Multiparty quantum-state sharing of an arbitrary two-particle state with Einstein-Podolsky-Rosen pairs

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A scheme for multiparty quantum state sharing of an arbitrary two-particle state is presented with Einstein-Podolsky-Rosen pairs. Any one of the N agents has the access to regenerate the original state with two local unitary operations if he collaborates with the other agents, say the controllers. Moreover, each of the controllers is required to take only a product measurement $\sigma_x \otimes \sigma_x$ on his two particles, which makes this scheme more convenient for the agents in the applications on a network than others. As all the quantum source can be used to carry the useful information, the intrinsic efficiency of qubits approaches the maximal value. With a new notation for the multipartite entanglement, the sender need only publish two bits of classical information for each measurement, which reduces the information exchanged largely.

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Quantum secret sharing (QSS), an important branch of quantum communication, is the generalization of classical secret sharing [1] into quantum scenario and has attracted a lot of attention [2–14]. There are three main goals in QSS: (1) it is used to distribute a private key among many users [2–8], similar to quantum key distribution (QKD) [15]; (2) it is a tool for sharing a classical secret directly [2–4,9,10], similar to quantum secure direct communication (QSDC) [16,17]; (3) it provides a secure way for sharing a quantum information (an unknown quantum state) [9–12], similar to the controlled teleportation [18–20]. Most existing QSS schemes are focused on creating a private key among several parties or splitting a classical secret. For example, an original QSS scheme [2] was proposed by Hillery, Bužek, and Berthiaume (HBB) in 1999 by using a three-particle or a four-particle Greenberger-Horne-Zeilinger (GHZ) state for distributing a private key among some agents and sharing a classical information.

In recent, an interest work was done by Li *et al.* [11] for sharing an unknown single qubit with a multipartite joint measurement. In their QSS protocol, the sender splits a qubit into m pieces for the m agents with m Einstein-Podolsky-Rosen (EPR) pairs, and any one of the agents can obtain the qubit with the help of the other agents. In 2004, Lance *et al.* [12] named the branch of quantum secret sharing for quantum information "quantum-state sharing" (QSTS). By far, there are no models for sharing an arbitrary multipartite state. In this paper, we will present a way for sharing an arbitrary two-particle state with $2N$ EPR pairs. Any one in the N agents can regenerate the original state when he collaborates with the others, say the controllers. Moreover, the controllers need only perform the single-particle measurements on their particles, and the receiver can reconstruct the original state with two local unitary operations.

The basic idea of this QSTS for an arbitrary two-particle state with two agents is shown in Fig. 1. Alice is the sender, Bob and Charlie are the two agents. Suppose that the unknown arbitrary two-particle state is described as

$$|\Phi\rangle_{xy} = \alpha|00\rangle_{xy} + \beta|01\rangle_{xy} + \gamma|10\rangle_{xy} + \delta|11\rangle_{xy}, \tag{1}$$

where x and y are the two particles in the state $|\Phi\rangle_{xy}$, and

$$|\alpha|^2 + |\beta|^2 + |\gamma|^2 + |\delta|^2 = 1. \tag{2}$$

At first, Alice shares the four EPR pairs a_1b_1, c_1d_1, a_2b_2 and c_2d_2 with Bob and Charlie, respectively. Here a_1 and b_1 are the two particles in an EPR pair, and similar notations for the other EPR pairs. An EPR pair is in one of the four Bell states shown as follows [23]:

$$|\psi^{\pm}\rangle = (1/\sqrt{2})(|0\rangle|1\rangle \pm |1\rangle|0\rangle),$$

$$|\phi^{\pm}\rangle = (1/\sqrt{2})(|0\rangle|0\rangle \pm |1\rangle|1\rangle), \tag{3}$$

where $|0\rangle$ and $|1\rangle$ are the eigenvectors of the operator σ_z. Without loss of generalization, we assume that all the EPR pairs are originally in the entangled state $|\phi^+\rangle = 1/\sqrt{2}(|0\rangle|0\rangle + |1\rangle|1\rangle)$.

