Cavity-mediated long-range interaction for fast multiqubit quantum logic operations

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Interactions among qubits are essential for performing two-qubit quantum logic operations. However, nature gives us only nearest neighbor interactions in simple and controllable settings. Here we propose a strategy to induce interactions among two atomic entities that are not necessarily neighbors of each other through their common coupling with a cavity field. This facilitates fast multiqubit quantum logic operations through a set of two-qubit operations. Using its explicit position dependence, this interaction can be employed for simulation of quantum spin systems. The ideas presented here are applicable to various quantum-information proposals for atom-based qubits such as trapped ions, atoms trapped in optical cavities, and optical lattices.

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I. INTRODUCTION

Quantum information science [1] has made rapid progress recently with scalable architectures proposed for atom based qubits through the ion-trap schemes [2] and for photonic qubits through the linear optical quantum-computing [3] and cavity-quantum-electrodynamics (QED) [4] schemes, along with various other proposals for quantum logic operations for atomic [5–7], photonic [8], and hybrid [9] qubits.

Two-qubit quantum logic operations require interaction between the physical entities used for encoding the qubits. Physical systems provide natural grounds for implementations of two-qubit operations as there are plenty of cases providing controllable interaction between two entities. However, direct multiqubit operations require controllable interactions between more than two entities at a time that are difficult to come by. Thus, the decomposition of multiqubit gates into their two-qubit and single-qubit counterparts is an essential step in quantum circuit design and implementations. Moreover, the majority of the two-body interactions have a spatially dependent interaction strength. Thus, in most cases only near-neighbor interactions are available.

To this end, we note that it is possible to induce interaction between atoms coupled collectively to the cavity vacuum [10]. This interaction has been shown to be useful to perform quantum logic operations by Gábris and Agarwal [5] in the case of two-level atoms trapped inside a cavity. Here we recognize that the interaction induced between the atoms through their common coupling with the cavity vacuum is essentially independent of the distance between the atoms. We employ the long-range nature of this interaction and develop a scheme to allow any two atoms from a chain of trapped neutral atoms to interact with each other. This interaction is then exploited to perform quantum logic operations between any two qubits. Our model employs metastable atomic states as qubits; thus qubit decoherence is not an issue. Moreover, the atom-cavity coupling is dispersive in nature; therefore, the cavity does not contain any real photons at any stage of the interaction. Thus, the cavity decay becomes a nonissue as well. It is well known that multiqubit quantum logic operations can be achieved through a sequence of single-qubit and two-qubit gates (e.g., controlled-NOT (CNOT) and controlled-sign (CS) gates) [11]. Thus, our scheme can be easily extended to perform multiqubit operations. In fact, as we show later, the quantum circuits for the multiqubit quantum operations as they are usually drawn, which involves several non-neighbor qubit operations, can be directly implemented through of a sequence of operations using our nonlocal scheme.

It is also instructive to recollect that an important hurdle for scalability has been identified for the ion-trap quantum-computing proposals [2]. Namely, physical motion of ions is required to ensure proximity among the qubits for the two-qubit logic operation. This dictates tremendous speed constraints on the current ion-trap quantum computer architecture. Very recently Tian *et al.* [9] have proposed a hybrid qubit approach, by coupling the ion-trap qubits with the superconducting ones, to cure this pathology of trapped-ion systems. Nevertheless, introduction of non-neighbor interactions would provide tremendous speed-up for the trapped-ion proposals. Furthermore, devising simple strategies for implementation of the error correcting codes [12] is essential, especially, in the context that the experimental demonstration has been possible only for a three-bit code [13] so far.

The article is organized as follows. As we are proposing a non-neighbor interaction for quantum logic operations, we present a simple analysis of how many operations are required in the conventional setting to carry out a nonlocal CNOT gate between the first and the Nth qubit from a set of total N qubits. Next, we present how distance-independent interaction can be induced between any two qubits from a

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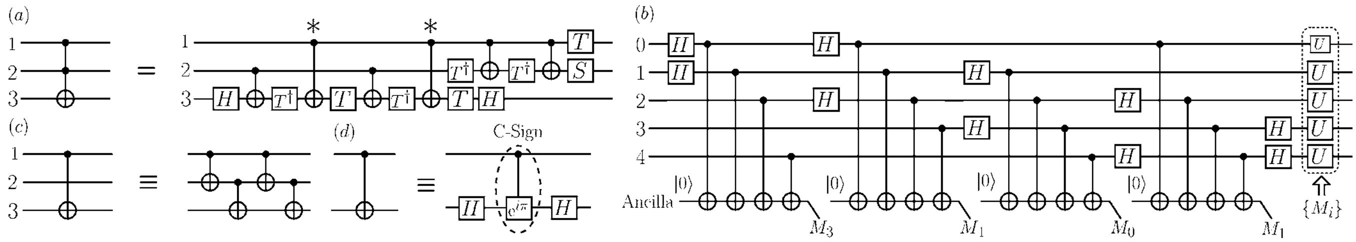

FIG. 1. Illustration of the need of non-neighbor interactions for fast quantum logic operations. (a) Decomposition of a three-qubit Toffoli gate into two-qubit CNOT gates and single-qubit unitary transformations [See Eq. (1)] adapted from Ref. [1]. The steps denoted by * involve non-neighbor interaction. (b) An error-correcting network for a five-bit encoding [12]. Error syndrome measurements ($\{M_3, M_4, M_0, M_1\}$) on the ancillas dictate the corrective unitaries (U) to be performed in the end to protect the encoded qubits from various errors. Notice the need for several non-neighbor two-qubit CNOT operations. (c) Decomposition of a non-neighbor two-qubit CNOT gate into near-neighbor two-qubit gates. (d) Equivalence between the CNOT and CS gates modulo single-qubit unitary transformations.

chain having a number of them. We give a set of operations required to perform the quantum phase gate operation such that the nonlocal nature is maintained. Then, we clarify advantages offered by our scheme and contrast it with several other quantum logic schemes for the ionic and neutral atom qubits. Finally, we present our conclusions. In the appendices, we provide calculational details leading to the quantum logic operations.

