_j are the corresponding phases. In the following, we are mostly interested in the fivefold symmetric configuration ($N_{\mathrm{b}}=5$, $\varepsilon_j=1$), similar to the Penrose tiling [23], which supports no translational invariance (see Fig. 1). The

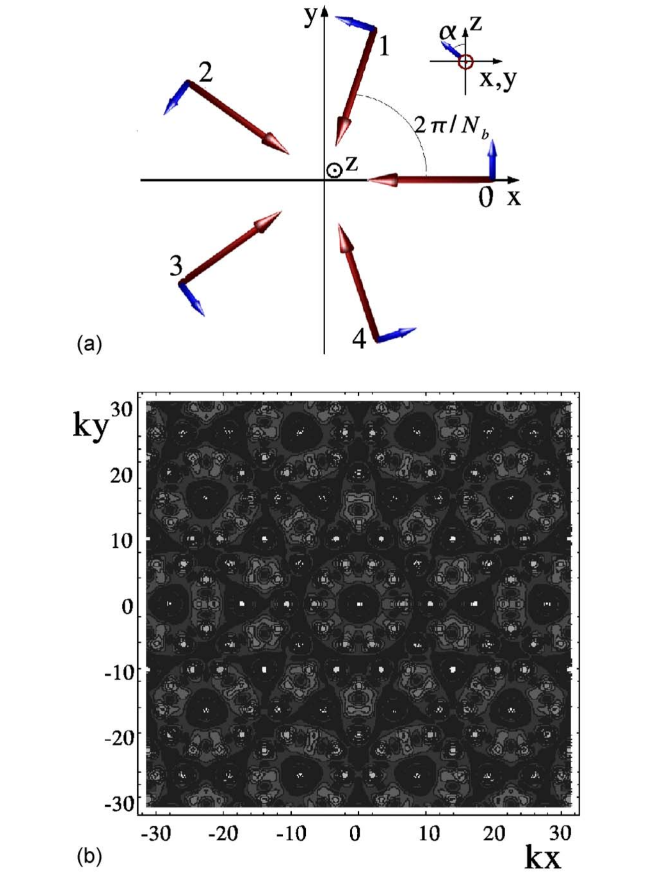

FIG. 1. (Color online) Left: The Laser arrangement (see the text for details). Right: A quasiperiodic lattice potential for $N_{\mathrm{b}}=5$, $V_0<0$ and $\alpha_j=0$. The white points correspond to potential minima.

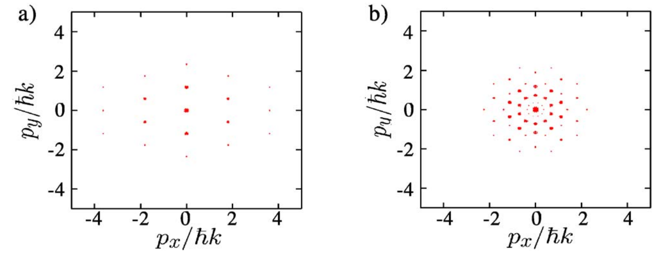

FIG. 2. (Color online) The matter-wave interference pattern of a BEC released from a combined OL and harmonic trap. (a) The periodic case; (b) The quasiperiodic case. Both correspond to ^{87}Rb and $V_0=-10E_{\mathrm{R}}$, where $E_{\mathrm{R}}=\hbar^2k^2/2M$ is the recoil energy.

lattice displays potential wells which are clearly not periodically arranged. We also consider the configuration obtained by switching off lasers 1 and 4, which results in an anisotropic periodic lattice.

