Theory for Raman superradiance in atomic gases

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A mean field theory for Raman superradiance (SR) with recoil is presented, where the typical SR signatures are recovered, such as quadratic dependence of the intensity on the number of atoms and inverse proportionality of the time scale to the number of atoms. A comparison with recent experiments and theories on Rayleigh SR and collective atomic recoil lasing (CARL) are included. The role of recoil is shown to be in the decay of atomic coherence and breaking of the symmetry of the SR end-fire modes.

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I. INTRODUCTION

Superradiance, first proposed by Dicke [1], is the enhanced radiation from a collection of coherently decaying dipoles. It has been studied extensively theoretically (see Ref. [2], and references therein) and has been observed in many different systems, including thermal gases [2], and Bose-Einstein condensates (BECs) [3–6]. There is mostly agreement now on the fact that the collectivity is responsible for superradiance, which is the same no matter whether the medium consists of Bosons or Fermions [7–9]. In the case of BEC, collectivity can be observed as matter wave stimulation, or "Bosonic enhancement" [7]. BEC is unique in that there is negligible Doppler broadening and the recoil momentum is measured easily and in fact was recently used to demonstrate BEC superradiance [3–6]. In particular, superradiance can be described by "collective atomic recoil laser" (CARL) equations in the bad cavity regime [3,10,11]. Collective gain can be observed with CARL in the sense that it depends nonlinearly on the density [12] and thus does not occur for atomic densities below a certain critical value [13].

Most experiments on superradiance were done using pulsed pump lasers to "instantaneously" invert a two-level system. The quantum stage of superradiance, where the radiation field builds up from vacuum fluctuations, can then be modeled to start only after the pump laser is turned off [2]. For this case, pump lasers obviously have to be strong; at the same time, experiments done with BECs use only weak pump fields. We therefore call the one "strong pump superradiance" and the other "weak pump superradiance." In the latter case, the quantum stage happens while the pump field is still on. In this article, we will focus on weak pump superradiance. Note that in this case the maximum instantaneous superradiance rate is limited by the pump laser intensity, while for strong pump superradiance no such limitation exists.

Mostly, earlier research concentrated on so-called Rayleigh superradiance [14–19] which happens for transitions between different center of mass (c.m.) states while the internal state remains unchanged [20]. We will here discuss Raman superradiance, where there are two different internal ground states for the pump and the superradiant transition. Recoil and different c.m. states are taken into account here as well, but are, as we will show, of lesser consequence. It turns out that Raman superradiance otherwise follows the same basic patterns as Rayleigh superradiance. Although superradiance with Raman pumping has been analyzed in Ref. [21], the recoil effect was ignored and the Raman pumping time was assumed to be short compared with the superradiance time. It will be shown in this paper that recoil induces the decay of Raman coherence and may make the superradiant modes asymmetric. In Ref. [22] an incoherent cw pump laser was considered numerically, also leading to superradiance. Recently, Cola *et al.* [23] presented a quantum theory to describe the Raman superradiance experiments with BECs [4–6]. In comparison, our analysis can be applied to both thermal atoms and BECs with emphasis on the effect of recoil. We also discuss the connection with CARL using stability analysis. In addition, we consider the asymmetry of superradiant modes as the pump laser setup is changed which helps to understand the underlying physics of superradiance.

This paper is organized as follows. In Sec. II we derive the dynamical equations to describe Raman superradiance. These equations are used to analyze the stability conditions in Sec. III. Numerical calculations in comparison with experiments are included in Sec. IV. Discussion and conclusion follow in Sec. V.

II. MODEL

We consider a three-level Λ-type atomic system with excited state $|1\rangle$ and two ground states $|2\rangle$ and $|3\rangle$ (Fig. 1). When the detuning of a pump laser is much larger than both, its Rabi frequency and the maximum Rabi frequency of the superradiant field, the interaction picture Hamiltonian of this system under dipole and rotating-wave approximation reads [24–26]

$$\begin{aligned} H &= \Psi_2^+ H_{\text{c.m.}} \Psi_2 + \Psi_3^+ (H_{\text{c.m.}} - \hbar\delta_3)\Psi_3 + H_f \\ &+ \sum_{\vec{q}} \hbar g^*_{3,\vec{q}} e^{-i(\vec{q}-\vec{k}_0)\vec{r}} \Psi_3^+ a^+_{\vec{q}3} \Psi_3 + \text{H.c.} \\ &+ \sum_{\vec{q}} \hbar g^*_{2,\vec{q}} e^{-i(\vec{q}-\vec{k}_0)\vec{r}} \Psi_2^+ a^+_{\vec{q}2} \Psi_3 + \text{H.c.}, \end{aligned} \tag{1}$$