II. ADVANTAGES OF THE NON-NEIGHBOR INTERACTIONS

To emphasize the speed-up obtainable through non-neighbor interactions, we revisit the design of multiqubit quantum logic gates through a sequence of single-qubit unitary transformations and two-qubit operations. For example, we consider the circuit representation of the three-qubit Toffoli gate in Fig. 1(a) in terms of the CNOT gates and single-qubit unitary transformations,

$$H = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}, \quad S = \begin{pmatrix} 1 & 0 \\ 0 & i \end{pmatrix}, \quad \text{and } T = \begin{pmatrix} 1 & 0 \\ 0 & e^{i\pi/4} \end{pmatrix}. \tag{1}$$

It needs at least two non-neighbor CNOT gates. In Fig. 1(b), we consider a five-bit error correction network [12]. It needs multiple interactions of each encoding qubit with the ancilla; therefore, with only near-neighbor interactions, there would be a dramatic increase in the number of operations needed for its implementation.

To quantify the number of extra operations required per non-neighbor two-qubit gate, we consider an example in Fig. 1(c) showing a decomposition of non-neighbor CNOT operations through multiple near-neighbor CNOT operations. Thus, to perform a non-neighbor two-qubit gate between the first and third qubit from a total set of three qubits, the optimal sequence of operations [as given in Fig. 1(c)] requires three more operations. It turns out, however, that this strategy is not optimal if it is extended in a straightforward manner to a set of total N qubits for performing a two-qubit operation between the first and the Nth qubit. The optimal strategy, using only local or near-neighbor interactions, is obtained by swapping the Nth qubit with the $(N-1)$th qubit, then swapping the $(N-1)$th qubit with the $(N-2)$th one and so on till the state of the Nth qubit is transferred to the second qubit. A simple calculation shows that this swapping operation requires $N-2$ operations. Then the two-qubit CNOT could be performed between the near-neighbor first and second qubits. Once again, $N-2$ SWAP operations would be needed to bring back the new state of the second qubit to the Nth qubit where it belongs. It is also known that a single SWAP operation requires three CNOT operations. Thus, to perform a two-qubit, nonlocal gate between the first and the Nth qubit, one requires $6(N-2)$ extra operations.

It has to be borne in mind, however, that each SWAP operation has to be completely error free; otherwise, the above procedure would introduce a tremendous amount of uncorrectable errors. Thus, one needs each CNOT operation implemented in a fault-tolerant manner, which would require, at the least say, ten error-correcting operations per CNOT gate. Thus, the actual number of operations required would be $60(N-2)$ operations. To illustrate, for a moderate number of qubits, say 10, a total of 480 extra operations would be needed just to perform a single two-qubit gate between the first and the tenth qubit.

In the above calculation, we have assumed that the error-correcting network needed to perform a single CNOT in a fault-tolerant manner does not require any nonlocal operations. However, it can be easily seen from Fig. 1(b) that this is not the case. Thus, the actual number of operations would be much higher than $60(N-2)$.

Therefore, as the complexity of the quantum circuit increases, more and more non-neighbor interactions would be needed and schemes based on the conventional approach would be very slow at best and very error prone at worst. Moreover, the number of operations needed to perform a single non-neighbor gate between the first and the last qubit would increase with the system size in the conventional setting. Therefore, the advantage offered by efficient nonlocal interaction cannot be overemphasized.

Just to note, the single-qubit operations of Eq. (1) can be attained through properly timed Raman pulses coupling the two atomic levels, $|0\rangle$ and $|1\rangle$, forming a qubit; this is routinely done in cavity-QED systems. Therefore, we restrict ourselves to the non-neighbor quantum phase gate operation.

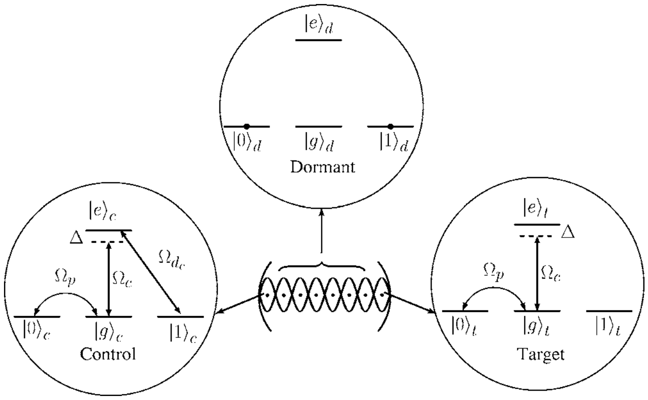

FIG. 2. Scheme for the non-neighbor two-qubit quantum phase gate with cavity-mediated interaction. The cavity mode is coupled to the $|g\rangle$-$|e\rangle$ transition for all the atoms with detuning Δ. The external drive fields couple different atomic states as per the Hamiltonian in Eq. (6). The external coupling could be direct or through Raman pulses as discussed in the context of Eq. (6). The dormant atoms maintain their general state $\alpha|0\rangle_d+\beta|1\rangle_d$ while quantum logic operation is being performed on the control and target atoms. The atomic energy levels $|0\rangle, |g\rangle$, and $|1\rangle$ are shown to be degenerate just for convenience. This is not a requirement for the success of the proposal.

III. NONLOCAL INTERACTION THROUGH THE CAVITY VACUUM

In the discussion to follow, we describe our scheme in detail and show how a distance-independent interaction can be introduced between a pair of atoms through their common interaction with the cavity vacuum.

To accomplish scalable architecture, it should be possible to perform two-qubit operations with equal ease in the presence of other qubits. To illustrate, having only two-level systems as qubits interacting dispersively with the cavity field as considered by Gábris and Agarwal [5], one needs twice as many operations to accomplish a two-qubit gate in the presence of an extra atom in the cavity, as this third atom also takes part in the collective interaction. Therefore, we employ collective coupling with the cavity with the choice of turning on the interaction as needed instead of having it on at all times.

