Comment on "Large enhancement of four-wave mixing by suppression of photon absorption from electromagnetically induced transparency"

Roberto Buffa,[1] Stefano Cavalieri,[2] and Marco V. Tognetti[1]

[1]*Dipartimento di Fisica, Università di Siena, and Istituto Nazionale per la Fisica della Materia, Via Roma 56, I-53100 Siena, Italy*

[2]*Dipartimento di Fisica, and European Laboratory for Nonlinear Spectroscopy, Università di Firenze, and Istituto Nazionale per la Fisica della Materia, Via G. Sansone 1, I-50019 Sesto Fiorentino, Italy*

(Received 24 August 2004; published 15 September 2005)

In this Comment we deal with a recent study of Ying Wu *et al.* [Phys. Rev. A **67**, 013811 (2003)] on a four-wave mixing process. We show that the authors' claim, that a large enhancement of the generation efficiency can be obtained by suppression of photon absorption from electromagnetically induced transparency, is not correct.

DOI: 10.1103/PhysRevA.72.037801 PACS number(s): 42.50.Gy, 42.65.Ky, 42.50.Hz

In their recent study [1] on a four-wave mixing (FWM) process, Ying Wu *et al.* state that a large FWM enhancement is obtained in their scheme when electromagnetically induced transparency (EIT) is used to suppress photon absorption. The authors indicate ^{85}Rb as a possible candidate for an experiment and, with an atomic density $N=10^{12}$ cm^{-3} and an interaction length $L=2$ mm, claim an enhancement of the FWM generation efficiency larger than 10^4, due to the presence of EIT. Here we will demonstrate that their claim is incorrect since the maximum FWM generation efficiency is actually obtained in the absence of EIT.

The scheme of Ying Wu *et al.* (see Fig. 1(a) of Ref. [1]) is a modified version of a similar scheme, discussed by Deng *et al.* [2] (see Fig. 1(b) of Ref. [1]), in which a giant enhancement of the generated FWM wave was claimed as a result of the presence of EIT. The difference between the two studies is that the intermediate state $|3\rangle$ is taken as real in Ref. [1] while it was considered as virtual in Ref. [2]. Due to this resonant mixing, Ying Wu *et al.* [1] show that their scheme results in a several orders of magnitude increase in the FWM efficiency in comparison with the scheme of Deng *et al.* [2]. Assuming that the claim of Deng *et al.* [2] was correct, Ying Wu *et al.* [1] conclude that their scheme provides a large enhancement of the FWM generation efficiency due to the presence of EIT. However, it has been recently shown [3] that the claim of Deng *et al.* [2] was not correct, since the efficiency of the FWM generation process in the scheme of Ref. [2] is actually reduced by the presence of EIT, and the maximum FWM signal is obtained in the absence of EIT. Therefore, while actually showing an increase in the FWM generation in comparison with the scheme of Deng *et al.* [2], the scheme of Ying Wu *et al.* [1] provides only another example in which the FWM generation efficiency is actually reduced by the presence of EIT. In fact, according to the formulas reported in Ref. [1], it is straightforward to show that, as in the scheme of Deng *et al.* [2], also in the scheme of Ying Wu *et al.* [1], the maximum FWM signal is obtained in the absence of EIT.

For an easy comparison, we use the notation of Ref. [1], where Ω_c is the field responsible for EIT and $\Omega_p(z,t)$ and $\Omega_m(z,t)$ represent, respectively, the pump and the generated FWM fields that propagate along the z axis. The main result of Ref. [1] is contained in Eq. (8), which reports the analytical expression of the Fourier transforms $W_p(z,\omega)$ and $W_m(z,\omega)$ of $\Omega_p(z,t)$ and $\Omega_m(z,t)$, respectively. In Fig. 1 the quantities $\int_{-\infty}^{+\infty}|W_p(z,\omega)|^2\,d\omega$ and $\int_{-\infty}^{+\infty}|W_m(z,\omega)|^2\,d\omega$, which are proportional to the pump energy and to the generated FWM energy, respectively, are reported versus the interaction length in the absence (dashed line) and in the presence (solid line) of the driving field ($\Omega_c=\gamma_2$). The atomic parameters used are those of ^{85}Rb as chosen by Ying Wu *et al.* [1],