with coupling constants $g^*_{2,\vec{k}} = i\sqrt{(\hbar kc/2\epsilon_0 V)}\hat{\epsilon}_2 \cdot \vec{d}_{12}(\Omega^*/\Delta)$ and $g^*_{3,\vec{k}} = i\sqrt{(\hbar kc/2\epsilon_0 V)}\hat{\epsilon}_3 \cdot \vec{d}_{13}(\Omega^*/\Delta)$, in what follows as-

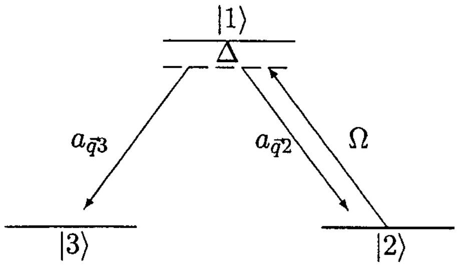

FIG. 1. Center of mass manifolds associated with three-internal-state atomic system. State $|2\rangle$ is the one-particle state of the initial BEC. Both Rayleigh transition and Raman transition are present with Raman field $a_{\vec{q}2}$ and Rayleigh field $a_{\vec{q}3}$. The pump laser Rabi frequency Ω is much smaller than the detuning $|\Delta|$.

sumed to be real. $\hat{\epsilon}_i$ is the polarization direction. While the c.m. Hamiltonian is $H_{\text{c.m.}}=-(\hbar^2/2m)\nabla^2$ with m being the mass and $\hbar$ being the Planck constant, the Hamiltonian of the optical fields is $H_f=\Sigma_{\vec{q}}\hbar qca^{+}_{\vec{q}2}a_{\vec{q}2}+\hbar qca^{+}_{\vec{q}3}a_{\vec{q}3}$, where $a_{\vec{q}2}(a_{\vec{q}3})$ is the field annihilation operator for the transition between $|1\rangle$ and $|2\rangle$ ($|3\rangle$), and $\vec{q}$ the momentum of the radiation field. Ψ_i is the atomic field operator, V the quantization volume, Ω and Δ the pump field Rabi frequency and detuning, δ_3 the ac Stark shift due to the pump laser, $d_{12}(d_{13})$ the dipole moment between $|1\rangle$ and $|2\rangle$ ($|1\rangle$ and $|3\rangle$). In Eq. (1), the second line describes the Rayleigh transition, the third line the Raman transition. The ratio of $g_{2,\vec{q}}/g_{3,\vec{q}}$ determines the branching ratio between Rayleigh and Raman superradiance [4,5]. While the Rayleigh superradiance has been studied extensively [14,15,27], this paper will focus on Raman superradiance.

Using Fock representation, Eq. (1) can be written as

$$H/\hbar=\sum_{j,k}\omega_k b^{+}_{jk}b_{jk}+\left(g_2\sum_{q,k}b^{+}_{2\bar{k}}a^{+}_{2q}b_{3k}+\text{H.c.}\right)+\sum_{q}\omega_q a^{+}_{q2}a_{q2}, \tag{2}$$

where $b_{jk}(j=2,3)$ annihilates an atom in state $|j\rangle$ with momentum k and energy $\omega_k=\hbar^2k^2/2m$. $b_{2\bar{k}}\equiv b_{2,k+k_0-q}$, $\omega_q=cq-\omega_0+\omega_{23}+\delta_3$, $\omega_0=ck_0$ and ω_{23} is the atomic energy difference between $|2\rangle$ and $|3\rangle$. For simplicity, the vector arrows from $\vec{q}$ and $\vec{k}$ have been dropped here. We assume that $g_{2,q}\approx g_2$ only weakly depends on q for the relevant range of modes. From this form it is obvious that the total population on $|2\rangle$ and $|3\rangle$ is conserved. The matter wave mode b_{3k} is coupled to different $b_{2\bar{k}}$ for different optical modes q. When the detuning is large, however, collective linewidth or multiple scattering can be neglected [28] and we can drop the coupling between different modes. In this article, we also neglect the depletion of BEC due to other modes. Raman transitions in different directions can thus be considered independently. From Eq. (2), Maxwell-Bloch equations can be derived:

$$\frac{d}{dt}A=-i\omega_k A-ig_2\sum_k\rho_{\bar{k}k}-\kappa A, \tag{3a}$$

$$\frac{d}{dt}\rho_{\bar{k}k}=-i(\omega_k-\omega_{\bar{k}})\rho_{\bar{k}k}-ig_2(1-2N_k)A, \tag{3b}$$

$$\frac{d}{dt}N_k=i(g_2\rho_{\bar{k}k}A-\text{c.c.}), \tag{3c}$$

where $A=\langle a_{q2}\rangle$, $\rho_{\bar{k}k}=\langle b^{+}_{2\bar{k}}b_{3k}\rangle$, $N_k=\langle b^{+}_{3k}b_{3k}\rangle$, with N_k being the number of atoms in state $|3k\rangle$. κ is the effective radiation field decay rate, if we neglect propagation in the mean field approximation [27,29]. This approximation works well when the medium is optically thin at the pump frequency, which is the case here since the pump field is far detuned from resonance. With L and D the length and diameter of the medium and λ the wavelength of the superradiant transition, the Fresnel number $F=D^2/L\lambda$ gives approximately the number of modes that fit in the medium in axial direction. If it is around or bigger than 1 as in the experiments [4,5], then $\kappa=c/2L$ for axial modes (also called "end-fire modes" [3]), which are the modes having largest gain for superradiance, and $\kappa_{\text{off}}\geqslant(c/2L)(1/F+1)$ for off-axial modes [29]. It will be shown in Sec. III that in experiments [4,5], κ dominates over all the other relevant characteristic rates and therefore makes the end-fire modes most likely to superradiate. In the following, we assume all superradiant modes to be axial.

In a BEC, only the $k=0$ state is present, and thus Eqs. (3) become

$$\frac{d}{dt}A=-i\omega_k A-ig_2N\rho_{\bar{0}0}-\kappa A, \tag{4a}$$

$$\frac{d}{dt}\rho_{\bar{0}0}=i\omega_r\rho_{\bar{0}0}-ig_2(1-2N_0)A, \tag{4b}$$

$$\frac{d}{dt}N_0=i(g_2\rho_{\bar{0}0}A-\text{c.c.}), \tag{4c}$$

where $\rho_{\bar{0}0}=\langle b^{+}_{2,\bar{0}}b_{3,0}\rangle$ with $b_{2,\bar{0}}=b_{2,k_0-q}$, the recoil energy $\hbar\omega_r=\hbar^2(k_0-q)^2/2m$, and N the total number of atoms in the system. Because we assume κ to be very large it follows from Eq. (4a) that

$$A\approx-ig_2N\rho_{\bar{0}0}/\kappa. \tag{5}$$

Substituting A into Eqs. (4b) and (4c), we arrive at

$$\frac{d}{d\tau}\rho_{\bar{0}0}=i\omega_r\rho_{\bar{0}0}-g_2^2(2N_0-1)\rho_{\bar{0}0},$$

$$\frac{d}{d\tau}N_0=-2g_2^2|\rho_{\bar{0}0}|^2. \tag{6}$$

Here, we scale the time such that $\tau=Nt$. It is therefore obvious that the timing of the resulting process scales with $1/N$, in the same way as in traditional superradiance [2]. From Eq. (5) we know that the output field amplitude A is proportional to N and thus the intensity is proportional to N^2. These are typical characteristics of superradiance. Note that without recoil $\omega_r=0$, Eqs. (6) are completely equivalent to Eqs. (6.36) of Ref. [2], which describe *standard superradiance*: a radiation cascade down the pseudospin ladder from $J_z=N/2$ to $J_z=-N/2$, giving a hyperbolic secant solution for the dependence of the upper-level population on time [2]. For BEC, the term with ω_r, which is due to recoil, only contributes to the phase evolution of Raman coherence, not to its decay, while for thermal atoms recoil does induce the decay of Raman coherence, as discussed in the next paragraph.