We consider a linear array of N atoms placed in a cavity. We note that the cavity supports a standing wave field with spatially dependent field amplitude. Therefore, it is important to trap atoms such that they all see the same field strength. Such an architecture can be achieved using the proposals for trapping atoms inside optical cavities [14], through the marriage of ion-trap and cavity-QED systems [15] or in the chain of neutral atoms trapped in standing wave fields [16]. The atoms are assumed to be identical and have a four-level internal structure as shown in Fig. 2. The states forming qubits could be taken as hyperfine sublevels of an electronic state or two states of a single hyperfine manifold. The preparation of the qubit state can be accomplished through efficient mixing of the well-developed optical pumping and adiabatic population transfer techniques. The states representing the qubit are chosen so that they do not directly interact with the cavity field. However, each atom has an extra pair of levels that can interact with the cavity field. Thus, once the ith and jth atoms are brought into these levels, they interact with the cavity field through the Hamiltonian

$$\mathcal{H} = \mathcal{H}_0 - \hbar\Omega_c \sum_{l=i,j} (|e_l\rangle\langle g_l| a_k + |g_l\rangle\langle e_l| a_k^\dagger), \tag{2}$$

with $|e_i\rangle$ and $|g_i\rangle$ being the levels of atom i close to resonance with the cavity field. Also, a_k is the cavity mode annihilation operator and Ω_c is the coupling strength of the atomic transition with the cavity field. Here the free Hamiltonian is

$$\mathcal{H}_0 = \hbar\nu_k a_k^\dagger a_k + \hbar\omega_{g_i}|g_i\rangle\langle g_i| + \hbar\omega_{e_i}|e_i\rangle\langle e_i|, \tag{3}$$

including only the relevant atomic energy levels. The atomic energies $(\hbar\omega_{g_i}, \hbar\omega_{e_i})$ are measured with respect to the ground state, $|0_i\rangle$, of the atoms. Here the position dependence of the Rabi frequency and the atomic dipole operators is not shown as the atomic positions (for example, antinodes of the standing wave field of the cavity) are chosen such that all atoms see the same cavity field strength. By including the position dependence of the cavity field and the atomic dipoles, this Hamiltonian (2) becomes tunable and can be employed for simulation of the quantum spin systems [17]. It is important to note that the cavity field is not directly resonant with the $|e\rangle$-$|g\rangle$ transitions and it is coupled only dispersively. Therefore, the cavity field is in its vacuum state and the atom does not get excited by the cavity field, or if it is in the excited state, it does not emit a photon in the cavity mode. Thus, through adiabatic elimination of the states corresponding to the presence of photons in the cavity mode, we arrive at the effective interaction Hamiltonian [10],

$$\mathcal{H}_{\text{eff}} = \hbar\eta\left(\sum_{k=i,j} |e_k\rangle\langle e_k| + |e_i\rangle\langle g_i| \otimes |g_j\rangle\langle e_j| + |e_j\rangle\langle g_j| \otimes |g_i\rangle\langle e_i|\right), \tag{4}$$

where $\eta = \Omega_c^2/\Delta$, and Δ is the cavity field detuning with respect to the atomic transition. We have assumed $\Delta \gg \Omega_c$ for arriving at this result. The first term leads to trivial phase factors that can be corrected in a straightforward manner and the last two terms lead to coupling between the atom through virtual exchange of a cavity photon. Thus, the cavity vacuum effectively induces interaction between two atoms, immaterial of their spatial position provided they both see the same cavity field strength. The dipole-dipole interaction usually falls off as reciprocal of the sixth power of the distance between the dipoles, whereas the interaction induced between the atomic dipoles through the cavity vacuum is independent of the distance between them. This long-range interaction can be employed to perform nonlocal quantum logic operations.

IV. THE NONLOCAL QUANTUM PHASE GATE

In this section, we provide the set of operations needed to accomplish direct two-qubit operation between any of the two qubits from a linear array of N qubits. We note that the two-qubit CNOT gate can be decomposed into Hadamard

TABLE I. Two-qubit non-neighbor CS gate operation: The effect of step (iii) on the state $|e_c,g_t\rangle$ is nontrivial and it leads to the state $e^{-i\eta t}[\cos(\eta t)|e_c,g_t\rangle - i\sin(\eta t)|g_c,e_t\rangle]$. Thus, with the choice of $\eta t=\pi$, it becomes as $|e_c,g_t\rangle$ as shown below. Moreover, the state $|e_c,1_t\rangle$ acquires a phase factor $e^{-i\eta t}=e^{-i\pi}=-1$ under step (iii). It can be noted that the first and the last column taken together correspond to the truth table of the CS gate.

Input	Step (i)	Step (ii)	Step (iii)	Step (iv)	Step (v)	Output
$\|0_c,0_t\rangle$	$\|g_c,g_t\rangle$	$\|g_c,g_t\rangle$	$\|g_c,g_t\rangle$	$\|g_c,g_t\rangle$	$\|0_c,0_t\rangle$	$\|0_c,0_t\rangle$
$\|0_c,1_t\rangle$	$\|g_c,1_t\rangle$	$\|g_c,1_t\rangle$	$\|g_c,1_t\rangle$	$\|g_c,1_t\rangle$	$\|0_c,1_t\rangle$	$\|0_c,1_t\rangle$
$\|1_c,0_t\rangle$	$\|1_c,g_t\rangle$	$\|e_c,g_t\rangle$	$\|e_c,g_t\rangle$	$\|1_c,g_t\rangle$	$\|1_c,0_t\rangle$	$\|1_c,0_t\rangle$
$\|1_c,1_t\rangle$	$\|1_c,1_t\rangle$	$\|e_c,1_t\rangle$	$-\|e_c,1_t\rangle$	$-\|1_c,1_t\rangle$	$-\|1_c,1_t\rangle$	$-\|1_c,1_t\rangle$

transformations [See Fig. 1(d), and matrix H from Eq. (1)] on the target qubit and a CS gate between the two qubits. Therefore, we only resort to implementing the CS gate operation given by

$$\sum_{j,k=0,1} c_{jk}|j,k\rangle \rightarrow \sum_{j,k=0,1} (e^{\pm i\pi})^{jk} c_{jk}|j,k\rangle. \quad (5)$$

The calculational details are given at length in Appendix II and the choice of the operations steps taken is justified.

Now we analyze several possible initial states of the atoms and their interaction with the cavity field. The results are summarized below. It can be seen that the states $|g_i,g_j,0_k\rangle, |g_i,a_j,0_k\rangle$, and $|a_i,g_j,0_k\rangle$ remain unaffected by the cavity field. Here state $a_{i,j}$ corresponds to some arbitrary atomic level a that does not interact with the cavity field. Also if the interaction time is taken to be $\eta t=\pi$, the states $|g_i,e_j,0_k\rangle$ and $|e_i,g_j,0_k\rangle$ return to their original atomic configurations, and the states $|e_i,a_j,0_k\rangle$ and $|a_i,e_j,0_k\rangle$ acquire a phase factor of $e^{-i\pi}$.

