PHYSICAL REVIEW A **72**, 050101(R) (2005)

Loophole-free Bell's experiments and two-photon all-versus-nothing violations of local realism

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(Received 27 July 2005; revised manuscript received 4 October 2005; published 17 November 2005)

We introduce an extended version of a previous all-versus-nothing proof of impossibility of Einstein-Podolsky-Rosen's local elements of reality for two photons entangled both in polarization and path degrees of freedom (A. Cabello, quant-ph/0507259), which leads to a Bell's inequality where the classical bound is 8 and the quantum prediction is 16. A simple estimation of the detection efficiency required to close the detection loophole using this extended version gives $\eta > 0.69$. This efficiency is lower than that required for previous proposals.

DOI: 10.1103/PhysRevA.72.050101 PACS number(s): 03.65.Ud, 03.67.Pp, 03.67.−a, 42.50.−p

If, from the result of one experiment, we can predict with certainty the result of a spacelike separated experiment, then, following Einstein, Podolsky, and Rosen (EPR) [1], there must be a local element of reality (LER) corresponding to the latter result. However, some predictions of local realistic theories are in conflict with those of quantum mechanics [2,3]. Experiments [4–6] have shown an excellent agreement with quantum mechanics and have provided solid evidence against LERs. So far, however, the results of these experiments still admit an interpretation in terms of LERs. A conclusive loophole-free experiment would require spacelike separation between the local experiments and a sufficiently large number of the prepared pairs' detections; otherwise, the possibility of communication at the speed of light between the particles cannot be excluded (locality loophole [7]), and neither can it be excluded that the nondetections correspond to local instructions like "if experiment *X* is performed, then do not activate the detector" (detection loophole [8]).

Photons are the best candidates for closing the locality loophole. For instance, the Innsbruck experiment [5] with polarization-entangled photons separated 400 m is not subject to the locality loophole; however, its detection efficiency ($\eta = 0.05$) was not high enough to close the detection loophole ($\eta \geqslant 0.83$ is required [9]). The detection efficiency for ions is much higher. For instance, in the Boulder experiment [6] with trapped beryllium ions, $\eta \approx 0.98$; however, the distance between ions (3 μm) was not enough to close the locality loophole.

There are several proposals for experiments for closing both loopholes [10]; however, most of them are very difficult to implement with current technology. The most promising approach for a loophole-free experiment is by using entangled photons and more efficient photodetectors [11]. Recent experiments with pairs of entangled photons have achieved $\eta = 0.33$ [12]. Closing the detection loophole with maximally entangled states and the Clauser-Horne-Shimony-Holt (CHSH) Bell's inequality [3] requires a detection efficiency $\eta > 2(\sqrt{2}-1) \approx 0.83$ [9]. By using nonmaximally entangled states and supplementary assumptions, η can be lowered to $\eta > 0.67$ [13]. However, these experiments are based on a different interpretation of EPR's condition for LERs [14].

The detection efficiency required for a loophole-free experiment on Bell's theorem of impossibility of EPR's LERs is related with the statistical strength of the proof tested in the experiment (i.e., with the amount of evidence against LERs provided by the corresponding experiment). In this respect, all-versus-nothing (AVN) proofs [15,16] provide stronger evidence against LERs than other proofs [17]. Specifically, a loophole-free experiment based on the three-observer version [18] of Greenberger, Horne, and Zeilinger's proof [15], would require $\eta > 0.75$ [19]. The negative side is that it requires three spacelike separated regions. The two-photon version [21–23] of the two-observer AVN proof [16] only requires a spacelike separation between two regions, but the detection efficiency needed for a loophole-free test is $\eta > 5/6 \approx 0.83$ [20].

In this paper we introduce a new AVN proof for two photons, and its corresponding Bell's inequality, which requires an efficiency $\eta > 0.69$ to close the detection loophole. This efficiency, although still higher than that achieved in recent experiments, is lower than that required for any previous proposal for a loophole-free experiment based on bipartite Bell's inequalities and the usual interpretation of EPR's condition. The new AVN proof is an extended version of a previous one [20].