Before the measurement, the state of the composite quantum system composed of the ten particles is

$$|\Psi\rangle_s \equiv |\Phi\rangle_{xy}|\phi^+\rangle_{a_1b_1}|\phi^+\rangle_{c_1d_1}|\phi^+\rangle_{a_2b_2}|\phi^+\rangle_{c_2d_2}. \tag{4}$$

Alice performs the three-particle GHZ state joint measurement M_1 on the particles x, a_1, and a_2 first, and then the M_2 on the particles y, c_1 and c_2. Bob takes the product measurement $M_B = \sigma_x \otimes \sigma_x$ on the particles b_1d_1, and then Charlie can recover the original state $|\Phi\rangle_{xy}$ with two local unitary operations $U_C = U_b \otimes U_d$ according to the results obtained by Alice and Bob, see Fig. 1.

Let us use an example to demonstrate the principle of this QSTS protocol with one controller. First, we introduce a new notation for the three-particle GHZ states.

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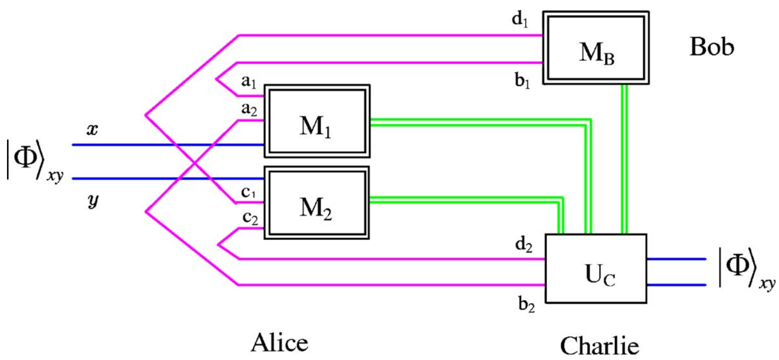

FIG. 1. (Color online) Multiparty quantum secret sharing for an arbitrary two-particle state with two agents. The single lines denote qubits, double lines denote classical data, similar to Ref. [21]. M_1, M_2 are the GHZ-state joint measurements on the particles xa_1a_2 and yc_1c_2, respectively; M_B is the product measurement $\sigma_x \otimes \sigma_x$ on the particles b_1d_1.

$$|G_{ij+}\rangle = (1/\sqrt{2})(|0ij\rangle + |1\bar{i}\bar{j}\rangle), \quad |G_{ij-}\rangle = (1/\sqrt{2})(|0ij\rangle - |1\bar{i}\bar{j}\rangle), \tag{5}$$

where $i,j \in \{0,1\}$, $\bar{i}=1-i$ and $\bar{j}=1-j$.

Suppose Alice gets the results $R_{xa_1a_2}=R_{yc_1c_2}=|G_{00+}\rangle$, which will occurs with the probability $\frac{1}{8}\times\frac{1}{8}=\frac{1}{64}$, then the state of the subsystem with the particles b_1, d_1, b_2, and d_2 becomes

$$|\Psi\rangle_{\rm sub} = \alpha|00\rangle_{b_1d_1}|00\rangle_{b_2d_2} + \beta|01\rangle_{b_1d_1}|01\rangle_{b_2d_2} + \gamma|10\rangle_{b_1d_1}|10\rangle_{b_2d_2} + \delta|11\rangle_{b_1d_1}|11\rangle_{b_2d_2}. \tag{6}$$

That is, the information of the state $|\Phi\rangle_{xy}$ is transferred to the state of the subsystem shared between Bob and Charlie. If they want to recover the quantum information $|\Phi\rangle_{xy}$, one of them performs $\sigma_x \otimes \sigma_x$ on his/her two particles and the other takes two local unitary operations on the two particles remained according to the information provided by the first one. For example, let us assume that Bob performs the $\sigma_x \otimes \sigma_x$ measurement on his two particles, and Charlie will reconstruct the original state when she collaborates with Bob. We can rewrite the state $|\Psi\rangle_{\rm sub}$ as

$$\begin{aligned}|\Psi\rangle_{\rm sub} = \tfrac{1}{2}[&|+x\rangle_{b_1}|+x\rangle_{d_1}(\alpha|00\rangle_{b_2d_2} + \beta|01\rangle_{b_2d_2} + \gamma|10\rangle_{b_2d_2} \\ &+ \delta|11\rangle_{b_2d_2}) + |+x\rangle_{b_1}|-x\rangle_{d_1}(\alpha|00\rangle_{b_2d_2} - \beta|01\rangle_{b_2d_2} \\ &+ \gamma|10\rangle_{b_2d_2} - \delta|11\rangle_{b_2d_2}) + |-x\rangle_{b_1}|+x\rangle_{d_1}(\alpha|00\rangle_{b_2d_2} \\ &+ \beta|01\rangle_{b_2d_2} - \gamma|10\rangle_{b_2d_2} - \delta|11\rangle_{b_2d_2}) + |-x\rangle_{b_1}| \\ &-x\rangle_{d_1}(\alpha|00\rangle_{b_2d_2} - \beta|01\rangle_{b_2d_2} - \gamma|10\rangle_{b_2d_2} + \delta|11\rangle_{b_2d_2})],\end{aligned} \tag{7}$$