Another important ingradient necessary for our proposal is selective addressing of the atoms. To achieve this, we consider a general interaction Hamiltonian,

$$\mathcal{H}_{\text{ext}} = -\hbar[\Omega_p e^{i\phi_p}\sum_{i=c,t}|g\rangle_i\langle 0|_i + \Omega_{d_c}e^{i\phi_{d_c}}|e\rangle_c\langle 1|_c + \text{H.c.}], \quad (6)$$

describing the application of external optical fields on certain atomic transitions. Here the Rabi frequencies could be directing couplings between the involved levels or they could be composite Rabi frequencies of a couple of Raman pulses coupling the involved levels through intermediate levels $|i_{1,2}\rangle$. In the composite case, the Rabi frequencies can be written as

$$\Omega_p = \frac{\Omega_{0,i_1}\Omega_{i_1,g}}{\delta_1}, \quad \Omega_{d_c} = \frac{\Omega_{1_c,i_2}\Omega_{i_2,e_c}}{\delta_2}. \quad (7)$$

Here $\Omega_{j,k}$ denotes the Rabi frequency of interaction of levels j and k with the corresponding light field applied with detuning δ_1 or δ_2 on the j-k transition. It can be noted that the composite Raman pulses are routinely used in the ion-trap quantum logic gates. We note that complete transfer of population, $|0\rangle \rightarrow |g\rangle$, can be accomplished through an application of a pulse with parameters $\Omega_p t=\pi/2$ and $\phi_p=3\pi/2$ and the inverse operation, $|g\rangle \rightarrow |0\rangle$, with $\Omega_p t=\pi/2$ and $\phi_p=\pi/2$. Similar considerations hold for the pulse with Rabi frequency Ω_{d_c} resonant on the $|1\rangle_c$-$|e\rangle_c$ transition of the control atom. The details of why a specific phase of the Rabi frequency is necessary to achieve population transfer is discussed in Appendix I.

Using the characteristics of the atom-cavity interaction and selective addressing through the external fields, as discussed in detail in Appendix II, we devise a set of operations for the CS gate:

(i) Operation $|0\rangle \rightarrow |g\rangle$ on both the target and control atoms through a pulse of Rabi frequency $\Omega_p e^{i\phi_p}$ with $\Omega_p t=\pi/2$ and $\phi_p=3\pi/2$.

(ii) Operation $|1\rangle_c \rightarrow |e\rangle_c$ through a pulse of Rabi frequency $\Omega_{d_c}e^{i\phi_{d_c}}$ with $\Omega_{d_c}t=\pi/2$ and $\phi_{d_c}=3\pi/2$ to move the control qubit to the excited state interacting with the cavity.

(iii) Interaction of the control and target atoms with the cavity for the time $t=\pi/\eta$.

(iv) Operation $|e\rangle_c \rightarrow |1\rangle_c$, to bring back the qubit state of the control qubit, through the same pulse as in step (ii) except for the phase $\phi_{d_c}=\pi/2$.

(v) Operation $|g\rangle \rightarrow |0\rangle$ on both control and target via the same pulse of step (i) and the phase $\phi_p=\pi/2$.

We note that steps (i), (v) and (ii), (iv) can be accomplished via appropriate terms in Eq. (6). The effect of these operations on various initial states of the two qubits is summarized at length in Table I. By choosing the interaction time t with the cavity-vacuum mode such that $\eta t=\pi$, the desired two-qubit CS gate operation is obtained.

The fidelity calculation for the above model of the two-qubit gate is summarized in Appendix III. Our numerical studies show a gate fidelity of 99%, with the cavity decay (κ), the spontaneous decay of level $|e\rangle(\gamma)$, and the detuning (Δ) taken to be $0.01\Omega_c, 0.0001\Omega_c$, and $10\Omega_c$, respectively. For experimental cavity parameters [14] ($\Omega_c=32\pi$ MHz and $\kappa=2.8\pi$ MHz), a modest $\gamma=0.001\Omega_c$ and $\Delta=10\Omega_c$, we obtain a fidelity of 93%. Once again, we would like to point out that all the states $|0\rangle, |1\rangle, |g\rangle$, and $|e\rangle$ for all the atoms could be taken to be metastable and the external coupling achieved through Eq. (6) could be achieved through Raman pulses. Therefore, atomic decoherence is not an issue. We have provided, above, the fidelity calculations just for completeness. In such a case, the atomic transition $|g\rangle$-$|e\rangle$ could be taken in the microwave range. The scheme could very well be applicable in the optical range, one only needs an appropriate metastable level $|e\rangle$ so that decoherence does not remain an issue.

To come back to the gate operations, employing Hadamard transformation, H from Eq. (1), on the target qubit before and after the CS gate, one obtains a CNOT gate as shown in Fig. 1(d). Combining several of these CNOT gates as shown in Figs. 1(a) and 1(b), the three-qubit Toffoli gate and five-bit error-correcting network can be directly constructed.

The cavity-atom interaction (2) also provides a single-step mechanism to create entanglement between distant atoms. For example, an initial state of two atoms $|e_i,g_j\rangle$ after the interaction with the cavity vacuum for the time $\eta t=\pi/4$ gives the entangled state

$$\frac{1}{\sqrt{2}}e^{-i\pi/4}(|e_i,g_j\rangle - i|g_i,e_j\rangle), \tag{8}$$

which can be transformed into any of the four Bell states by one-qubit unitary transformations.

V. ADVANTAGES OF THE CURRENT PROPOSAL AND ITS CONNECTION WITH PRIOR PROPOSALS

In this section, we mention the advantages of our proposal and contrast it briefly with some of the other cavity-QED and ion-trap quantum phase gate proposals in the literature.

It is interesting to contrast the proposed scheme with the ones in the literature for atomic qubits. Pellizzari *et al.* [18] have proposed a scheme for the implementation of controlled unitary gates through adiabatic passage on a Raman transition. Their proposal hinges on transferring the qubit from two atoms to an extra pair of levels within a single atom and requires the atoms to be close to each other. The atomic level scheme for two-qubit operations requires three doubly degenerate, i.e., in effect six, energy levels for each atom. Another contrasting feature of this proposal is that it induces interaction among atomic qubits by the exchange of a real cavity photon; thus, it is susceptible to the cavity decay.