(a) Equilibrium properties. The stationary BEC wave function ψ_0 is obtained from evolution in imaginary time of the 2D Gross-Pitaevskii equation (GPE)

$$i\hbar\partial_t\psi=[-\hbar^2\vec{\nabla}^2/2M+V_{\mathrm{ho}}+V_{\mathrm{latt}}+g_{2\mathrm{D}}|\psi|^2]\psi, \qquad (2)$$

where $g_{2\mathrm{D}}=\sqrt{8\pi\hbar^3\omega_z/M}a_{\mathrm{sc}}$, with a_{sc} the s-wave scattering length. In order to elucidate the long-range order properties of the BEC, we compute the momentum distribution of ψ_0. In the periodic case [Fig. 2(a)], as expected, the momentum distribution displays discrete peaks corresponding to combinations of elementary basis vectors of the reciprocal lattice, $n_1\vec{\kappa}_1+n_2\vec{\kappa}_2$ with integer coefficients n_1 and n_2. As obtained in previous experimental works [24,25], this confirms the periodic long-range order of ψ_0. The quasiperiodic case [Fig. 2(b)] is more intriguing, resulting in a more complex structure. The momentum distribution also displays sharp peaks, being the signature of a long-range order which is quasiperiodic rather than periodic [26]. As in the periodic case, the positions of the peaks are linear combinations of integer numbers of $N_b=5$ wave vectors: $\Sigma_{j=0}^{N_b-1}n_j\vec{\kappa}_j$, where $\vec{\kappa}_0=\vec{k}_1-\vec{k}_0$ and $\vec{\kappa}_j=\mathcal{R}[\phi_j]\vec{\kappa}_0$ is the wave vector obtained by a rotation of angle $\phi_j=2\pi j/N_b$ of $\vec{\kappa}_0$. The reciprocal lattice thus clearly shows a fivefold rotation symmetry incompatible with any translation invariance [27]. This resembles the Penrose tiling [23] and the solid state QCs observed via Bragg diffraction [14].

The discussed momentum distribution can be *directly imaged* via matter-wave interferometry after a time-of-flight expansion [28]. Indeed, although the interactions are crucial for determining local populations of each potential well, they do not contribute significantly to the free BEC expansion after release from the trap [24]. Such measurements, standard in periodic OLs [24,25], can be easily extended to quasiperiodic ones.

(b) Quantum transport. Certainly, not all physical properties of optical QCs can be directly interpolated from the behavior of periodic lattices. Indeed, solid QCs show intriguing dynamical properties that are not yet completely understood [15]. In the following, we investigate dynamical properties of quasiperiodic lattices.

(b1) Coherent diffusion. Starting from the equilibrium wave function ψ_0, we consider the situation in which the harmonic trap is switched off at $t=0$, letting the BEC evolve in the OL. The BEC expansion is then computed using a Crank-Nicholson algorithm for the real time-dependent GPE (2). Figure 3 (inset) shows the time evolution of $\langle x^2\rangle$ and $\langle y^2\rangle$ of the interacting BEC along x and y, respectively. In the periodic case, the condensate expands coherently as one ex-

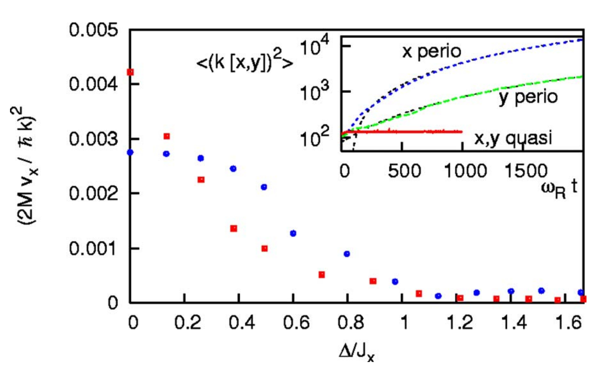

FIG. 3. (Color online) The crossover from ballistic to localization regimes with (squares) and without (circles) interactions, for $V_0=-7.5E_{\mathrm{R}}$. Inset: Coherent diffusion in periodic and quasiperiodic lattices for $V_0=-5E_{\mathrm{R}}$. Fits to $\langle r_j^2(t)\rangle=r_{j0}^2+2D_jt+v_j^2t^2$ in the periodic case are also shown.

