

## Generating entangled photon pairs from a cavity-QED system

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We propose a scheme for the controlled generation of Einstein-Podolsky-Rosen entangled photon pairs from an atom coupled to a high  $Q$  optical cavity, extending the prototype system as a source for deterministic single photons. A thorough theoretical analysis confirms the promising operating conditions of our scheme as afforded by currently available experimental setups. Our result demonstrates the cavity QED system as an efficient and effective source for entangled photon pairs, and shines new light on its important role in quantum information science.

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In the Schrödinger picture, a quantum state of a system represents all the knowledge we can obtain. For a composite system, its wave function or density matrix describes not only the state of each part, but also the correlations between the different parts. The notion of entanglement of a quantum state for a composite system describes the inseparable correlations between different parts that are beyond the classical domain. It has been widely attributed that entanglement is a valuable resource for quantum computing and quantum information. Many current efforts are directed on the controlled generation and detection of entangled states.

Paradoxically, almost all states of composite systems in nature are entangled, as a result of interactions among different system parts. The more useful entangled states are those that can be easily and economically manipulated as in a coupled system of many qubits. They are the so-called Einstein-Podolsky-Rosen (EPR) state of two qubits [1], Greenberger-Horne-Zeilinger (GHZ) state [2] and W state [3] of three qubits, maximally entangled states [4] and cluster states [5] of many qubits, etc. The simplest of them is the EPR state. In a two qubit (spin- $\frac{1}{2}$ ) system, it is commonly denoted as

$$|\text{EPR}\rangle = \frac{1}{\sqrt{2}}(|\uparrow\rangle_1|\downarrow\rangle_2 - |\downarrow\rangle_1|\uparrow\rangle_2), \quad (1)$$

and displays maximal entanglement. In general, indices 1 and 2 refer to the two qubits, and in our scheme they refer to the first and second emitted cavity photons.

Among all physical realizations of qubits, photons are especially useful as they can be directly used for quantum communication. The standard process of producing entangled photon pairs uses nonlinear optical crystals in the so-called parametric down-conversion process, where a single pump photon spontaneously decays into an entangled pair composed of a signal and an idler photon [6,7]. Despite improvement over the years with brighter sources for parametric down converted photons, inherently, the pump photon decay process is stochastic, thus coincidence counting has to be used. In this paper, we propose a scheme for deterministically generating entangled photon pairs from an atom coupled to a high  $Q$  optical cavity. Our work is prompted by

the rapid development of a deterministic single photon source with a trapped ion and/or atom in a high  $Q$  optical cavity [8–13] and the recent theory for atom-photon entanglement generation and distribution [14]. To our knowledge, this is the first proposal for deterministically generating EPR photon pairs from a single atom in a cavity.

Cavity QED is a unique architecture for implementing quantum computing technology as it allows for coherent exchange of quantum information between material qubits (atoms/ions) and photonic qubits. However, it is a daunting task to reach this goal as this requires a coherent light-matter coupling at the single photon level, or the so-called *strong coupling* limit. Several important quantum logic protocols have been developed within this limit that forms the basis of the ongoing experimental efforts for quantum computing and communication with cavity QED systems [11,12,15,16]. Another important application of cavity QED is the “photon gun” protocol, whereby a single atom leads to a deterministic cavity photon in the bad cavity limit [9].

The experimental setup for our system is sketched in Fig. 1. As an atom falls through an optical cavity, it interacts first with the cavity mode field, then with a classical pump field  $\pi$ -polarized with respect to the cavity axis and propagating along a perpendicular direction. We utilize two orthogonal polarizations of the same resonant cavity mode and assume the atomic levels to be that of a  $F=1 \rightarrow F'=1$  transition as shown in Fig. 2. Each Zeeman state is denoted as  $|Fm_F\rangle$ , further simplified as  $|g_{m_F}\rangle = |Fm_F\rangle$  and  $|e_{m_{F'}}\rangle = |F'm_{F'}\rangle$ . Similar model systems were invoked previously in Ref. [17],

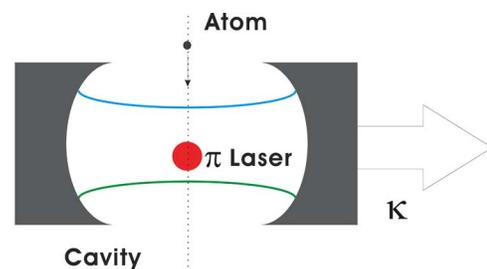


FIG. 1. (Color online). The illustration of the proposed cavity QED setup.