Further, the proposal by Cirac and Zoller [19] for cold, trapped ions achieves nonlocal operations through a collective excitation of the vibrational motion of the ions with lasers. In the context of multiqubit operations, Goto and Ichimura [6] propose a cavity-QED-based scheme to perform a multiqubit unitary gate by adiabatic passage. The gate operation mechanism is completely different and hinges on the presence of cavity photons; therefore, it is susceptible to the cavity decay.

Physically our scheme is close to the one considered by Gábris and Agarwal [5], which uses two-level atoms interacting with the cavity. The physical closeness comes in the sense that the qubit-qubit interaction is mediated by the cavity vacuum. However, the sequence of operations proposed by them is very different and, therefore, requires a higher number of operations for multiqubit gates as opposed to the possibility of direct circuit implementation available in the present scheme. Another proposal using cavity-mediated interaction is by Zheng and Guo [20]; however, it does not employ the long-range nature of this interaction. Jané *et al.* [21] utilize the cavity field for quantum logic with two three-level atoms and mention that the atoms could be arbitrarily positioned in the cavity field at integral wavelength separation, but do not consider the simultaneous presence of more than two atoms in the cavity field. As it is already clear from the proposal in Ref. [5], even though the interaction could be introduced by similar means, the actual set of operations performing the two-qubit gates are very crucial in determining if two-qubit gates can be performed in the presence of other qubits or not. The nonlocal interactions can be easily introduced if the presence of other qubits does not require alteration of the gate operation sequence.

Thus, compared to several prior proposals for multiqubit gate operations among the atomic qubits, our scheme is not susceptible to the cavity decay as it only uses the nonlocal coupling available through the interaction with the cavity vacuum. Moreover, the two-qubit gate operations required do not depend on the presence of a large number of other qubits in the system. Therefore, the sequence of operations can be applied in succession to the qubits of interest allowing direct quantum circuit implementation of any multiqubit quantum logic operation and error-correction networks. The atomic qubits are implemented through metastable atomic levels; therefore, qubit decoherence is a nonissue as well.

VI. CONCLUSION

To summarize, we have demonstrated a non-neighbor two-body interaction between atomic qubits through their collective coupling with the cavity-vacuum mode. This non-neighbor interaction can be employed to obtain implementations of the two-qubit universal quantum gates. Thus, we provide an architecture for performing fast quantum logic operation in the presence of other qubits. As selective coupling between any two qubits becomes available, regardless of their spatial position, multiqubit operations can be quickly performed through a sequential application of laser pulses to the appropriate atoms. Several advantages offered by our scheme include practically no decoherence, as only the metastable atomic states are used and the cavity is always in the vacuum state. The proposal is fairly general and can be applied to a variety of sytems using atomic qubits, such as ion traps, trapped atoms or ions in optical lattices or cavities. Most importantly this approach provides a strategy, using current experimental techniques, to surpass the pathology of the ion-trap quantum-computing proposals that require movement of the ions to facilitate two-body interactions.

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APPENDIX A: PHASE-FREE POPULATION TRANSFER

In this appendix, we show how to obtain population transfer of the type $|b\rangle \rightarrow |a\rangle$ with the usual π pulses. Normally application of a π pulse achieves population transfer but imparts an extra phase factor to the final state. While carrying out quantum logic operations, these phase factors become relevant and it is better to avoid them. Here we show how the extra phase factors could be eliminated to obtain a clean state transfer by appropriately phased optical pulses. This technique is essential for performing several of the operations needed to obtain the quantum phase gate as described in the main body of the paper.

Consider the interaction of a two-level atom (lower level $|b\rangle$ and upper level $|a\rangle$ with an optical field of Rabi frequency $\Omega(\exp i\phi)$ resonant on the transition. This can be described by the interaction Hamiltonian

$$H = -\hbar\Omega[|a\rangle\langle b|\exp(i\ \phi) + |b\rangle\langle a|\exp(-i\ \phi)]. \tag{A1}$$

Thus for a general wave function $|\Psi\rangle = a(t)|a\rangle + b(t)|b\rangle$, the population rate equations can be written through the Schrödinger equation

$$|\dot{\Psi}\rangle = -\frac{i}{\hbar}H|\Psi\rangle, \tag{A2}$$

as

$$\dot{a}(t) = i\ \Omega\ \exp(i\ \phi)b(t) \tag{A3}$$

$$\dot{b}(t) = i\ \Omega\ \exp(-i\ \phi)a(t). \tag{A4}$$

The general solution of this set of coupled equations, in terms of the initial values $a(0)$ and $b(0)$, can be written as

$$a(t) = a(0)\cos(\Omega t) + i\ b(0)\ \exp(i\ \phi)\ \sin(\Omega t), \tag{A5}$$

$$b(t) = b(0)\cos(\Omega t) + i\ a(0)\ \exp(-i\ \phi)\ \sin(\Omega t). \tag{A6}$$

Therefore, to obtain a clean state transfer of the kind $|b\rangle \rightarrow |a\rangle$, we initially have $a(0)=0$, $b(0)=1$, and we need $\Omega t = \pi/2$ and $i\exp(i\ \phi)=1$, i.e., $\phi=-\pi/2$ or $\phi=3\pi/2$. Also to obtain $|a\rangle \rightarrow |b\rangle$, we have $a(0)=1$, $b(0)=0$, and we need $\Omega t = \pi/2$ and $i\exp(-i\ \phi)=1$, i.e., $\phi=\pi/2$.

APPEXDIX B: DETAILS OF THE STEPS REQUIRED FOR THE QUANTUM PHASE-GATE OPERATION

In this appendix, we consider the interaction of the atoms with the cavity vacuum and show how quantum phase-gate operation could be achieved through this interaction. For the purpose of this discussion, we limit ourselves to only the levels nearly resonant with the cavity, namely, $|e_i\rangle$ the excited state and $|g_i\rangle$ the ground state for the ith atom in the cavity.

The interaction the two atoms with the cavity is governed by the Hamiltonian

$$\mathcal{H} = \hbar\nu_k a_k^\dagger a_k + \hbar\omega_{e_1}|e_1\rangle\langle e_1| + \hbar\omega_{e_2}|e_2\rangle\langle e_2| - \hbar g(|e_1\rangle\langle g_1| + |e_2\rangle \times\langle g_2|)a_k - \hbar g(|g_1\rangle\langle e_1| + |g_2\rangle\langle e_2|)a_k^\dagger. \tag{B1}$$

Note that we have taken both the atoms to be exactly identical and all the energies are measured with respect to the ground state of the atoms. In the main body of the paper, we have used Ω_c for the cavity-atom coupling strength as opposed to g here.