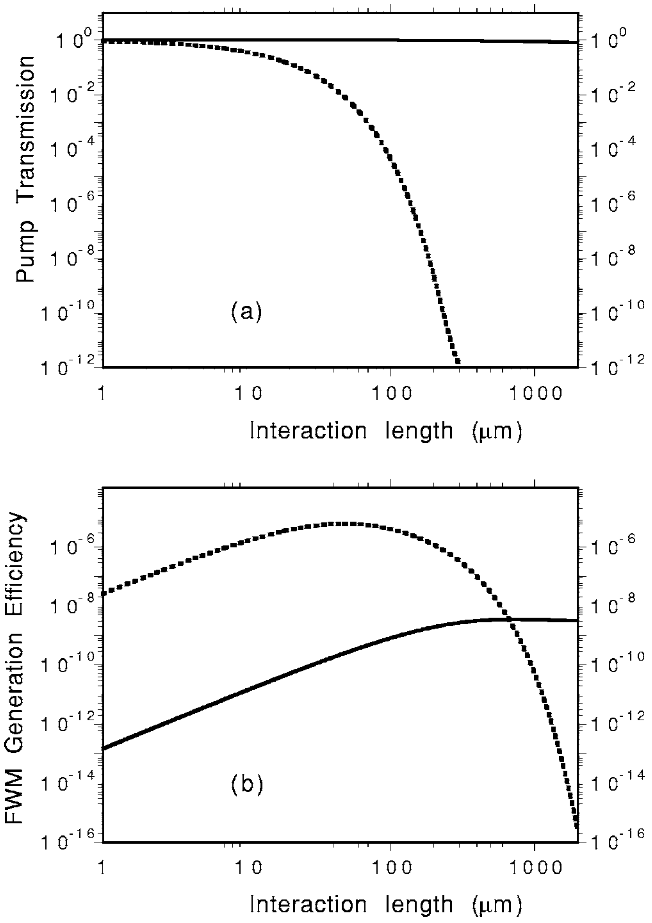

FIG. 1. Pump (a) and generated FWM (b) energies versus the interaction length in the absence (dashed line) of EIT ($\Omega_c=0$) and in the presence (solid line) of EIT ($\Omega_c=\gamma_2$).

$N=10^{12}$ cm^{-3}, $\Omega_1=\Omega_2=0.01\gamma_2$, and the power spectrum of the input pump pulse $|W_p(0,\omega)|^2$ has been taken as Gaussian, with a (FWHM) spectral width $\delta\omega=5\times10^{-2}\gamma_2=\gamma_4/3$.

The interpretation of the behavior shown by Fig. 1 is straightforward, as already discussed in Ref. [3]. In the presence of EIT (solid line), the pump field $\Omega_p(z,t)$ propagates with negligible absorption and provides an almost constant source term for the FWM field $\Omega_m(z,t)$. This saturates at a value for which the source term—proportional to $\Omega_p(z,t)$—compensates for the loss term—proportional to $\Omega_m(z,t)$. In the absence of EIT (dashed line), the pump field $\Omega_p(z,t)$ propagates with a strong absorption and provides a strongly decreasing source term for the FWM field $\Omega_m(z,t)$. The optimal interaction length L_{max} (≈ 50 μm) is in between the absorption length L_p at the frequency of the pump field and the absorption length L_m at the frequency of the generated field. For interaction lengths L much longer than L_m ($L=2$ mm in the case of Ref. [1]) the FWM field $\Omega_m(z,t)$ is completely absorbed and, as claimed by Ying Wu *et al.* [1], the FWM energy generated in the presence of EIT is many orders of magnitude larger than that generated in the absence of EIT. However, it is clear that this "illusory" large enhancement is due only to the wrong choice of the density length product ($N\times L$) that provides—in the absence of EIT—a completely opaque medium that completely absorbs the generated FWM field. As shown by Fig. 1, with the same interaction length $L=2$ mm but a reduced atomic density $N\approx 2.5\times10^{10}$ cm^{-3}, the FWM generation efficiency in the absence of EIT results many orders of magnitude larger than the FWM generation efficiency obtained with $N=10^{12}$ cm^{-3} in the presence of EIT.