pects from tunnel couplings between adjacent lattice sites. In order to infer a convenient fitting functional for the expansion of the interacting BEC, let us recall that in free space for large times $\langle r_j^2(t)\rangle \propto v_j^2 t^2$ with $v_j \propto 1/M$ [28]. In periodic lattices, the inertia is enhanced, and the expansion is expected to be as in free space but substituting the atomic mass M by an effective mass $M^* > M$. We thus expect $v_j \propto 1/M^*$ and $M/M^* \propto J$ where J is the site-to-site tunneling rate [29]. Numerical computations for various depths of the lattice potential V_0 show that v_j^2 decreases exponentially with V_0 as expected from the well-known exponential decay of J. Anisotropic ballistic expansion of the BEC reflects the anisotropy of our periodic lattice.

The behavior of the BEC in the quasiperiodic lattice is dramatically different, since after a short transient the BEC localizes [30] (inset of Fig. 3). This behavior strongly contrasts with the results obtained in the context of laser cooling where a similar classical normal expansion ($\langle r_j^2(t)\rangle \sim 2\widetilde{D}_j t$) was found for both periodic and quasiperiodic OLs [19]. Here, spatial localization is a coherent effect induced by quasidisorder due to the lack of periodicity. Indeed, the BEC populates localized (Wannier-like) states centered on each lattice site. In the periodic case these states have all the same energy and are strongly coupled through quantum tunneling. On the contrary, in the quasiperiodic lattice, the sites have different energies. In particular, the typical difference of depths of adjacent sites (denoted Δ below) can be of the order of magnitude of (but smaller than) the potential depth. The tunneling is not resonant and the BEC localizes.

The remarkable flexibility of OLs [18] allows for the accurate study of the competition between tunneling and quasidisorder. By ramping up gradually the intensity of lasers 1 and 4 while keeping constant 0, 2, and 3, one turns continuously from an anisotropic periodic lattice to a fivefold symmetric quasiperiodic one, and hence from ballistic expansion to spatial localization. For small intensity of the control lasers 1 and 4, the quasiperiodicity is mainly *compositional* (the sites are still periodically displaced but the on-site energies are different from site to site). We define the quasidisorder Δ parameter as the variance of the differences of onsite energies in adjacent sites. From the previous discussion and to compare to the results of the nondegenerate case [19] we fit

$$\langle r_j^2(t)\rangle = \langle r_{j0}^2\rangle + 2D_j t + v_j^2 t^2. \qquad (3)$$

In the considered range of parameters, all calculations fit well with Eq. (3) with a negligible diffusive term $2D_j t$. We characterize the expansion along the x direction through the ballistic velocity v_j. The behavior of v_x versus the quasidisorder parameter Δ is shown in Fig. 3 for the interacting BEC and it is compared to the noninteracting case. For the latter, we simultaneously switched off the interactions at $t=0$ [32]. In both cases, as expected, coherent diffusion dramatically decreases when quasidisorder increases. Spatial localization occurs for $\Delta \gtrsim J_x$, with J_x the tunneling rate between adjacent sites along the x direction [33]. This supports the interpretation that competition between coherent tunneling and inhomogeneities turns into localization as soon as tunneling becomes nonresonant.

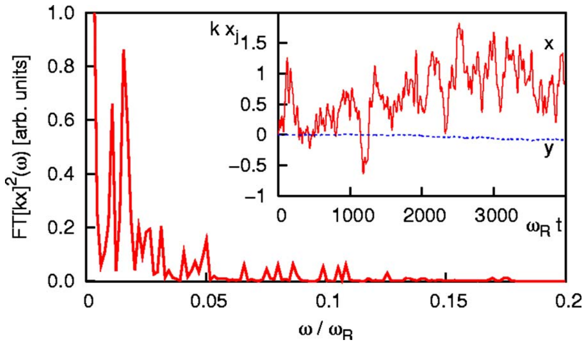

FIG. 4. (Color online) The Fourier transform of the mean position of a BEC in a periodic or quasiperiodic lattice. Inset: The time evolution of a BEC in a tilted quasiperiodic lattice. All beams have the same intensity, $V_0=-2E_R$ and $V_{tilt}=0.002E_R \times kx$.