Now we consider possible initial states for the two atoms and explore their dynamical evolution individually to look for possible conditions under which the system returns to its initial state after interacting with the cavity for some time τ.

(1) $|I_1\rangle = |g_1\rangle|g_2\rangle|0_k\rangle$

It can be easily seen that $\mathcal{H}|I_1\rangle = 0$. Thus, this state does not evolve in time.

(2) $|I_2\rangle = |e_1\rangle|g_2\rangle|0_k\rangle$

We can see that this state is coupled to the states $|g_1\rangle|e_2\rangle|0_k\rangle$ and $|g_1\rangle|g_2\rangle|1_k\rangle$ Thus, we can consider a general state $|\Psi_2\rangle = a(t)|e_1\rangle|g_2\rangle|0_k\rangle + b(t)|g_1\rangle|e_2\rangle|0_k\rangle + c(t)|g_1\rangle|g_2\rangle|1_k\rangle$ with $a(t=0)=1$, and $b(t=0)=c(t=0)=0$ and study its dynamics. With the Schrödinger equation

$$i\hbar\dot{\Psi} = \mathcal{H}\Psi, \tag{B2}$$

we obtain the following set of equations:

$$\dot{a}(t) = i\ g\ c(t) - i\ \omega_e a(t),$$

$$\dot{b}(t) = i\ g\ c(t) - i\ \omega_e b(t),$$

$$\dot{c}(t) = -i\ \nu_k c(t) + i\ g\ a(t) + i\ g\ b(t). \tag{B3}$$

In the rotated frame defined by $a(t)=\exp(-i\ \omega_e t)\tilde{a}(t)$, $b(t) = \exp(-i\ \omega_e t)\tilde{b}(t)$, and $c(t)=\exp(-i\ \omega_e t)\tilde{c}(t)$, the time derivatives can be written as

$$\dot{a}(t) = -i\ \omega_e \exp(-i\ \omega_e t)\tilde{a}(t) + \exp(-i\ \omega_e t)\dot{\tilde{a}}(t),$$

$$\dot{b}(t) = -i\ \omega_e \exp(-i\ \omega_e t)\tilde{b}(t) + \exp(-i\ \omega_e t)\dot{\tilde{b}}(t),$$

$$\dot{c}(t) = -i\ \omega_e \exp(-i\ \omega_e t)\tilde{c}(t) + \exp(-i\ \omega_e t)\dot{\tilde{c}}(t). \tag{B4}$$

Thus, the new rate equations, dropping the ~, take the following forms:

$$\dot{a}(t) = i\ g\ c(t),$$

$$\dot{b}(t) = i\ g\ c(t),$$

$$\dot{c}(t) = i(\omega_e - \nu_k)c(t) + i\ g[a(t) + b(t)]. \tag{B5}$$

Noting that $\Delta = \omega_e - \nu_k \gg g$, we can set $\dot{c}(t)=0$ to obtain the steady state value

$$c(t) = -\frac{g}{\Delta}[a(t) + b(t)]. \tag{B6}$$

Substituting this result in the other rate equations, we obtain

$$\dot{a}(t) = -i\frac{g^2}{\Delta}[a(t) + b(t)], \tag{B7}$$

$$\dot{b}(t) = -i\frac{g^2}{\Delta}[a(t) + b(t)]. \tag{B8}$$

Thus, with the initial condition $a(0)=1$, we obtain the solution

$$a(t) = \exp\left(-i\frac{g^2}{\Delta}t\right)\cos\left(\frac{g^2}{\Delta}t\right),$$

$$b(t) = -i\exp\left(-i\frac{g^2}{\Delta}t\right)\sin\left(\frac{g^2}{\Delta}t\right). \tag{B9}$$

Thus, for $(g^2/\Delta)t=\pi$, we have only the initial state populated. That is, the final state is $\exp(-i\ \pi)\cos\pi|e_1\rangle|g_2\rangle|0_k\rangle$. However, because this is the state in the rotated frame, we should go back to the lab frame which takes the form: $\exp(-i\ \omega_e t)|e_1\rangle|g_2\rangle|0_k\rangle$, i.e., $\exp(-i\ \pi\omega_e\Delta/g^2)|e_1\rangle|g_2\rangle|0_k\rangle$, since $t=\pi\Delta/g^2$.

(3) $|I_3\rangle=|g_1\rangle|e_2\rangle|0_k\rangle$

In this case, the roles of $a(t)$ and $b(t)$ are reversed, thus, at a time t satisfying $(g^2/\Delta)t=\pi$, we once again obtain the initial state back, i.e., $\exp(-i\ \pi)\cos\pi|g_1\rangle|e_2\rangle|0_k\rangle$. And in the lab frame, it becomes $\exp(-i\ \pi\omega_e\Delta/g^2)|g_1\rangle|e_2\rangle|0_k\rangle$.

(4) $|I_4\rangle=|e_1\rangle|e_2\rangle|0_k\rangle$

Once again it can be seen that this state will be coupled to $|g_1\rangle|e_2\rangle|1_k\rangle$, $|e_1\rangle|g_2\rangle|1_k\rangle$, and these states can be coupled to the state $|g_1\rangle|g_2\rangle|2_k\rangle$ states through the Hamiltonian under consideration. Thus, we can take

$$|\Psi_4\rangle = a(t)|e_1\rangle|e_2\rangle|0_k\rangle + b(t)|g_1\rangle|e_2\rangle|1_k\rangle + c(t)|e_1\rangle|g_2\rangle|1_k\rangle + d(t)|g_1\rangle|g_2\rangle|2_k\rangle.$$