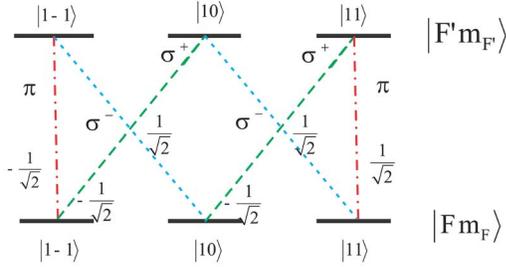


FIG. 2. (Color online). The proposed coupling scheme: the vertical dot-dashed lines denote the  $\pi$ -polarized pump field, while the tilted lines denote the left (dotted lines for  $\sigma^-$ ) and right (dashed lines for  $\sigma^+$ ) polarized cavity fields. The various CG coefficients are also indicated.

where Lange *et al.* proposed a scheme for generating GHZ photon multiplets by an adiabatic passage and in Refs. [18–20], where the entanglement of two modes in one or two cavities were investigated.

We assume an initially empty cavity and prepare the atom in state  $|e_0\rangle$  before it enters the cavity mode [21]. The atom's passage through the cavity can now be divided into two parts. In the first part, the atom only interacts with the cavity mode, and does not enter the area of the  $\pi$ -polarized pump field. Thus, the excited atom emits a first photon, entangled with the atom in ground states as discussed before [14]. In the second part, the atom is subsequently excited by the pump, and emits a second photon and swaps its entanglement with the first photon (already outside the cavity) to the second photon. The whole process now generates an entangled photon pair. We now analyze the above protocol including both atomic and cavity decays. In the interaction picture to the Hamiltonian

$$H_0 = \hbar\omega_C \sum_{m_{F'}} |e_{m_{F'}}\rangle \langle e_{m_{F'}}| + \hbar\omega_C (a_L^\dagger a_L + a_R^\dagger a_R) \quad (2)$$

of the atom plus the cavity, our system dynamics is governed by  $H = H_1 + H_2$ , with

$$H_1 = -\hbar\Delta \sum_{m_{F'}} |e_{m_{F'}}\rangle \langle e_{m_{F'}}| + \frac{1}{2}\hbar[\Omega(t)A_{10}^\dagger + \text{H.c.}], \quad (3)$$

$$H_2 = \frac{1}{2}\hbar g(t)(A_{11}^\dagger a_R + A_{-1}^\dagger a_L + \text{H.c.}), \quad (4)$$

where the second term of  $H_1$  is from the interaction of the atom with the pump, and  $H_2$  denotes the interaction of the atom with the left and right circular polarized cavity modes.  $\Delta = \omega_C - \omega_A$  is the detuning and for simplicity the pump field is assumed to be resonant with the cavity. The atom field coupling coefficients are

$$g(t) = -\mathcal{E}(\vec{R}) \langle F' || d_1 || F \rangle, \quad (5)$$

$$\Omega(t) = -E(\vec{R}) \langle F' || d_1 || F \rangle, \quad (6)$$

where  $\langle F' || d_1 || F \rangle$  is the reduced dipole matrix element according to the Wigner-Eckart theorem, and  $\mathcal{E}(\vec{R})$  and  $E(\vec{R})$ , are respectively, the spatial profiles of the cavity mode and

the pump field. The atomic raising operators are defined as

$$A_{1q}^\dagger = \sum_{m_F, m_{F'}} \langle F' m_{F'} | F m_F 1 q \rangle |F' m_{F'}\rangle \langle F m_F|. \quad (7)$$

According to the previous prescription, the interaction parameters take the following time dependence:

$$g(t) = gh(t)h(T-t), \quad (8)$$

$$\Omega(t) = \Omega h(t-t_1)h(t_2-t), \quad (9)$$

with the Heaviside step function  $h(t) = 0$  for  $t < 0$  and  $h(t) = 1$  for  $t \geq 0$ . The different times have the following meaning: the atom enters the cavity at time 0, arrives at the pump laser at time  $t_1$ , leaves the pump at time  $t_2$ , and finally exits the cavity at time  $T$ .