where $|+x\rangle=(1/\sqrt{2})(|0\rangle+|1\rangle)$ and $|-x\rangle=(1/\sqrt{2})(|0\rangle-|1\rangle)$ are the two eigenstates of the measuring basis σ_x. Provided that Bob agrees to cooperate with Charlie, Charlie can recover the unknown state by performing the unitary operations $U_0 \otimes U_0$, $U_0 \otimes U_1$, $U_1 \otimes U_0$, and $U_1 \otimes U_1$ on the particles b_2 and d_2 if the outcomes obtained by Bob are $|+x\rangle_{b_1}|+x\rangle_{b_2}$, $|+x\rangle_{b_1}|-x\rangle_{b_2}$, $|-x\rangle_{b_1}|+x\rangle_{b_2}$, and $|-x\rangle_{b_1}|-x\rangle_{b_2}$, respectively. Here $U_0 \equiv I$, $U_1 \equiv \sigma_z$, $U_2 \equiv \sigma_x$ and $U_3 \equiv i\sigma_y$, and I is the identity matrix and $\sigma_i (i=x,y,z)$ are the Pauli matrices.

For the other cases, the relation between the results of the measurements done by Alice and Bob and the local unitary operations with which Charlie reconstructs the unknown

TABLE I. The relation between the local unitary operations and the results $R_{xa_1a_2}$, $R_{yc_1c_2}$, R_{b_1}, and R_{d_1}. $\Phi_{b_2d_2}$ is the state of the two particles hold in the hand of Charlie after all the measurements are done by Alice and Bob; U_C are the local unitary operations with which Charlie can reconstruct the unknown state $|\Phi\rangle_{xy}$.

$V_{xa_1a_2}$	$V_{yc_1c_2}$	$P_{xa_1a_2}\otimes P_{b_1}$	$P_{yc_1c_2}\otimes P_{d_1}$	$\Phi_{b_2d_2}$	U_C
0	0	+	+	$\alpha\|00\rangle+\beta\|01\rangle+\gamma\|10\rangle+\delta\|11\rangle$	$U_0\otimes U_0$
0	0	+	−	$\alpha\|00\rangle-\beta\|01\rangle+\gamma\|10\rangle-\delta\|11\rangle$	$U_0\otimes U_1$
0	0	−	+	$\alpha\|00\rangle+\beta\|01\rangle-\gamma\|10\rangle-\delta\|11\rangle$	$U_1\otimes U_0$
0	0	−	−	$\alpha\|00\rangle-\beta\|01\rangle-\gamma\|10\rangle+\delta\|11\rangle$	$U_1\otimes U_1$
0	1	+	+	$\alpha\|01\rangle+\beta\|00\rangle+\gamma\|11\rangle+\delta\|10\rangle$	$U_0\otimes U_2$
0	1	+	−	$\alpha\|01\rangle-\beta\|00\rangle+\gamma\|11\rangle-\delta\|10\rangle$	$U_0\otimes U_3$
0	1	−	+	$\alpha\|01\rangle+\beta\|00\rangle-\gamma\|11\rangle-\delta\|10\rangle$	$U_1\otimes U_2$
0	1	−	−	$\alpha\|01\rangle-\beta\|00\rangle-\gamma\|11\rangle+\delta\|10\rangle$	$U_1\otimes U_3$
1	0	+	+	$\alpha\|10\rangle+\beta\|11\rangle+\gamma\|00\rangle+\delta\|01\rangle$	$U_2\otimes U_0$
1	0	+	−	$\alpha\|10\rangle-\beta\|11\rangle+\gamma\|00\rangle-\delta\|01\rangle$	$U_2\otimes U_1$
1	0	−	+	$\alpha\|10\rangle+\beta\|11\rangle-\gamma\|00\rangle-\delta\|01\rangle$	$U_3\otimes U_0$
1	0	−	−	$\alpha\|10\rangle-\beta\|11\rangle-\gamma\|00\rangle+\delta\|01\rangle$	$U_3\otimes U_1$
1	1	+	+	$\alpha\|11\rangle+\beta\|10\rangle+\gamma\|01\rangle+\delta\|00\rangle$	$U_2\otimes U_2$
1	1	+	−	$\alpha\|11\rangle-\beta\|10\rangle+\gamma\|01\rangle-\delta\|00\rangle$	$U_2\otimes U_3$
1	1	−	+	$\alpha\|11\rangle+\beta\|10\rangle-\gamma\|01\rangle-\delta\|00\rangle$	$U_3\otimes U_2$
1	1	−	−	$\alpha\|11\rangle-\beta\|10\rangle-\gamma\|01\rangle+\delta\|00\rangle$	$U_3\otimes U_3$