The equations of motion can be written as

$$\dot{a}(t) = i\ g[b(t) + c(t)] - 2i\ \omega_e a(t),$$

$$\dot{b}(t) = -i\ \nu_k b(t) + i\ g[a(t) + d(t)] - i\ \omega_e b(t),$$

$$\dot{c}(t) = -i\ \nu_k c(t) + i\ g[a(t) + d(t)] - i\ \omega_e c(t),$$

$$\dot{d}(t) = -2i\ \nu_k d(t) + i\ g[b(t) + c(t)]. \tag{B10}$$

Once again moving to the rotated frame and sticking to the same notation, we obtain

$$\dot{a}(t) = i\ g[b(t) + c(t)],$$

$$\dot{b}(t) = i\ \Delta\ b(t) + i\ g[a(t) + d(t)],$$

$$\dot{c}(t) = i\ \Delta\ c(t) + i\ g[a(t) + d(t)],$$

$$\dot{d}(t) = 2i\ \Delta\ d(t) + i\ g[b(t) + c(t)]. \tag{B11}$$

Now with the assumption that $\Delta \gg g$, we can obtain at steady state

$$d(t) = -\frac{g}{2\Delta}[b(t) + c(t)]. \tag{B12}$$

By substituting the steady state value of $d(t)$ in the rate equations for $b(t)$ and $c(t)$ and solving them at steady state gives

$$b(t) = -\frac{g\Delta}{\Delta^2 - g^2}a(t) \tag{B13}$$

$$c(t) = -\frac{g\Delta}{\Delta^2 - g^2}a(t). \tag{B14}$$

Using these solutions, we obtain

$$\dot{a}(t) = -2i\frac{g^2\Delta}{\Delta^2 - g^2}a(t), \tag{B15}$$

which has a solution

$$a(t) = \exp\left(-2i\frac{g^2\Delta}{\Delta^2 - g^2}t\right) = \exp\left(-2i\frac{g^2}{\Delta}t\right). \tag{B16}$$

Here the last term is obtained by ignoring g^2 compared to Δ^2 in the denominator and simplifying. Thus at time t given by $g^2t/\Delta=\pi$ the final state is given by $|e_1\rangle|e_2\rangle|0_k\rangle$ without any phase factor. However, after undoing the transformation to the rotated frame it becomes $\exp(-i\ 2\pi\omega_e\Delta/g^2)|e_1\rangle|e_2\rangle|0_k\rangle$.

Thus it is clear that having these states as direct qubit combinations would not give us the phases needed to construct the quantum phase gate. We consider the effect of cavity interactions on some special states, where the target qubit, i.e., the second atom has been shifted to a state $|a\rangle$ whenever it starts with the excited state $|e\rangle$. This gives us the following possibilities for the two atom states: (i) $|g_1\rangle|g_2\rangle$, (ii) $|g_1\rangle|a_2\rangle$, (iii) $|e_1\rangle|g_2\rangle$, and (iv) $|e_1\rangle|a_2\rangle$. Now we arrange the level a such that it does not interact with the cavity. We have already seen the evolution of possibilities (i) and (iii). It only remains to be seen how the states (ii)$\equiv|g_1\rangle|a_2\rangle|0_k\rangle$ and (iv)$\equiv|e_1\rangle|a_2\rangle|0_k\rangle$ evolve. It is easy to see that the state (ii) does not evolve; however, the state (iv) can be shown to acquire a phase factor $\exp(-i\ \pi\omega_e\Delta/g^2)\exp(-i\ g^2t/\Delta)$, which for $\eta=g^2t/\Delta=\pi$ is -1. The extra phase factor $\exp(-i\ \pi\omega_e\Delta/g^2)$ can be eliminated trivially.

Using this, we propose our scheme for the two-qubit gates as discussed in the text. We choose the qubit states to be $|0\rangle$ and $|1\rangle$, which are not coupled to the cavity mode. Only when the interaction with the cavity is required (state-transfer pulses are employed, through the Hamiltonian in Eq. (6) to arrive from the initial qubit state to one of the (i), (ii), (iii), or (iv) states discussed above). Then the cavity interaction gives appropriate phase factors to the appropriate two-qubit states facilitating the quantum phase gate. Then the qubits are transferred back from the levels interacting with the cavity to the long-lived states $|0\rangle$ and $|1\rangle$. This justifies the five steps needed to perform the two-qubit operation as discussed in the text.

To summarize the results of the atom-cavity interaction, we see that

$$|g_1\rangle|g_2\rangle|0_k\rangle \to |g_1\rangle|g_2\rangle|0_k\rangle,$$

$$|e_1\rangle|g_2\rangle|0_k\rangle \to \exp(-i\ \eta t)\cos\eta t|e_1\rangle|g_2\rangle|0_k\rangle - i\exp(-i\ \eta t)\sin\eta t|g_1\rangle|e_2\rangle|0_k\rangle,$$

$$|g_1\rangle|e_2\rangle|0_k\rangle \to \exp(-i\ \eta t)\cos\eta t|g_1\rangle|e_2\rangle|0_k\rangle - i\exp(-i\ \eta t)\sin\eta t|e_1\rangle|g_2\rangle|0_k\rangle,$$

$$|e_1\rangle|e_2\rangle|0_k\rangle \to \exp(-2i\ \eta t)|e_1\rangle|e_2\rangle|0_k\rangle,$$

$$|g_1\rangle|a_2\rangle|0_k\rangle \to |g_1\rangle|a_2\rangle|0_k\rangle,$$

$$|e_1\rangle|a_2\rangle|0_k\rangle \to \exp(-i\ \eta t)|e_1\rangle|a_2\rangle|0_k\rangle,$$

where $\eta = g^2/\Delta$. Thus, for $\eta t = \pi$, we obtain

$$|g_1\rangle|g_2\rangle|0_k\rangle \to |g_1\rangle|g_2\rangle|0_k\rangle,$$

$$|e_1\rangle|g_2\rangle|0_k\rangle \to |e_1\rangle|g_2\rangle|0_k\rangle,$$

$$|g_1\rangle|e_2\rangle|0_k\rangle \to |g_1\rangle|e_2\rangle|0_k\rangle,$$

$$|e_1\rangle|e_2\rangle|0_k\rangle \to |e_1\rangle|e_2\rangle|0_k\rangle,$$

$$|g_1\rangle|a_2\rangle|0_k\rangle \to |g_1\rangle|a_2\rangle|0_k\rangle,$$

$$|e_1\rangle|a_2\rangle|0_k\rangle \to (-1)|e_1\rangle|a_2\rangle|0_k\rangle. \quad \text{(B17)}$$

In the text, the auxiliary level $|a_2\rangle$ is actually the qubit state $|1\rangle$, which is not coupled to the cavity.

This discussion also explains the results presented in Table I.