Dissipations are essential to our protocol as they allow the excited atom to decay and the cavity photons to emit. Their effects on the dynamics can be included straightforwardly using a master equation

$$\dot{\rho} = -i[H, \rho] + D\rho + C\rho, \quad (10)$$

where the cavity and atom dissipative terms are

$$D\rho = \kappa \sum_{\xi=R,L} (2a_\xi \rho a_\xi^\dagger - a_\xi^\dagger a_\xi \rho - \rho a_\xi^\dagger a_\xi), \quad (11)$$

$$C\rho = \frac{\gamma}{2} \sum_q (2A_{1q} \rho A_{1q}^\dagger - A_{1q}^\dagger A_{1q} \rho - \rho A_{1q}^\dagger A_{1q}). \quad (12)$$

$2\kappa$  is the one side decay rate of the cavity, while the other side of the cavity is assumed perfectly reflecting.  $\gamma$  is the decay rate for the atomic excited state  $|e_{m_{F'}}\rangle$ . In a more complete model study, we have constructed the wave function for the system including the free fields outside the cavity. We find that our results are completely verified within the rotating wave approximation [22].

The initial state of our system is now  $|\psi(0)\rangle = |e_0, 0_L, 0_R\rangle$ , and the rate of cavity photon emission with polarization  $\xi$  at time  $t$  is  $p_\xi(t) = 2\kappa \langle a_\xi^\dagger(t) a_\xi(t) \rangle$ . The probability of cavity photon emission is then  $P_\xi(t) = 2\kappa \int_0^t dt' \langle a_\xi^\dagger(t') a_\xi(t') \rangle$ . We require that the cavity photon emit quickly as soon as it is generated, thus the preferred operating condition is close to the bad cavity limit ( $\gamma \ll g^2/\kappa \ll \kappa$ ) as for a single photon source [9,10,14]. In fact, we find it is desirable to operate with  $\kappa \sim g \gg \gamma$ , a compromise of the *bad cavity* with the *strong coupling* limit due to the necessity of coherently pumping the atom to the excited state for a second photon.

To gain more insight, we describe the dynamic evolution using the non-Hermitian effective Hamiltonian

$$H_{\text{eff}} = H - i\kappa(a_L^\dagger a_L + a_R^\dagger a_R) - i\frac{\gamma}{2} \sum_q |e_q\rangle \langle e_q|. \quad (13)$$

Such an approach is appropriate as re-excitations of the decayed atom due to emitted photons are negligible, and reabsorptions of the cavity photons can be neglected due to their fast decays to the outside of the cavity. For  $t \in [0, t_1]$ , the system state is approximately

$$|\psi(t)\rangle = d_0(t)|e_0, 0_L, 0_R\rangle + c_{-1}(t)|g_{-1}, 0_L, 1_R\rangle + c_1(t)|g_1, 1_L, 0_R\rangle. \quad (14)$$

The effective Schrödinger equation becomes

$$i\frac{d}{dt}\begin{pmatrix} d_0(t) \\ c_{-1}(t) \\ c_1(t) \end{pmatrix} = \begin{pmatrix} -\Delta - i\frac{\gamma}{2} & -\frac{g}{\sqrt{2}} & \frac{g}{\sqrt{2}} \\ -\frac{g}{\sqrt{2}} & -i\kappa & 0 \\ \frac{g}{\sqrt{2}} & 0 & -i\kappa \end{pmatrix} \begin{pmatrix} d_0(t) \\ c_{-1}(t) \\ c_1(t) \end{pmatrix},$$

which leads to the solution

$$c_1(t) = -\frac{ig}{\sqrt{2}} \frac{e^{s_1 t} - e^{s_2 t}}{s_1 - s_2}, \quad (15)$$

on making use of the property  $c_1(t) = -c_{-1}(t)$ .  $s_1$  and  $s_2$  are the roots of equation

$$2s^2 + (2\kappa + \gamma - i2\Delta)s + \gamma\kappa + 2g^2 - i2\kappa\Delta = 0, \quad (16)$$

where an identical coupling is assumed for both polarization modes, thus  $p_R(t) = p_L(t) = 2\kappa|c_1(t)|^2$ . When the duration before the pump excitation is so long that the excited atom (in  $|e_0\rangle$ ) completely decays into ground states and the cavity photon completely leaks out, the final state of the atom plus the cavity modes becomes

$$\rho(t_1) = \frac{1}{2}|g_{-1}, 0_L, 0_R\rangle\langle g_{-1}, 0_L, 0_R| + \frac{1}{2}|g_1, 0_L, 0_R\rangle\langle g_1, 0_L, 0_R|, \quad (17)$$

as the first photon is traced out after being emitted into modes outside the cavity.