For thermal atoms, Eq. (3b) describes quantum diffusion as well as generation of Raman coherence. In particular, the term $-i(\omega_k-\omega_{\bar{k}})\rho_{\bar{k}k}$ in Eq. (3b) shows that coherence stored in different levels experiences quantum diffusion, since the term will have different values for different k. To understand how the quantum diffusion works, we assume Raman coherence has been generated uniformly for all levels, which means $\rho_{\bar{k}k}(0)=\rho(0)p_k$ with $\rho(0)$ being the coherence for one level and p_k the probability distribution of atom at level k. If we set field amplitude A to zero, the solution of Eq. (3b) is $\rho_{\bar{k}k}(t)=\rho(0)p_k e^{-i(\omega_k-\omega_{\bar{k}})t}$. The coherence is then $\rho(t)=\Sigma_k\rho_{\bar{k}k}(t)=\rho(0)\Sigma_k p_k e^{-i(\omega_k-\omega_{\bar{k}})t}$. To proceed, we need to specify p_k at temperature T. Here, either Bose-Einstein distribution for Bosons or Fermi-Dirac distribution for Fermions are appropriate. For simplicity, however, we assume Lorentzian distribution $p_k=(1/\pi)[\delta p/(k^2+\delta p^2)]$ with $\delta p^2/2m=k_BT/2$, which describes the atoms well even at subrecoil temperatures [30]. The summation can be approximated by an integral and it follows that the Raman coherence decays exponentially

$$\rho(t)=\int\frac{1}{\pi}\frac{\delta p}{k^2+\delta p^2}\rho_{\bar{k}k}(t)dk=\rho(0)e^{i\omega_\Gamma t}e^{-\Gamma t}, \tag{7}$$

where $\Gamma=2k_0\sqrt{k_BT/m}\sin\theta/2$, $\omega_\Gamma=2\hbar k_0^2\sin^2\theta/2$, and θ is the angle between $\vec{q}$ and $\vec{k}_0$. It is clear now that the decay rate Γ depends on the pump laser direction $\hat{k}_0$ relative to the superradiant pulse direction $\hat{q}$. Thus, Eq. (3b) can be rewritten as

$$\frac{d}{dt}\rho=(i\omega_\Gamma-\Gamma)\rho-ig_2(1-2\rho_{33})A, \tag{8}$$

where $\rho_{33}=\Sigma_k N_k$ is the population in state $|3\rangle$. If we would use a Gaussian rather than Lorentzian density of states, the inverse $1/e$ decay time would be $\sqrt{2}\Gamma$ rather than Γ. As an example, for Rb at the Doppler limit temperature of 143 μK, $\Gamma=1.35\times10^6$ s^{-1}.

Comparing the result for thermal atoms in Eq. (8) with the result for a BEC in Eq. (4b), we see that thermal distribution contributes additional coherence decay, otherwise these equations are the same as expected. We can therefore generalize the results to

$$\frac{d}{dt}A=-i\omega_kA-ig_2N\rho-\kappa A, \tag{9a}$$

$$\frac{d}{dt}\rho=i\omega_\Gamma\rho-ig_2(1-2\rho_{33})A-\kappa_R\rho, \tag{9b}$$

$$\frac{d}{dt}\rho_{33}=i(g_2\rho A-\text{c.c.}), \tag{9c}$$

where the total coherence decay $\kappa_R=\kappa_R'+\Gamma$. κ_R' can be introduced phenomenologically to contain collisions, magnetic gradients, etc., and $\Gamma=0$ for BECs. These equations are now analogous to Eqs. (13)–(15) in Ref. [23], but can be applied to both BEC and thermal atoms.

III. LINEAR STABILITY ANALYSIS

In this section, we will determine the necessary conditions for Raman superradiance to happen, which is easiest using linear stability analysis [20,31,32]. Obviously, $A=0$, $\rho=0$, and $\rho_{33}=1$ give a stationary solution of Eqs. (9). Rewriting Eqs. (9) for $A=0+\delta A$, $\rho=0+\delta\rho$, and $\rho_{33}=1+\delta\rho_{33}$ leads to a two-dimensional linear system with the characteristic equation

$$S^2+[i(\omega_k-\omega_\Gamma)+\kappa+\kappa_R]S+(-i\omega_\Gamma+\kappa_R)\kappa-Ng_2^2+\omega_k\omega_\Gamma+i\omega_k\kappa_R=0. \tag{10}$$

(The third equation is equivalent to zero in this case and can be dropped.) In comparison with the cubic instability equation for Rayleigh superradiance [20], this is a quadratic equation. The physical reason for such a change is that for a Rayleigh transition, the initial and final internal states are the same and thus only atoms with different c.m. states may contribute to the gain [see Eq. (49) of Ref. [20]]; for a Raman transition, the initial and final internal states are different and thus all atoms contribute to the gain regardless of the c.m. states.