APPENDIX C: FIDELITY CALCULATION

In this appendix, we briefly discuss the fidelity calculation for the proposed nonlocal quantum phase gate.

We notice that when the initial state of the two qubits is $|e_c, g_t\rangle$, there could be decoherence during the time the atoms are interacting with the cavity. If level, $|e_c\rangle$ decays spontaneously. As we have discussed in the text, this state can be taken to be a metastable state of the atom, then decoherence does not really become an issue. Nevertheless, we carry out the analysis to see the possible effect of atomic decay and cavity decay on the gate fidelity.

Here we would like to estimate the effect of decoherence caused by spontaneous emission from the level $|e\rangle$ to level $|1\rangle$. It can be noted that by clever choice of the quantum numbers for level $|0\rangle$ one can suppress spontaneous emission decay from level $|e\rangle$ to $|0\rangle$. Therefore, we only need to consider spontaneous emission from level $|e\rangle$ to level $|1\rangle$. We should note that the initial state $|e_c, g_t, 0_k\rangle$ couples to the states $|g_c, e_t, 0_k\rangle$ and $|g_c, g_t, 1_k\rangle$. Noting that spontaneous emission affects the level $|e_t\rangle$ and the cavity decay κ affects the cavity state of $|1_k\rangle$, we need to include more states in our state space, namely, $|1_c, g_t, 0_k, \gamma\rangle$, $|g_c, 1_t, 0_k, \gamma\rangle$, and $|g_c, g_t, 0_k, \gamma\rangle$. We term them collectively as $|0_k, \gamma\rangle$, where γ corresponds to either the spontaneously emitted photon by the atoms or the photon decayed from the cavity. It is important to note that the state $|0_k, \gamma\rangle$ is dynamically decoupled from the rest of the states of interest. The decay mechanism is illustrated in Fig. 3. As it can be seen from Fig. 3, the state after the decay of the atoms of the cavity is dynamically decoupled from the atom-cavity interaction Hamiltonian; thus, studying the dynamics of the amplitude equations is sufficient, and complete density matrix treatment is not required. Let the general state of the two atoms and cavity state be

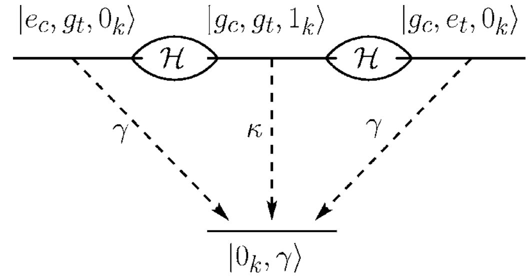

FIG. 3. The decoherence mechanism for the two-qubit gate during the interaction with the cavity through Hamiltonian $\mathcal{H}$. We note that the state after the spontaneous decay of the atom or the cavity decay is dynamically decoupled from the atom-cavity interaction Hamiltonian $\mathcal{H}$.

$$|\Psi\rangle = a(t)|e_c, g_t, 0_k\rangle + b(t)|g_c, e_t, 0_k\rangle + c(t)|g_c, g_t, 1_k\rangle + d(t) \times |g_c, g_t, 0_k, \gamma\rangle. \quad \text{(C1)}$$

The evolution of the state under the influence of the atom-cavity interaction Hamiltonian and the decay mechanisms gives the dynamical equations

$$\dot{a}(t) = -\frac{\gamma}{2}a(t) + i\Omega_c c(t),$$

$$\dot{b}(t) = -\frac{\gamma}{2}b(t) + i\Omega_c c(t),$$

TABLE II. Fidelity for various system parameters at time $t = \pi/\eta = \Delta\pi/\Omega_c^2$. All parameters are given in the units of Ω_c. Common parameters are $\Delta = 10\Omega_c$. We calculate the fidelity with the adiabatic elimination analytical results and through a complete numerical procedure without the adiabatic elimination of the $|1_k\rangle$ state of the cavity field.

γ	κ	F	F' (Complete Numerical)
0.001	0.1	0.939334	0.924248
0.01	0.1	0.707988	0.698862
0.1	0.1	0.0418878	0.0430447
0.1	0.01	0.0430785	0.0466712
0.01	0.01	0.728113	0.727542
0.001	0.01	0.966035	0.960429
0.001	0.001	0.968768	0.965277

$$\dot{c}(t) = \left(i\ \Delta - \frac{\kappa}{2}\right)c(t) + i\Omega_c[a(t) + b(t)]. \qquad \text{(C2)}$$

Once again, we adiabatically eliminate the state $|g_c, g_t, 1_k\rangle$ with the assumption that $\Delta \gg \Omega_c \gg \kappa$ to arrive at

$$c(t) = -\frac{\Omega_c}{\Delta + i\ \kappa/2}[a(t) + b(t)]. \qquad \text{(C3)}$$

Substituting $c(t)$ in the other equations, we obtain

$$\dot{a}(t) = -\frac{\gamma}{2}a(t) - i\frac{\Omega_c^2}{\Delta + i\ \kappa/2}[a(t) + b(t)],$$

$$\dot{b}(t) = -\frac{\gamma}{2}b(t) - i\frac{\Omega_c^2}{\Delta + i\ \kappa/2}[a(t) + b(t)]. \qquad \text{(C4)}$$

The solution of the above equations with the initial condition $a(0)=1$, $b(0)=0$ is given by

$$a(t) = \frac{1}{2}\exp\left(-\frac{t\gamma\Delta}{2\Delta + i\ \kappa}\right)\left[\exp\left(-i\frac{t\gamma\kappa}{2(2\Delta + i\ \kappa)}\right) + \exp\left(-i\frac{t(\gamma\kappa + 8\Omega^2)}{2(2\Delta + i\ \kappa)}\right)\right],$$

$$b(t) = -\frac{1}{2}\exp\left(-\frac{t\gamma\Delta}{2\Delta + i\ \kappa}\right)\left[\exp\left(-i\frac{t\gamma\kappa}{2(2\Delta + i\ \kappa)}\right) - \exp\left(-i\frac{t(\gamma\kappa + 8\Omega^2)}{2(2\Delta + i\ \kappa)}\right)\right]. \qquad \text{(C5)}$$

Now we choose several values for the parameters and determine the fidelity

$$F = |a(t)|^2. \qquad \text{(C6)}$$

The results for different values of γ and κ measured in the units of Ω_c are summarized in Table II.

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