quantum information $|\Phi\rangle_{xy}$ is shown in Table I. Here $V_{xa_1a_2}$ and $V_{yc_1c_2}$ represents the bit value of the results of the GHZ state joint measurements on xa_1a_2 and yc_1c_2, respectively. Define

$$V_{|G_{ij\pm}\rangle} \equiv j, \quad P_{|G_{ij\pm}\rangle} \equiv \pm, \quad P_{|\pm x\rangle} \equiv \pm, \tag{8}$$

where $i,j \in \{0,1\}$. In detail, $V_{xa_1a_2}=1$ and $P_{xa_1a_2}=-$ if the result of the three-particle joint measurement on particles xa_1a_2 is $R_{xa_1a_2}=|G_{01-}\rangle$ or $R_{xa_1a_2}=|G_{11-}\rangle$; $P_{b_1}=-$ when $R_{b_1}=|-x\rangle$. $U_i\otimes U_j$ means that Charlie performs U_i and U_j on the two particles b_2 and d_2, respectively, here $i,j=0,1,2,3$.

Table I shows that the unknown state $|\Phi\rangle_{xy}$ can be shared by Bob and Charlie completely, and they can reconstruct the state with two single-qubit measurements along the x direction and two local unitary operations. They need not do Bell state measurement on the particles, which makes this QSTS protocol more convenient for the agents than that in Ref. [20]. Moreover, Alice needs only to publish two bits of classical information for her agents to recover the state $|\Phi\rangle_{xy}$.

It is straightforwardly to generalize this QSTS scheme to the case with N agents, say Bob$_i(i=1,2,\ldots,N-1)$ and Charlie. As the symmetry, we still assume that Charlie is the agent who will recover the unknown state with the help of the $N-1$ controllers, Bob. For the end, Alice should share $2N$ EPR pairs $|\psi\rangle_{a_ib_i}$ and $|\psi\rangle_{c_id_i}$, $(i=1,2,\ldots,N)$ with the N agents. In this time, the state of the composite quantum system is

$$|\Phi\rangle_S \equiv |\Phi\rangle_{xy}\prod_{i=1}^{N} \otimes |\phi^+\rangle_{a_ib_i}\otimes|\phi^+\rangle_{c_id_i}. \tag{9}$$

Define a set of orthogonal vectors as

$$|G_{\underbrace{ij\ldots k}_{N}+}\rangle = \frac{1}{\sqrt{2}}(|0\underbrace{ij\ldots k}_{N}\rangle + |1\underbrace{\bar{i}\bar{j}\ldots\bar{k}}_{N}\rangle),$$

$$|G_{\underbrace{ij\ldots k}_{N}-}\rangle = \frac{1}{\sqrt{2}}(|0\underbrace{ij\ldots k}_{N}\rangle - |1\underbrace{\bar{i}\bar{j}\ldots\bar{k}}_{N}\rangle), \tag{10}$$

where $i,j,k\in\{0,1\}$, $\bar{i},\bar{j}$ and $\bar{k}$ are the counterparts of the binary numbers i, j, and k, respectively.