The above quadratic equation has two roots for S, S_+, and S_-. Since S is the exponent of the state vector $[(\delta A,\delta\rho)=(\delta A(0),\delta\rho(0))\exp St]$, the zero solution becomes unstable if at least one of S_+ or S_- has a positive real part. The larger real part (let us call the respective root $S_+=S_+'+iS_+''$) is therefore defined as an instability factor. If $S_+'>0$, then the system is dynamically unstable, from which the threshold pump intensity can be derived. From Eq. (10), it can be easily seen that S_+' depends nonlinearly on the number of atoms N. Note that nonlinear dependence on N is the essence of collective instability [20]. In the bad cavity regime as is the case for the experiments [4,5], κ is large, the system therefore depends linearly on the atomic density and therefore may display superradiant behavior. In a good cavity, however, κ is much smaller and thus the collective gain depends on the density nonlinearly [11,12].

Since ω_Γ and ω_k can be shown to have only a minor effect on S_+' under the experimental conditions of Refs. [4,5], we set $\omega_\Gamma=\omega_k=0$. In this case, the instability factor is

$$S_+'=\frac{-(\kappa+\kappa_R)+\sqrt{(\kappa+\kappa_R)^2+4(Ng_2^2-\kappa\kappa_R)}}{2}. \tag{11}$$

In particular, for vanishing coherence decay $\kappa_R=0$, the system is unstable and therefore superradiant for any pump laser power. This is different from the case of Rayleigh superradiance which always has a nonzero threshold pump laser intensity [20]. In the case of thermal atoms, however, κ_R can

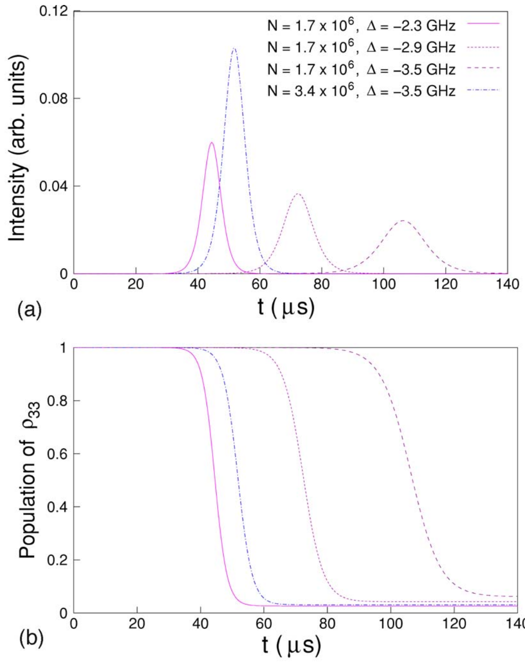

FIG. 2. (Color online)Effect of number of atoms and detuning on the evolution of (a) the intensity $|A|^2$ and (b) the population ρ_{33} as a function of time. Parameters used in the calculations are from Ref. [5]. The $1/N$ dependence of the delay time and the N^2 dependence of the maximum intensity can be clearly seen. The finite population left in state $|3\rangle$ is due to the decay of Raman coherence.

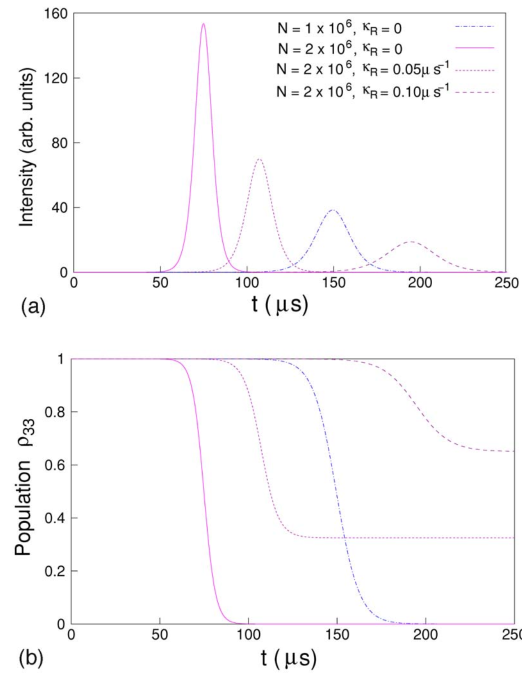

FIG. 3. (Color online) Effect of Raman coherence decay rate κ_R on the evolution of (a) the intensity $|A|^2$ and (b) the population ρ_{33} as a function of time. $\kappa_R > 0$ is responsible for a longer superradiance delay time and a lower maximum intensity. For this figure, $N = 2\times 10^6$. Other parameters are the same as in Fig. 2.

be considerable, and the threshold pump intensity is quite high in the bad cavity limit. This explains why collective gain was not observed in Refs. [4,5]. It should be possible experimentally to minimize the decay due to quantum diffusion if the pump laser is collinear with the sample. Raman superradiance or collective gain might perhaps be observed in this case even in thermal atoms.