For  $t \in (t_1, t_2]$ , the initial state (17) is now completely mixed, so we can evolve its different decompositions, respectively. In the first case for  $|\psi(t_1)\rangle = |g_1, 0_L, 0_R\rangle$ , the state can be approximately expanded as

$$|\psi(t)\rangle = d_1(t)|e_1, 0_L, 0_R\rangle + c_0(t)|g_0, 0_L, 1_R\rangle + c_1(t)|g_1, 0_L, 0_R\rangle, \quad (18)$$

and governed by an effective Schrödinger equation

$$i\frac{d}{dt}\begin{pmatrix} d_1(t) \\ c_0(t) \\ c_1(t) \end{pmatrix} = \begin{pmatrix} -\Delta - i\frac{\gamma}{2} & -\frac{g}{\sqrt{2}} & \frac{\Omega}{\sqrt{2}} \\ -\frac{g}{\sqrt{2}} & -i\kappa & 0 \\ \frac{\Omega}{\sqrt{2}} & 0 & 0 \end{pmatrix} \begin{pmatrix} d_1(t) \\ c_0(t) \\ c_1(t) \end{pmatrix}.$$

The solutions for  $d_1(t)$  and  $c_0(t)$  are again analytic and expressed in terms of the roots  $s_1$ ,  $s_2$ , and  $s_3$  of equation

$$2s^3 + (2\kappa + \gamma - i2\Delta)s^2 + (\Omega^2 + \kappa\gamma + g^2 - i2\Delta\kappa)s + \Omega^2\kappa = 0. \quad (19)$$

Finally after the atom passes through the  $\pi$ -polarized pump field, we find

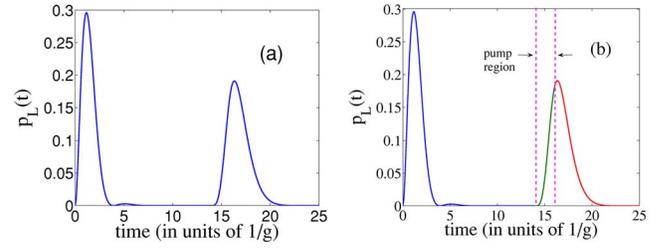


FIG. 3. (Color online). The emission rate of a cavity photon with left circular polarization at time  $t$  based on solving (a) the master equation (10); and (b) the analytic solution to the non-Hermitian Hamiltonian (13).

$$c_0(t) = c_0(t_2) \left( \frac{s_1 - s_3}{s_1 - s_2} e^{s_1(t-t_2)} + \frac{s_2 - s_3}{s_2 - s_1} e^{s_2(t-t_2)} \right),$$

where  $s_1$  and  $s_2$  are the roots of the equation

$$2s^2 + (2\kappa + \gamma - i2\Delta)s + g^2 + \kappa\gamma - i2\kappa\Delta = 0, \quad (20)$$

and  $s_3 = -\gamma/2 + i\Delta - ig\sqrt{2}d_1(t_2)/[2c_0(t_2)]$ . The emission rate of a second photon with right circular polarization during  $[t_1, T]$  is therefore  $p_R(t) = \kappa|c_0(t)|^2$ .

We now compare the above analytical analysis with numerical solutions from the master equation (10) and the effective non-Hermitian Hamiltonian (13). For the parameter ranges considered, the two numerical approaches give the same results as the analytical one. We find that a high fidelity EPR entangled photon pair of the form (1) ( $\uparrow \rightarrow 1_L$  and  $\downarrow \rightarrow 1_R$ ) is generated with a high efficiency. Like nonclassical photon pairs from an atomic ensemble [23], these two photons are distinguishable from their temporal order [24], and can be individually addressed to confirm their EPR correlation.