It has to be stressed that the study of FWM schemes in the presence of EIT stems from the original work of Harris *et al.* [4]. One must primarily distinguish between (1) processes in which the driving field responsible for the EIT "actively participates" in the mixing generation and that cannot occur in the absence of that field (like that studied by Harris *et al.* [4]), and (2) those in which the driving field responsible for the EIT just "assists" the mixing generation and that can occur also in the absence of that field (like that studied by Ying Wu *et al.* [1] or by Deng *et al.* [2]). For processes of type (1), near-unity photon flux conversion efficiency has been observed experimentally [5], while, for processes of type (2), no experimental evidence of any FWM efficiency enhancement has been reported in the literature. The comparison between these two kind of processes is the most enlightening.

Taking cw excitation, and following textbooks of nonlinear optics, it is straightforward to obtain the linear and nonlinear susceptibilities of the five-level scheme shown in Fig. 1(a) of Ref. [1] in their most general form. From here, one can reduce to (i) the scheme of Ying Wu *et al.* [1] by taking Ω_p, Ω_1, and Ω_2 below the saturation levels and obtain

$$\chi_p^{(1)}=N\frac{D_{02}^2\omega_p}{2\hbar c\varepsilon_0}\frac{\Delta\tilde{\omega}_p}{\Delta\tilde{\omega}_p\Delta\tilde{\omega}_c-\Omega_c^2}, \tag{1a}$$

$$\chi_m^{(1)}=N\frac{D_{04}^2\omega_m}{2\hbar c\varepsilon_0}\frac{1}{\Delta\tilde{\omega}_4}, \tag{1b}$$

$$\chi_m^{(3)}=N\frac{D_{02}D_{23}D_{34}D_{04}\omega_m}{8\hbar^3 c\varepsilon_0}\frac{\Delta\tilde{\omega}_p}{\Delta\tilde{\omega}_3\,\Delta\tilde{\omega}_4(\Delta\tilde{\omega}_p\,\Delta\tilde{\omega}_c-\Omega_c^2)} \tag{1c}$$

(from which the scheme of Deng *et al.* [2] is obtained by just taking large detunings $\Delta\omega_3\gg\gamma_3$); or (ii) the scheme of Harris *et al.* [4] by taking $\Omega_c=0$ and large detunings $\Delta\omega_p\gg\gamma_2$ to obtain

$$\chi_p^{(1)}=N\frac{D_{02}^2\omega_p}{2\hbar c\varepsilon_0}\frac{1}{\Delta\omega_p}, \tag{2a}$$

$$\chi_m^{(1)}=N\frac{D_{04}^2\omega_m}{2\hbar c\varepsilon_0}\frac{\Delta\tilde{\omega}_3}{\Delta\tilde{\omega}_3\,\Delta\tilde{\omega}_4-\Omega_2^2}, \tag{2b}$$

$$\chi_m^{(3)}=N\frac{D_{02}D_{23}D_{34}D_{04}\omega_m}{8\hbar^3 c\varepsilon_0}\frac{1}{\Delta\omega_p(\Delta\tilde{\omega}_3\,\Delta\tilde{\omega}_4-\Omega_2^2)}. \tag{2c}$$