When κ is much larger than any other frequency in Eq. (10), i.e., $\kappa \gg \sqrt{Ng_2^2}$, ω_k, ω_r, as in the experiments [4,5], the instability factor can be simplified to

$$S'_+ \simeq \frac{1}{\kappa}\{Ng_2^2 - \kappa_R \kappa\}. \tag{12}$$

In this case, S'_+ is linear in N, which means experiments in Refs. [4,5] would be purely in the superradiant regime.

IV. NUMERICAL SIMULATIONS

To compare our theory with experiments, we solve Eqs. (9) for both BEC and thermal atoms. In the simulations, we use the initial value of $\rho(0) = (2/N)^{1/2}$, which is determined by quantum noise [29,33]. Other parameters are calculated using the data in Ref. [5]: $g_2 = 0.5\times 10^6\ \mathrm{s}^{-1}$, $\kappa = 1.76 \times 10^{12}\ \mathrm{s}^{-1}$. ω_r is negligible in this context.

In Fig. 2 we show that the intensity of superradiance is proportional to N^2 and the superradiance delay time is proportional to $1/N$ at least as long as there is no Raman coherence decay, i.e., if we assume a BEC. The numerical delay time 75–150 μS also reproduces the experimental data well [4,5].

Figure 3 shows that when the Raman coherence decay rate κ_R is increased, the radiation intensity decreases and the delay time increases. This is similar to two-level superradiance: dipole-dipole interaction decreases the coherence between atoms and thus competes with superradiance. Because of the effective population mixing caused by Raman coherence decay there is always a finite number of atoms in the Rayleigh lower state $|3\rangle$ at any time for a finite κ_R. For thermal atoms at Doppler cooling limit $T = 143\ \mu$K, $\Gamma = 1.35 \times 10^6\ \mathrm{s}^{-1}$, and the instability factor S'_+ is smaller than zero and no superradiance happens.

The roles of photon and atomic coherence are intertwined for superradiance. Collectivity can be attributed to either photons or atoms, or both. In the case of weak pump superradiance, the pulse exits the medium and thus decays much faster than the (atomic) Raman coherence. Thus the intensity of the superradiant pulses is small, and stimulation of photons by photons is not critical in this case. For example, if in the calculation the pump laser is turned off before all the atoms have radiated and then turned on again, superradiance continues nearly at the same point it was interrupted. This is true for an interruption that lasts longer than the photon coherence time (which is here just the escape time of the photons of about 1 ps), but shorter than the Raman coherence time, which is between infinity and 1 ms in our simulations. The conclusion is that atomic coherence is more important than stimulated emission in this case for superradiance to happen.

It was claimed in Ref. [5] that the output photon number, N_p, enhances the superradiance $\dot{N}_r \propto (N_r + N_p + 1)$, where N_r is the number of atoms having superradiated. Indeed, it was assumed that $N_p = N_r$ [5], then $\dot{N}_r \propto (2N_r + 1)$. However, no cavity was used in Ref. [5], which means the average N_p is small within the sample and can be neglected, as is done in Refs. [3,4]. Note that the collecting of photons in the (ring) cavity modifies the rate [34,35]. But in a high-Q cavity, the coupling between atoms and field is strong and a perturbation approach of Fermi's golden rule as used in Ref. [5] may not apply. Detailed analysis of this is beyond this paper.