In the numerical results shown, we have used dimensionless parameters  $\Delta=0$ ,  $g=1.0$ ,  $\gamma=0.01$ ,  $\kappa=1.2$ ,  $\Omega=1.2$ ,  $t_1=14$ ,  $t_2=16$ , and  $T=25$ . Such a set of parameters can be realized for corresponding physical parameters of  $\gamma=(2\pi)0.2$  (MHz),  $g=(2\pi)20$  (MHz),  $\kappa=(2\pi)24$  (MHz), and  $T=200$  (ns) [14]. In Fig. 3, two single photon pulses are seen to be emitted sequentially from the cavity. The results from the two numerical approaches agree well with each other. In Fig. 4(a), the probability of generating these two single photons are displayed. We see that they are better than 98%. In Fig. 4(b) the occupations of different atomic states are shown. Exactly as expected, the first photon is generated from the decay of state  $|e_0\rangle$  to  $|g_1\rangle$  or  $|g_{-1}\rangle$ ; and the second photon is generated from each of these two states when pumped by the  $\pi$ -polarized laser.

Based on our extensive simulation with other parameters, we hope to emphasize three points [25]. First there exists an optimal  $\kappa$ , which leads to the fastest photon emission. When  $\kappa \ll g$ , oscillations emerge, a signature of the strong coupling. When  $\kappa \gg g$ , the probability of emission actually decays linearly and the emission time becomes longer because of the increased bandwidth of the cavity, thus correspondingly reduced emission strength of the atom into the cavity. Second there is also an optimal time for the atom to pass the pump

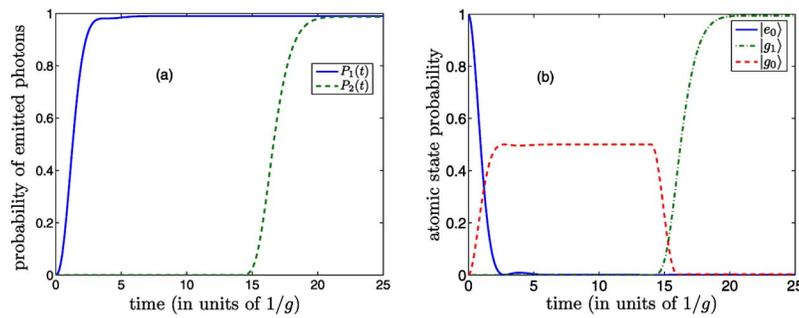


FIG. 4. (Color online). (a) The probability of the first and second photon emission  $P_1(t)$  and  $P_2(t)$ ; (b) The time-dependent atomic state populations.

laser. When that duration is too short, the atom cannot be completely emptied from states  $|g_1\rangle$  and  $|g_{-1}\rangle$ . However, if it is too long, oscillations between states  $|e_{\pm 1}\rangle$  and  $|g_{\pm 1}\rangle$  arise. More conveniently, it will be desirable to use a trapped atom inside the cavity [26], and replace the transit time over the pump region with a temporal pump pulse. Third, the probability of generating the first photon depends on the atomic decay rate  $\gamma$ . One solution to overcome such a dependence is to use an auxiliary starting state and an additional laser coupled to state  $|e_0\rangle$  as in the single photon source protocol [9,10,14]. The  $\pi$ -polarization laser then is applied to swap this entanglement from the atom to the second cavity photon. The entanglement of the emitted photon pairs can be easily detected from polarization correlations between the first and second photons using a time resolved detection scheme [24].

Finally, we want to emphasize that dissipations are crucial in our protocol, not only because they lead to the output of cavity photons. Without dissipations, the atom cannot enter the state  $|g_0\rangle$ , because the quantum amplitudes for the

two paths ( $|e_0\rangle \rightarrow |g_{-1}\rangle \rightarrow |e_{-1}\rangle \rightarrow |g_0\rangle$  and  $|e_0\rangle \rightarrow |g_1\rangle \rightarrow |e_1\rangle \rightarrow |g_0\rangle$ ) interfere destructively. This balance is broken due to dissipations.

In summary, we have proposed a simple but efficient scheme to deterministically generate EPR entangled photon pairs from an atom coupled to a high Q optical cavity. Our scheme has the potential for realizing a deterministic source for entangled photon pairs. The optical nonlinearity of the parametric down conversion is rather weak and due to many atoms (emitters), in contrast to the strong resonant interaction of a single atom as utilized here in a cavity QED. Thus the down converted photon pairs are inherently probabilistic. If  $p$  (small) is the probability of a single photon pair, double pairs, triple pairs  $\dots$ , occur with (smaller) probabilities  $p^2, p^3, \dots$ . In our scheme, however, the photon pair is deterministic. Each passing atom leads to a single pair, as in a push-button device.

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