In the quantum communication, Alice performs first the joint measurement on the $N+1$ particles x, $a_1,\ldots$, and a_N, then on the $N+1$ particles y, $c_1,\ldots$, and c_N. When the agents want to reconstruct the unknown state $|\Phi\rangle_{xy}$, each of the controllers, Bob$_i$ performs $\sigma_x\otimes\sigma_x$ on his two particles b_i and d_i, i.e.,

$$|\Phi\rangle_S = \Psi_{xa_1\ldots a_N}\otimes\Psi_{yc_1\ldots c_N}\otimes\left(\prod_{i=1}^{N}\Psi_{b_i}\right)\otimes\left(\prod_{i=1}^{N}\Psi_{d_i}\right), \tag{11}$$

where

$$\Psi_{xa_1\ldots a_N},\Psi_{yc_1\ldots c_N}\in\{|G_{\underbrace{ij\ldots k}_{N}+}\rangle,|G_{\underbrace{ij\ldots k}_{N}-}\rangle\}$$

are the results of the joint measurements done by Alice. In more detail, the state of the quantum system (without being normalized) can be rewritten as

$$\begin{aligned}|\Psi\rangle_S = \sum_{\substack{i,j,\ldots,k\\m,n,\ldots,l}}\{&|G_{ij\cdots k+}\rangle|G_{mn\cdots l+}\rangle(\alpha|ij\cdots k\rangle|mn\cdots l\rangle\\
&+\beta|ij\cdots k\rangle|\bar{m}\bar{n}\cdots\bar{l}\rangle+\gamma|\bar{i}\bar{j}\cdots\bar{k}\rangle|mn\cdots l\rangle+\delta|\bar{i}\bar{j}\cdots\bar{k}\rangle\\
&\times|\bar{m}\bar{n}\cdots\bar{l}\rangle)+|G_{ij\cdots k+}\rangle|G_{mn\cdots l-}\rangle(\alpha|ij\cdots k\rangle|mn\cdots l\rangle\\
&-\beta|ij\cdots k\rangle|\bar{m}\bar{n}\cdots\bar{l}\rangle+\gamma|\bar{i}\bar{j}\cdots\bar{k}\rangle|mn\cdots l\rangle-\delta|\bar{i}\bar{j}\cdots\bar{k}\rangle\\
&\times|\bar{m}\bar{n}\cdots\bar{l}\rangle)+|G_{ij\cdots k-}\rangle|G_{mn\cdots l+}\rangle(\alpha|ij\cdots k\rangle|mn\cdots l\rangle\\
&+\beta|ij\cdots k\rangle|\bar{m}\bar{n}\cdots\bar{l}\rangle-\gamma|\bar{i}\bar{j}\cdots\bar{k}\rangle|mn\cdots l\rangle-\delta|\bar{i}\bar{j}\cdots\bar{k}\rangle\\
&\times|\bar{m}\bar{n}\cdots\bar{l}\rangle)+|G_{ij\cdots k-}\rangle|G_{mn\cdots l-}\rangle(\alpha|ij\cdots k\rangle|mn\cdots l\rangle\\
&-\beta|ij\cdots k\rangle|\bar{m}\bar{n}\cdots\bar{l}\rangle-\gamma|\bar{i}\bar{j}\cdots\bar{k}\rangle|mn\cdots l\rangle+\delta|\bar{i}\bar{j}\cdots\bar{k}\rangle\\
&\times|\bar{m}\bar{n}\cdots\bar{l}\rangle)\},\end{aligned} \tag{12}$$

where $|i\rangle=(1/\sqrt{2})[|+x\rangle+(-1)^i|-x\rangle]\{i,j,\ldots,k,m,n,\ldots,l\}$ are $2N$ binary numbers, and $\bar{m}$ is the counterpart of m, i.e., $\bar{m}=1-m$. As the symmetry, the measurements done by the controllers can be expressed by the operation M,

$$M=[(\langle+x|)^{N-1-t}(\langle-x|)^t]_1\otimes[(\langle+x|)^{N-1-q}(\langle-x|)^q]_2, \tag{13}$$

where $[(\langle+x|)^{N-1-t}(\langle-x|)^t]_1$ is the measurement operation related to the state of the quantum subsystem b_i (i.e, $\Pi_{i=1}^N\Psi_{b_i}$), and $[(\langle+x|)^{N-1-q}(\langle-x|)^q]_2$ is related to d_i, t and q are the numbers that the controllers obtain the result $|-x\rangle$ when they measure the particle b_i and d_i, respectively. After the measurements done by Alice and the $N-1$ controllers, the relation between the state of the particles b_Nd_N and the results of the measurements can be expressed as