To understand the effect of interactions that can help ($\Delta \lesssim 0.2J_x$ in Fig. 3) or hinder ($\Delta \gtrsim 0.2J_x$ in Fig. 3) diffusion, note first that two phenomena contribute to localization: (i) initial inhomogeneities (due to disorder and harmonic confinement) that appear in the dynamics through the interaction term $g_{2D}|\psi_0|^2$ [see Eq. (2)] and (ii) inhomogeneities associated to quasidisorder. Because of these inhomogeneities, quantum tunneling is not resonant and thus less efficient. However, during diffusion, the interaction energy is converted into kinetic energy and this tends to fasten the expansion. For small quasidisorder, the second phenomenon dominates so that interactions contribute to expansion whereas for larger quasidisorder, the inhomogenities significantly hinder tunneling so that interactions contribute to localization. The nontrivial interplay between disorder and interactions will be the subject of further research.

(b2) Quasiperiodic Bloch oscillations. One of the most appealing predictions of the quantum theory of solids [27] is that homogeneous static forces induce oscillatory rather than constant motion in periodic structures [34]. The corresponding Bloch oscillations have already been observed in superlattice superconductors [35] and on cold atoms in OLs [1,2]. It is a fundamental question whether such a phenomenon also exists in less ordered systems like QCs. Arguments based on general spectral properties of QCs [36] and numerical simulations of 1D Fibonacci lattices [37] support the existence of Bloch oscillations in quasiperiodic lattices. However, to the best of our knowledge, this effect has never been observed experimentally. Using accelerated lattices [1,2] or gravity [38] (we consider the latter), this question can be addressed experimentally in the discussed arrangement (Fig. 1). Starting from ψ_0 we switched off the harmonic trap and the interactions at time $t=0$ and tilt the quasiperiodic lattice in the x direction [39]. The latter evolution of the quantum gas is shown in the inset of Fig. 4. We find noisylike oscillatory motion in the (tilted) x direction and no motion in the (nontilted) y direction. The oscillations in the x direction are clearly not periodic [40]. However, they definitely have an ordered structure, which is evidenced by the appearance of discrete sharp peaks in the time Fourier transform of the BEC mean position $\langle x(t)\rangle$ (Fig. 4), corresponding to a quasiperiodic motion [26].

The Bloch-like quasiperiodic oscillations can be interpreted as follows. Both in periodic and quasiperiodic lattices, the BEC wave function extends over many lattice wells, and can be decomposed into a sum of localized (Wannier-like) states. Due to the applied external force, these energy states are arranged in a Wannier-Stark ladder. In periodic lattices, the energy separation ΔE between the ladder states is fixed, leading to periodic Bloch oscillations of period $\propto \hbar / \Delta E$. However, for quasiperiodic lattices, a discrete set of different (noncommensurate) differences of on-site energies in adjacent wells occurs (i.e., a nonequally spaced Wannier-Stark ladder) leading to quasiperiodic (instead of periodic) oscillations. Purely random potentials would result in a continuous set of differences of on-site energies leading, as expected, to the disapearance of any sort of Bloch oscillations.

Summarizing, we have investigated the physics of BECs trapped in optical QCs. We have shown that (i) the equilibrium BEC wave function displays long-range quasiperiodic order and that (ii) quantum transport shares properties with both ordered and disordered systems. On the one hand, because of quasidisordered inhomogeneities, diffusion turns from ballistic to localization when quasiperiodicity is switched on. On the other hand, because of coherence extending over several lattice sites, quasiperiodic Bloch oscillations occur in quasiperiodic BECs.

The discussed arrangement can be easily generated using standard techniques, offering an exciting tool for controlled studies of the transition between periodic, quasiperiodic, and fully disordered systems in cold gases, a major topic of current experimental research [11–13]. In addition, the system can be used to address experimentally some unsolved issues on QCs, as, e.g., the application of the renormalization theory to the 2D case [15].

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