Consider two photons entangled both in polarization and in path degrees of freedom [21–24] prepared in the state

$$|\psi\rangle = \tfrac{1}{2}(|Hu\rangle_1|Hu\rangle_2 + |Hd\rangle_1|Hd\rangle_2 + |Vu\rangle_1|Vu\rangle_2 - |Vd\rangle_1|Vd\rangle_2), \tag{1}$$

where $|H\rangle_j$ and $|V\rangle_j$ represent horizontal and vertical polarization, and $|u\rangle_j$ and $|d\rangle_j$ denote two orthonormal path states for photon-j. Consider also 6 local observables on photon-j: three for polarization degrees of freedom, defined by the operators

$$X_j = |H\rangle_j\langle V| + |V\rangle_j\langle H|, \tag{2}$$

$$Y_j = i(|V\rangle_j\langle H| - |H\rangle_j\langle V|), \tag{3}$$

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$$Z_j = |H\rangle_j\langle H| - |V\rangle_j\langle V|, \tag{4}$$

and three for path degrees of freedom,

$$x_j = |u\rangle_j\langle d| + |d\rangle_j\langle u|, \tag{5}$$

$$y_j = i(|d\rangle_j\langle u| - |u\rangle_j\langle d|), \tag{6}$$

$$z_j = |u\rangle_j\langle u| - |d\rangle_j\langle d|. \tag{7}$$

Each of these observables can take two values: −1 or 1. Each observer randomly chooses to measure either a polarization observable, a path observable, or a polarization observable and a path observable on his/her photon. The choice of measurement and the measurement itself on photon-1 are assumed to be spacelike separated from those on photon-2.

We will prove that these observables satisfy EPR's condition for LER, namely, "*if, without in any way disturbing a system, we can predict with certainty (i.e., with probability equal to unity) the value of a physical quantity, then there exists an element of physical reality corresponding to this physical quantity*" [1]. Z_1 and z_1 (Z_2 and z_2) are EPR's LERs because their values can be predicted with certainty from spacelike separated measurements of Z_2 and z_2 (Z_1 and z_1), respectively, because state (1) satisfies the following equations:

$$Z_1Z_2|\psi\rangle = |\psi\rangle, \tag{8}$$

$$z_1z_2|\psi\rangle = |\psi\rangle. \tag{9}$$

X_1 and x_1 (X_2 and x_2) are EPR's LERs because their values can be predicted with certainty from spacelike separated measurements of X_2z_2 and Z_2x_2 (X_1z_1 and Z_1x_1), respectively, because state [1] satisfies

$$X_1X_2z_2|\psi\rangle = |\psi\rangle, \tag{10}$$

$$x_1Z_2x_2|\psi\rangle = |\psi\rangle, \tag{11}$$

$$X_1z_1X_2|\psi\rangle = |\psi\rangle, \tag{12}$$

$$Z_1x_1x_2|\psi\rangle = |\psi\rangle. \tag{13}$$

Analogously, Y_1 and y_1 (Y_2 and y_2) are EPR's LERs because state (1) satisfies

$$Y_1Y_2z_2|\psi\rangle = -|\psi\rangle, \tag{14}$$

$$y_1Z_2y_2|\psi\rangle = -|\psi\rangle, \tag{15}$$

$$Y_1z_1Y_2|\psi\rangle = -|\psi\rangle, \tag{16}$$

$$Z_1y_1y_2|\psi\rangle = -|\psi\rangle. \tag{17}$$

We will prove that two compatible observables on the same photon, like X_1 and z_1, are independent EPR's LERs in the sense that the measurement of one of them does not change the value of the other (and therefore there is no need for any additional assumptions beyond EPR's condition for LER itself; see [25] for a similar discussion). A suitable measurement of X_1 does not change $v(x_1)$ because $v(x_1)$ can be predicted with certainty from a spacelike separated measurement of Z_2 and x_2, see Eq. (11), and this prediction is not affected by whether X_1 is measured before x_1, or X_1 and x_1 are jointly measured. Therefore, EPR's condition is enough to guarantee that x_1 has a LER [i.e., a value $v(x_1)$] which does not change with a measurement of X_1. A similar reasoning applies to any other local observable involved in the proof.

In addition, state (1) satisfies the following equations:

$$X_1x_1Y_2y_2|\psi\rangle = |\psi\rangle, \tag{18}$$

$$X_1y_1Y_2x_2|\psi\rangle = |\psi\rangle, \tag{19}$$

$$Y_1x_1X_2y_2|\psi\rangle = |\psi\rangle, \tag{20}$$

$$Y_1y_1X_2x_2|\psi\rangle = |\psi\rangle. \tag{21}$$

To be consistent with Eqs. (10)–(21), local realistic theories predict the following relations between the values of the LERs:

$$v(X_1) = v(X_2)v(z_2), \tag{22}$$

$$v(x_1) = v(Z_2)v(x_2), \tag{23}$$

$$v(X_1)v(z_1) = v(X_2), \tag{24}$$

$$v(Z_1)v(x_1) = v(x_2), \tag{25}$$

$$v(Y_1) = -v(Y_2)v(z_2), \tag{26}$$

$$v(y_1) = -v(Z_2)v(y_2), \tag{27}$$

$$v(Y_1)v(z_1) = -v(Y_2), \tag{28}$$

$$v(Z_1)v(y_1) = -v(y_2), \tag{29}$$

$$v(X_1)v(x_1) = v(Y_2)v(y_2), \tag{30}$$

$$v(X_1)v(y_1) = v(Y_2)v(x_2), \tag{31}$$

$$v(Y_1)v(x_1) = v(X_2)v(y_2), \tag{32}$$

$$v(Y_1)v(y_1) = v(X_2)v(x_2). \tag{33}$$

However, it is impossible to assign the values −1 or 1 to the observables in a way consistent with Eqs. (22)–(33), and therefore the predictions of quantum mechanics cannot be reproduced by EPR's LERs. Indeed, the assignation is impossible even for each of eight possible subsets of four equations. For instance, the product of Eqs. (22) and (26) [or the product of Eqs. (24) and (28)] leads to $v(X_1)v(Y_1) = -v(X_2)v(Y_2)$; while the product of Eqs. (30) and (32) [or the product of Eqs. (31) and (33)] leads to $v(X_1)v(Y_1) = v(X_2)v(Y_2)$. Analogously, the product of Eqs. (23) and (27) [or the product of Eqs. (25) and (29)] leads to $v(x_1)v(y_1)$

$=-v(x_2)v(y_2)$; while the product of Eqs. (30) and (31) [or the product of Eqs. (32) and (33)] leads to $v(x_1)v(y_1)=v(x_2)v(y_2)$. Note that if we explicitly write down the eight sets, the four Eqs. (30)–(33) would appear twice as frequently as the eight Eqs. (22)–(29).

In a real experiment, measurements are imperfect and the observed correlation functions fail to attain the values assumed in the ideal case. Therefore, it is convenient to translate the contradiction of the AVN proof into a Bell's inequality. This inequality naturally follows from the observation that the relevant features of the AVN proof derive from the fact that state (1) is an eigenstate of the operator

$$\beta = X_1X_2z_2 + x_1Z_2x_2 + X_1z_1X_2 + Z_1x_1x_2 - Y_1Y_2z_2 - y_1Z_2y_2 - Y_1z_1Y_2 - Z_1y_1y_2 + 2X_1x_1Y_2y_2 + 2X_1y_1Y_2x_2 + 2Y_1x_1X_2y_2 + 2Y_1y_1X_2x_2. \tag{34}$$

As can be easily checked, in any model based on LERs the expected value of β must satisfy

$$|\langle\beta\rangle| \leq 8. \tag{35}$$

However, the quantum prediction for the state (1) is

$$\langle\psi|\beta|\psi\rangle = 16, \tag{36}$$

which is indeed the maximum possible violation of inequality (35). The difference between the maximal violation of the Bell's inequality and its upper bound is $16-8=8$ for the inequality presented here, while it is just $2\sqrt{2}-2\approx 0.8$ for the CHSH inequality [3], $4-2=2$ for the three-qubit version of Mermin's inequality [26], and $9-7=2$ for the Bell's inequality derived from the two-observer AVN proof [16].

The simplest way to estimate the detection efficiency required to avoid the detection loophole for a Bell experiment based on this AVN proof, and a good estimation of the required efficiency for a test of the inequality (35), is to see it as a game in the spirit of Vaidman's game [27] and Brassard's "quantum pseudo-telepathy" [28]. Consider a team of two players, Alice and Bob, each of them isolated in a booth. Each of them is asked one out of eight possible questions: (i) "What are $v(X)$ and $v(z)$?," (ii) "What are $v(Z)$ and $v(x)$?," (iii) "What are $v(Y)$ and $v(z)$?," (iv) "What are $v(Z)$ and $v(y)$?," (v) "What are $v(X)$ and $V(x)$?," (vi) "What are $v(X)$ and $v(y)$?," (vii) "What are $v(Y)$ and $v(x)$?," and (viii) "What are $v(Y)$ and $v(y)$?" If one player is asked a question from (i) to (iv), then the other is asked the same question; if one is asked (v), the other is asked (viii); if one is asked (vi), the other is asked (vii). Therefore, the possible scenarios are (i)–(i), meaning that both Alice and Bob are asked (i), (ii)–(ii), (iii)–(iii), (iv)–(iv), (v)–(viii), (vi)–(vii), (vii)–(vi), and (viii)–(v). Each player must give one of the following answers: "-1 and -1," "-1 and 1," "1 and -1," or "1 and 1." Since $v(X)$ represents a LER, Alice's answer to "What is $v(X)$?" must be the same regardless of the scenario in which is asked. The same applies for all 12 LERs used in the game. Alice and Bob win if the product of the answers satisfies the corresponding equation in Eqs. (22)–(33). Let us assume that all questions are asked with the same frequency. This is equivalent to assuming that, from the 12 possible scenarios considered in Eqs. (22)–(33), those of Eqs. (30)–(33) occur twice as frequently than those of Eqs. (22)–(29). Assuming this, it is easy to see that an optimal classical strategy allows the players to win this game in 3/4 of the rounds. For instance, a simple optimal classical strategy is that the players always use the following set of local answers:

$$G := \left\{ \begin{array}{cc|cc} v(X_1) & v(x_1) & v(X_2) & v(x_2) \\ v(Y_1) & v(y_1) & v(Y_2) & v(y_2) \\ v(Z_1) & v(z_1) & v(Z_2) & v(z_2) \end{array} \right\} = \left\{ \begin{array}{cc|cc} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{array} \right\}. \tag{37}$$

This strategy always wins except for scenarios (iii)–(iii) and (iv)–(iv) [i.e., it satisfies all Eqs. (22)–(33), except Eqs. (26)–(29)]. However, the players can win all the rounds if they share pairs of photons in the state (1) and give as answers the results of the corresponding measurements [i.e., if one is asked question (i), he/she gives as answers the results of measuring X and z on his/her photon].

In a real experiment to test the quantum predictions, the low efficiency of detectors opens the possibility that nondetections correspond to local instructions like "if X is measured, then the photon will not activate the detector." This allows us to construct a model with local instructions which simulates the observed data by taking advantage of those rounds in which one photon goes undetected.

Therefore, to estimate the efficiency required for a loophole-free test consider a modified version of the previous game, including the possibility of each player not answering in a fraction $1-\eta$ of the rounds. If any of the players gives no answers, that round is not taken into account. This new rule opens the possibility of the players also sharing a fraction of sets of local instructions like

$$B_1 := \left\{ \begin{array}{cc|cc} 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{array} \right\}, \tag{38}$$

or

$$B_2 := \left\{ \begin{array}{cc|cc} 1 & 1 & 1 & 1 \\ 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 \end{array} \right\}, \tag{39}$$

where the 0s in B_1 means that, if Alice and Bob are using a set B_1, Alice will not give any answer to questions which include "What is $v(Y_1)$?" or "What is $v(y_1)$?," i.e., to questions (iii), (iv), (vi), (vii), and (viii). Analogously, the 0s in B_2 means that, if Alice and Bob are using a set B_2, Bob will not answer questions which include "What is $v(Y_2)$?" or "What is $v(y_2)$?"

Suppose the players are using sets of predefined answers (i.e., suppose the observed data are adequately described by a local realistic theory). For instance, sets like G with a frequency $1-p$, sets like B_1 with a frequency $p/2$, and sets like B_2 with a frequency $p/2$. This p is related to the efficiency of the photodetector corresponding to photon-j, η_j, by the relation

$$\eta_j = 1 - p + \frac{p}{2} f_j + \frac{p}{2}, \tag{40}$$

where f_1 (f_2) is the probability that Alice (Bob) answers [i.e., she (he) does not get the instruction 0 in her (his) set] when they are using a B_1 (B_2) set. In our case, $f_j = 3/8$.

Let us calculate the minimum detection efficiency required to discard the possibility that nature is using this particular set of predefined answers. To emulate the quantum probability of winning the game ($P_Q = 1$ in our case), the minimum p must satisfy

$$P_Q = (1-p) P_G + \frac{p}{2} P_{B_1} + \frac{p}{2} P_{B_2}, \tag{41}$$

where P_G is the probability of winning the game when the players use a G set, and P_{B_j} is the probability of winning when the players use a B_j set and both answer the questions. In our example, $P_G = 3/4$ and $P_{B_j} = 1$. Introducing the values in Eqs. (40) and (41), we arrive at the conclusion that our model can simulate the quantum predictions if $\eta_j \leqslant 11/16 \approx 0.69$. An exhaustive examination of all possible sets like G and B_j shows that the previously presented model is indeed optimal and, therefore, we conclude that LERs cannot simulate the quantum predictions if

$$\eta_j > 11/16. \tag{42}$$

If we do a similar analysis for a similar game based only on Eqs. (22), (26), (30), and (32), we arrive at the conclusion that closing the detection hole in this case would require $\eta_j > 3/4$ [20].

This work was sparked by some experiments made in Hefei [22] and Rome [23] and a talk given by P.G. Kwiat in Vienna, and was initiated during the Benasque Center for Science Quantum Information Workshop. The author thanks A. Broadbent, E. Galvão, P.G. Kwiat, A. Lamas-Linares, J.-Å. Larsson, C.-Y. Lu, S. Lu, E. Santos, and H. Weinfurter for useful comments, and acknowledges support by Projects Nos. BFM2002-02815 and FQM-239.

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