$$\begin{aligned}M|\Psi\rangle_S=\sum_{\substack{i,j,\ldots,k\\m,n,\ldots,l}}\{&|G_{ij\cdots k+}\rangle|G_{mn\cdots l+}\rangle\otimes e^{-\theta_1}(\alpha|kl\rangle+(-1)^q\beta|k\bar{l}\rangle\\
&+(-1)^t\gamma|\bar{k}l\rangle+(-1)^{t+q}\delta|\bar{k}\bar{l}\rangle)+|G_{ij\cdots k+}\rangle|G_{mn\cdots l-}\rangle\\
&\otimes e^{-\theta_2}(\alpha|kl\rangle+(-1)^{q+1}\beta|k\bar{l}\rangle+(-1)^t\gamma|\bar{k}l\rangle\\
&+(-1)^{t+q+1}\delta|\bar{k}\bar{l}\rangle)+|G_{ij\cdots k-}\rangle|G_{mn\cdots l+}\rangle\otimes e^{-\theta_3}(\alpha|kl\rangle\\
&+(-1)^q\beta|k\bar{l}\rangle+(-1)^{t+1}\gamma|\bar{k}l\rangle+(-1)^{t+q+1}\delta|\bar{k}\bar{l}\rangle)\\
&+|G_{ij\cdots k-}\rangle|G_{mn\cdots l-}\rangle\otimes e^{-\theta_4}(\alpha|kl\rangle+(-1)^{q+1}\beta|k\bar{l}\rangle\\
&+(-1)^{t+1}\gamma|\bar{k}l\rangle+(-1)^{t+q}\delta|\bar{k}\bar{l}\rangle)\},\end{aligned} \tag{14}$$

where $e^{-\theta_i}(i=1,2,3,4)$ is an integer phase related to the state of quantum system b_Nd_N, $\Psi_{b_Nd_N}$, and it does not affect the result of the final state $\Psi_{b_Nd_N}$ after all the measurements are completed.

Similar to the notations discussed above, we define

$$V_{|G_{ij\cdots k\pm}\rangle}\equiv k, \quad P_{|G_{ij\cdots k\pm}\rangle}\equiv\pm. \tag{15}$$

The relation between the results of the measurements and the local unitary operations with which Charlie reconstructs the unknown quantum information is as same as that in Table I with just a little modification. That is, $V_{xa_1a_2}$, $V_{yc_1c_2}$, $P_{xa_1a_2}\otimes P_{b_1}$, and $P_{yc_1c_2}\otimes P_{d_1}$ are replaced with $V_{xa_1\cdots a_N}$, $V_{yc_1\cdots c_N}$, $P_{xa_1\cdots a_N}\otimes(-1)^t$, and $P_{yc_1\cdots c_N}\otimes(-1)^q$, respectively.

Same as the case with two agents, Alice need only publish two bits of classical information for each $(N+1)$-particle GHZ state measurement. The controllers are required only to perform two single-particle measurements along the x direction, σ_x for their particles and the receiver can obtain the arbitrary two-particle state $|\Phi\rangle_{xy}$ with two local unitary operations if she collaborates with the other $N-1$ agents. As all the quantum source are used to carry the useful information and no particles are abandoned in this scheme, the intrinsic efficiency for qubits approaches the maximal value. The security of this QSTS scheme depends on the process that Alice shares the EPR pairs with the agents. The ways for sharing a sequence of EPR pairs securely between two remote men have been discussed in Refs. [17,20,22]. So this QSTS scheme can be made to be secure.

Quantum state sharing is the extension of quantum secret sharing, and is used to split an unknown quantum state. For sharing a classical information, single photons can be used as the quantum source for setting up the quantum channel [6–8]. For splitting an unknown state, the quantum source has to be an entangled quantum system. Although big process has been made for producing entanglement, the efficiency is still low, in particular for multipartite entanglement [24]. With the present techniques, the EPR pairs may be one of the optimal entangled quantum sources for quantum state sharing and quantum teleportation [25]. On the other hand, the disadvantage of this scheme is that the joint measurement done by the sender, Alice becomes more difficult with the increase of the agents. With the development of technology, it is likely easy for measuring a multipartite entanglement.

In summary, we have presented a way for quantum state sharing of an arbitrary two-particle state with $2N$ EPR pairs. Any one in the N agents can recover the original state with two local unitary operations if he collaborates with the other agents, the $N-1$ controllers who are required only to perform two single-particle measurements along the x direction σ_x without Bell state joint measurements, which makes it more convenient for the agents in its applications than others. Certainly, Alice has to perform two multipartite joint measurements on her particles. Another advantage is that all the particles can be used to carry the useful information and the intrinsic efficiency for qubits approaches the maximal value. With the new notations for GHZ state, Alice need only publish four bits of classical information for recovering the original state, which reduces the information exchanged largely.

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