Now we consider the effect of the field decay rate κ on determining the direction of the superradiant field modes. If the Fresnel number F is around 1 as in the experiments [4,5], the decay rate of the off-axial modes is much bigger than that of the axial mode, and thus only the axial mode superradiates. Since the decay rates of the Raman coherence for the axial modes and their directly neighboring modes are almost equal to each other, the field decay rate determines the radiation direction. On the other hand, if F is much bigger than 1, the off-axial modes do not have a decay rate much different from the axial ones, and thus they may also superradiate. In this case, the quantum fluctuation stage determines which modes are fired. The random dots in the simulation of Ref. [14] show the effect of the fluctuations in this case. In general, many modes might fire simultaneously as long as the population in state $|3\rangle$ is not depleted. If the delay time of one mode is shorter than the sum of the delay and superradiance time of the axial mode, then this mode also superradiates. The same is obviously also true for the competition between Raman and Rayleigh superradiance [4].

Let us consider the symmetry of the superradiating modes. The two axial modes in opposite directions in the experiments [3–5] show identical behavior: the recoil pattern is symmetric. One of the reasons for this is that the field decay rate for these modes is the same. Also the recoil induced decay κ_R is zero for a BEC. This is also true for an initially fully inverted two level system [2]. However, if the pump laser is parallel to the sample axis (longitudinal pumping), the recoil induced decay for thermal atoms can be cancelled if the superradiance mode is parallel to the pump laser. This breaks the symmetry of the two axial modes and privileges the parallel mode over the antiparallel one. In Fig. 3 we see that the mode with small κ_R is stronger than other modes and may suppress superradiance for them by depleting the population ρ_{33}. The broken symmetry indicates that the equivalence of a three-level system with a far detuned pump laser and a two-level system does not hold in this case. Note that if the atoms are not fully inverted, the symmetry could also be broken due to stored coherence [36]. However, for BECs, since recoil does not contribute to the decay of Raman coherence significantly, two superradiant modes would still fire even with longitudinal pump.

V. DISCUSSION AND CONCLUSION

Rayleigh superradiance does not happen without recoil. In comparison to this, recoil is not critical for Raman superradiance to happen, which means that atomic bunching and density grating pictures do not apply for explaining Raman superradiance, as they do for Rayleigh superradiance. Interference between pump laser and superradiance output [27] equally does not apply in a case where both transitions radiate light with different polarization. We therefore believe that collective effects, which might be called Bosonic stimulation in the case of Bosons, are the main players in Raman superradiance.

Interesting is the relationship between Rayleigh and Raman superradiance. States with different momentum may be considered to be orthogonal [14] in the same way as different internal levels, and thus the Rayleigh transition can be looked upon as a Raman transition between *different* motional states [32,37,38]. Indeed, the gain coefficients have a similar functional dependence on the atomic density [13]. In particular, in the case of thermal atoms with a pump laser not parallel to the sample axis, i.e., with large κ_R, it can be shown from Eq. (11) that the instability factor S'_+ depends linearly on N. This was the regime discussed in Refs. [32,37,38] in which the Raman transition is considered to be in the (linear) single-atom gain regime [39]. Although atom statistics are not critical for superradiance [27], the Fermi momentum k_F in Rayleigh scattering is replaced by the relative momentum difference in Raman scattering, thus the problem with a very short coherence time in the case of fermions due to recoil might be overcome [8]. As is done for Rayleigh superradiance [40], also atom-atom interaction can be included, and will be presented in a forthcoming publication.

Finally, we would like to differentiate two concepts: collectivity and collective gain or collective instability. Collectivity means that all atoms in the system contribute to the same mode [41], while collective gain or collective instability [12] means that the gain depends on the number of atoms N nonlinearly. While the experiments are in the noncollective gain regime, collectivity still plays a major role in Raman superradiance. Raman superradiance therefore shows that it is the collective effect rather than "Bosonic stimulation" that is responsible for superradiance [7–9]. It was claimed [13] that if the pump laser makes the two-photon detuning for superradiant mode zero, and thus the Rayleigh transition corresponds to a Raman transition between different c.m. states there would be a single-atom gain instead of collective gain [39]. However, we tried to show that even in a pure Raman transition, collective gain is still possible if a cavity is included.

To conclude, we developed a mean field theory for Raman superradiance. Raman superradiance does not necessarily have an intrinsic threshold for pump laser intensity even if the decay of the optical field is included. We found that recoil induced decay of Raman coherence may break the symmetry of the two axial modes if the atoms are pumped longitudinally, in which case it is possible to realize Raman superradiance even in thermal atoms while at the same time it might not be possible to realize Rayleigh superradiance. We also note that both the Rayleigh and Raman superradiance experiments were done in the regime where the pump laser is far detuned, such as not to populate the excited state. What happens in the case of a resonant pump laser is under investigation presently.

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