The comparison between Eqs. (1) and (2) shows how the dramatic difference between these schemes can be ascribed to the different role played by constructive and destructive interference of the EIT process. In the scheme of Harris *et al.* [4] [Eqs. (2)], absorption at the pump frequency is suppressed by detuning the pump field far from resonance, while absorption at the FWM frequency is suppressed by EIT induced by the field Ω_2. The nonlinear coefficient $\chi_m^{(3)}$ [Eq. (2c)] does not show the effect of destructive interference caused by EIT and the ratio $\chi_m^{(3)}/\chi_m^{(1)}$ results independent of the value of the field Ω_2 responsible for EIT. On the contrary, in the scheme of Ying Wu *et al.* [1] [Eqs. (1)], absorption at the pump frequency is suppressed by EIT induced by the field Ω_c. Unfortunately, so doing, the nonlinear coefficient $\chi_m^{(3)}$ [Eq. (1c)] experiences the same kind of destructive interference and the ratio $\chi_m^{(3)}/\chi_m^{(1)}$ decreases at increasing values of the field Ω_c responsible for EIT.

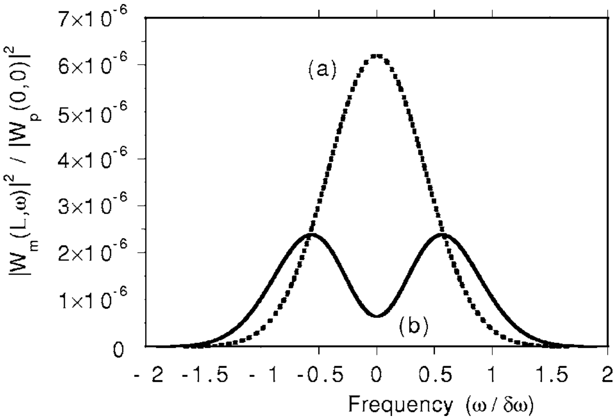

FIG. 2. Power spectrum of the generated FWM field $|W_m(L,\omega)|^2$ in the absence (dashed line), and in the presence (solid line) of EIT. For curve (a), $L=50$ μm, corresponding to the maximum value of the dashed curve of Fig. 1(b); for curve (b), $L=730$ μm, corresponding to the maximum value of the solid curve of Fig. 1(b). The data of curve (b) (solid line) have been multiplied by a factor of 10^3.

The spectral behavior of the nonlinear coefficient $\chi_m^{(3)}$ reflects into the power spectrum of the generated FWM field. In Fig. 2 the quantity $|W_m(L,\omega)|^2$, calculated in the absence (dashed line) and in the presence (solid line) of EIT, is reported. For curve (a), $L=50\ \mu$m, corresponding to the maximum value of the dashed curve of Fig. 1(b); for curve (b), $L=730\ \mu$m, corresponding to the maximum value of the solid curve of Fig. 1(b). The data of curve (b) (solid line) have been multiplied by a factor 10^3. While the power spectrum of the generated FWM field in the absence of EIT (a) reproduces the power spectrum of the input pump pulse $|W_p(0,\omega)|^2$, the effect of destructive interference in the presence of EIT in the scheme of Ying Wu *et al.* [1] appears evident (b).

[1] Y. Wu, J. Saldana, and Y. Zhu, Phys. Rev. A **67**, 013811 (2003).

[2] L. Deng, M. Kozuma, E. W. Hagley, and M. G. Payne, Phys. Rev. Lett. **88**, 143902 (2002).

[3] R. Buffa, S. Cavalieri, and M. V. Tognetti, Phys. Rev. Lett. **93**, 129401 (2004).

[4] S. E. Harris, J. E. Field, and A. Imamoglu, Phys. Rev. Lett. **64**, 1107 (1990).

[5] A. J. Merriam, S. J. Sharpe, Hui Xia, D. A. Manuszak, G. Y. Yin, and S. E. Harris, IEEE J. Sel. Top. Quantum Electron. **5**, 1502 (1999).